ECE 410F MIDTERM EXAM - Version A

Monday, October 28, 1996

11:00am - 12:00pm

NAME:

STUDENT NUMBER:

Instructions

1. You may use a non-programmable calculator.
2. You may use one double-sided page of notes.
3. **Write all answers on these 8 pages.**
4. You may find the following formulae useful. For the second order system

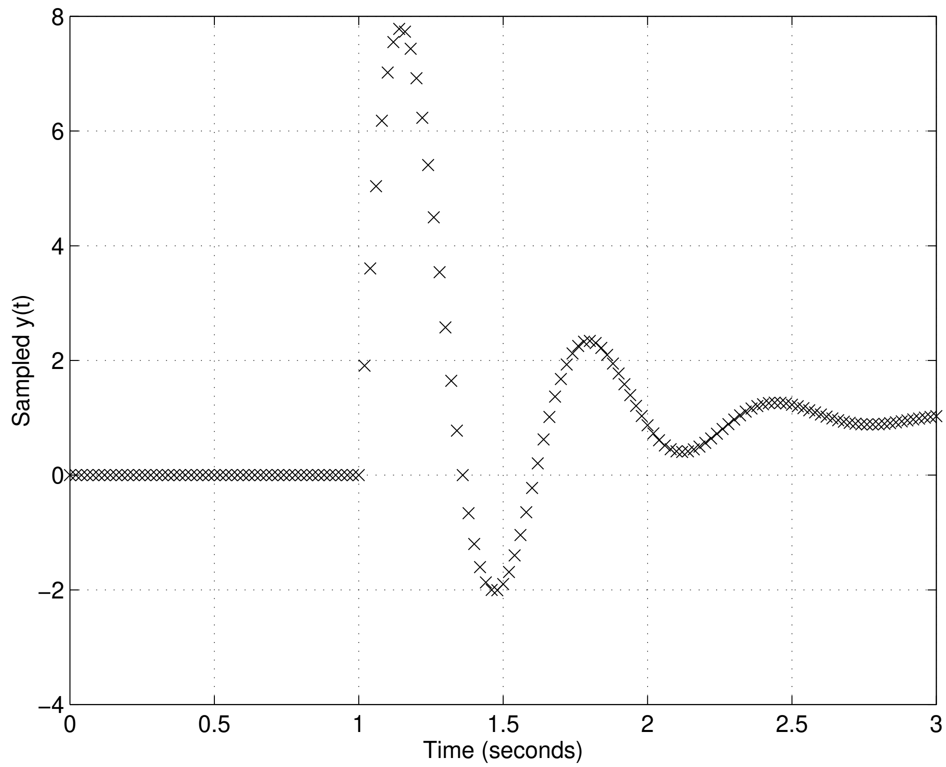
$$H(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2},$$

$$\begin{aligned} \text{percent overshoot} &= 100e^{-\frac{\delta\pi}{\sqrt{1-\delta^2}}} && (1) \\ \text{damped frequency} &= \omega_d = \omega_n \cdot \sqrt{1-\delta^2} && (2) \\ \text{settling time} &= t_s = \frac{3}{\delta\omega_n} && (3) \end{aligned}$$

Question 1	_____	of 20
Question 2	_____	of 20
Question 3	_____	of 30
Question 4	_____	of 30
TOTAL	_____	of 100

The following two problems are multiple choice. Full marks will be awarded if you **circle** the correct answer. Partial credit will be awarded if you **do not** circle an answer, but you eliminate some candidate answers by placing an **X** through them. Zero marks will be awarded if the wrong answer is circled.

Problem 1a. (10 points) Below is a sampled step response for a plant $P_1(s)$. (The unit step input occurs at $t = 1$ second.) The sample rate is 50 Hz.



