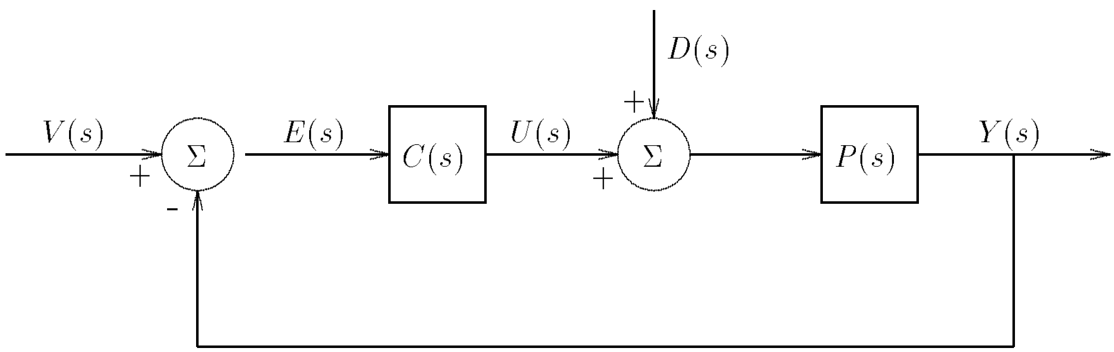
ECE 410F MIDTERM EXAM - Version A
Monday, October 30, 1995
11:00am - 12:00pm
PLEASE WRITE YOUR ANSWERS TO PROBLEMS **1** and **2** ON THIS SHEET
GOOD LUCK !

NAME:

STUDENT NUMBER:

Problem 1. (20 points). Circle **True** or **False** for the following statements, which pertain to the system diagram below. Assume the (closed-loop) system $\frac{Y(s)}{V(s)}$ is stable.



1. TRUE or FALSE: If $\frac{Y(s)}{V(s)}$ is non-minimum phase, then either $P(s)$ or $C(s)$ has a zero in the open right-half plane.

2. TRUE or FALSE: If $C(s)$ has a pole at $s = 0$, then $\frac{Y(s)}{V(s)}$ also has a pole at $s = 0$.