A good model for P_1 is

1. $P_1(s) = \dfrac{100}{s^2+5s+100}$

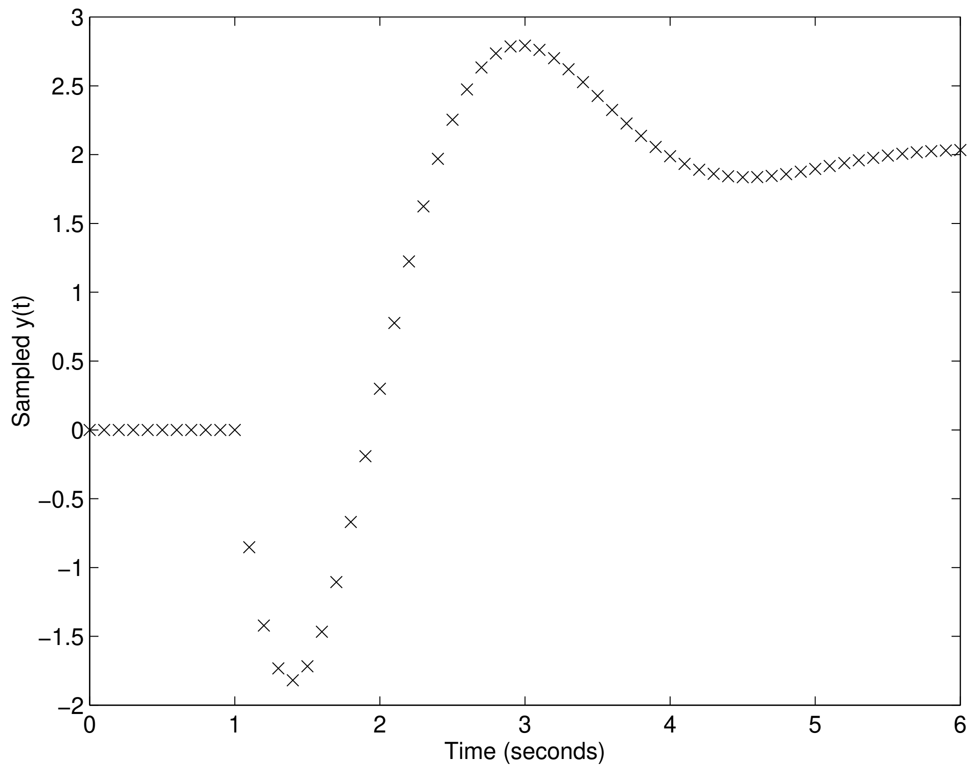
2. $P_1(s) = \dfrac{100(s+1)}{s^2+5s+100}$

3. $P_1(s) = \dfrac{100(s-1)}{s^2+5s+100}$

4. $P_1(s) = \dfrac{(s-10)}{s^2+s+10}$

5. $P_1(s) = \dfrac{10}{s^2+s+10}$

Problem 1b. (10 points) Below is a sampled step response for a plant $P_2(s)$. (The unit step input occurs at $t = 1$ second.) The sample rate is 10 Hz.



A good model for P_2 is

1. $P_2(s) = \frac{10}{s^2+2s+5}$

2. $P_2(s) = \frac{10(s+1)}{s^2+2s+5}$

3. $P_2(s) = \frac{-10(s-1)}{s^2+2s+5}$

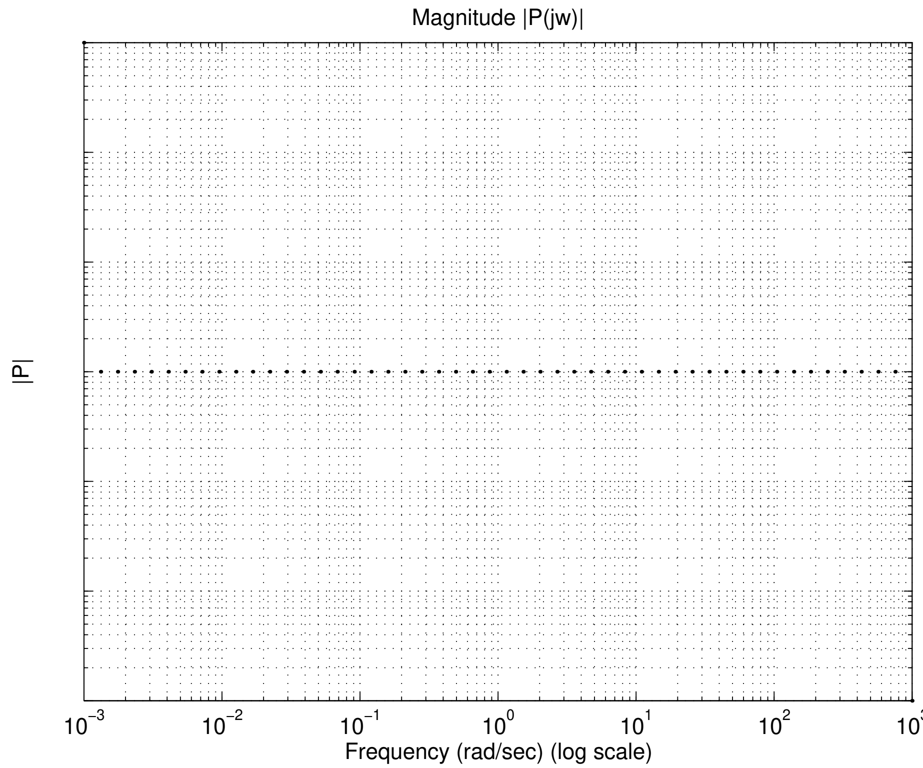
4. $P_2(s) = \frac{10(s-5)}{s^2+s+1}$

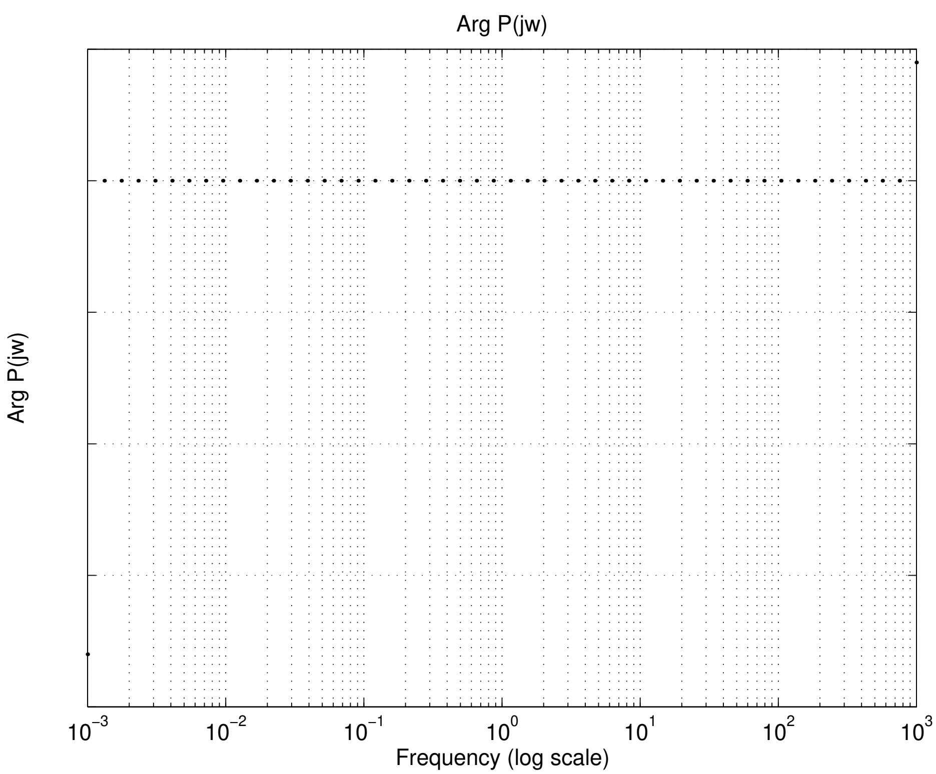
5. $P_2(s) = \frac{10(s+5)}{s^2+s+1}$

Problem 2. (20 points) On the axes below, draw the asymptotic approximations to the Bode magnitude and phase for the system

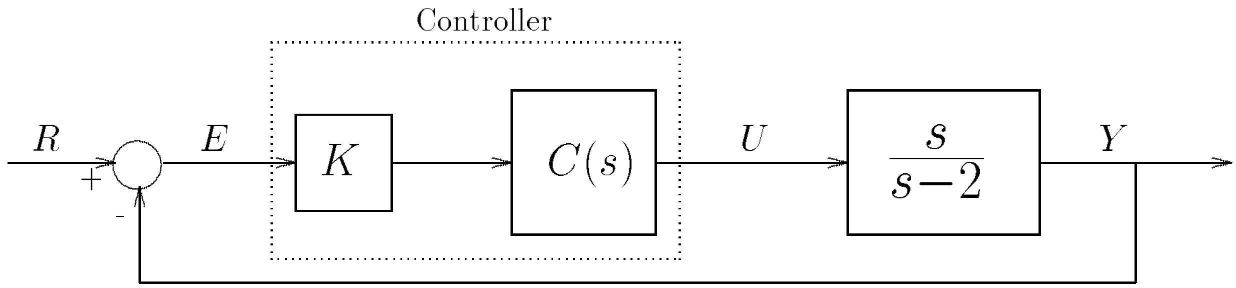
$$P(s) = \frac{9000(s^2+s+1)}{s(s-1)(s^2+6s+900)}$$

Note: You may wish to practice in your exam booklet first — Neatness counts!





Problem 3. (30 points) Consider the following closed-loop control system.



Using a root locus drawn on the axes below, design $C(s)$ and K to stabilize the closed-loop system. The controller must satisfy the following specifications.

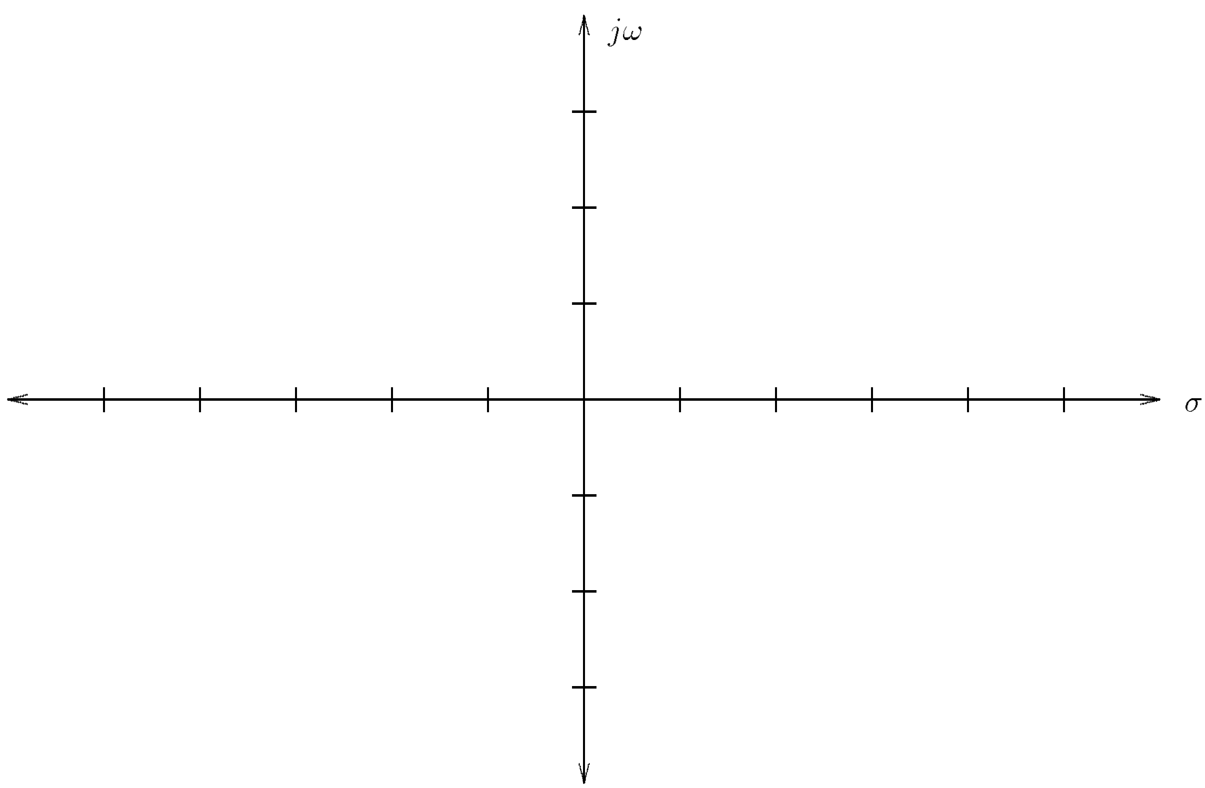
- $C(s)$ is of the form
$$C(s) = \frac{1}{(s+a_0)(s+a_1)}$$
where a_0 and a_1 are **nonzero integers**, i.e., all the poles of $C(s)$ are **real, nonzero integers.**

- K is a real number, $-\infty < K < \infty$.

To receive full credit you must

1. (10 points) Place the two poles of $C(s)$ on the plot below;
2. (10 points) Draw the resulting root locus;
3. (10 points) Determine **exactly** the range of K that assures closed-loop stability (using a Routh array, for example).

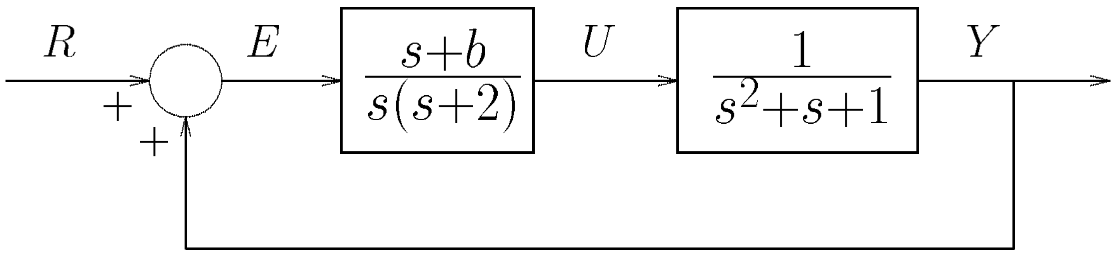
Note: Use the exam booklet for rough sketches, and then **neatly** draw your final design below. Do Part 3 on the next page.



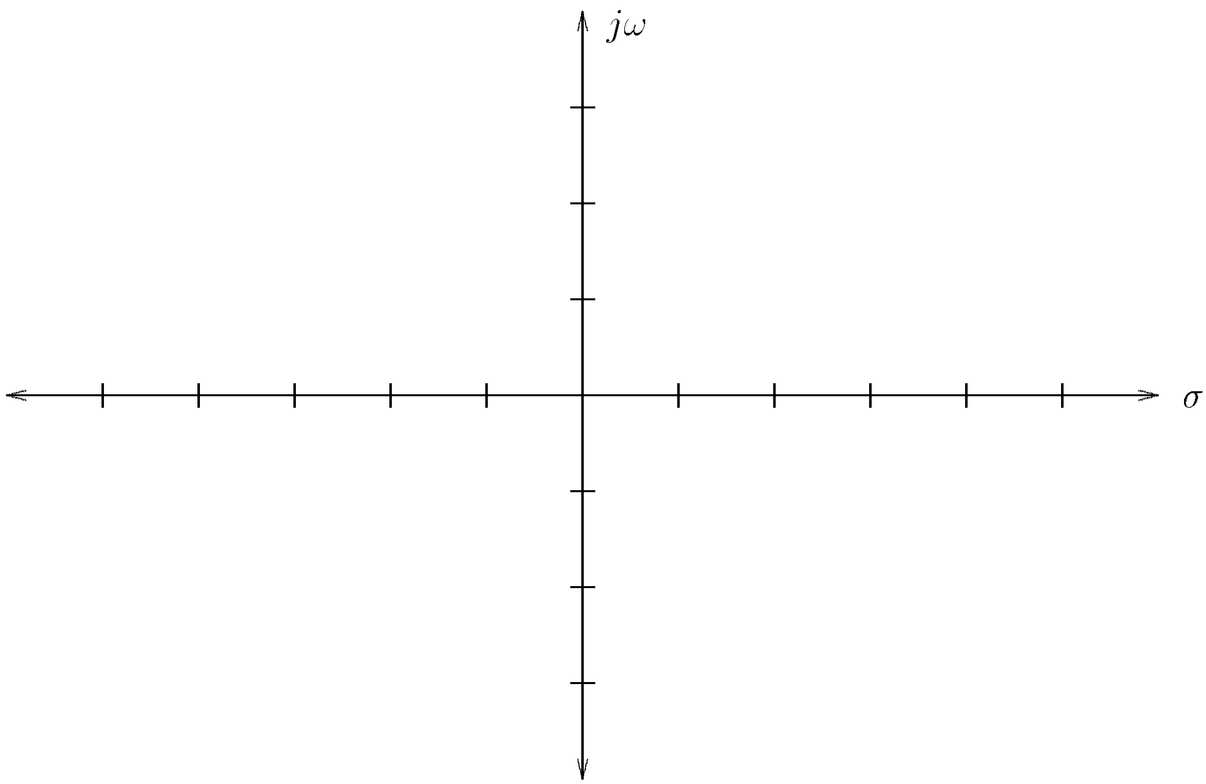
Problem 3. (cont.)

Range of K? ________

Problem 4. (30 points) Consider the following ***positive feedback* system**.



1. (20 points) Plot the root locus for $b < 0$ for the following system. **Note the *positive* feedback.**



2. (5 points) For what range of b is the closed-loop system stable? (Show work on next page.)

Range of b? ________

3. (5 points) If b is within the range specified in 2, and if $r(t)$ is a unit step, then what is

$$\lim_{t\to\infty} e(t) = \quad \_\_\_\_\_\_\_\_$$

Problem 4. (cont.) Show work for parts 2 and 3 here.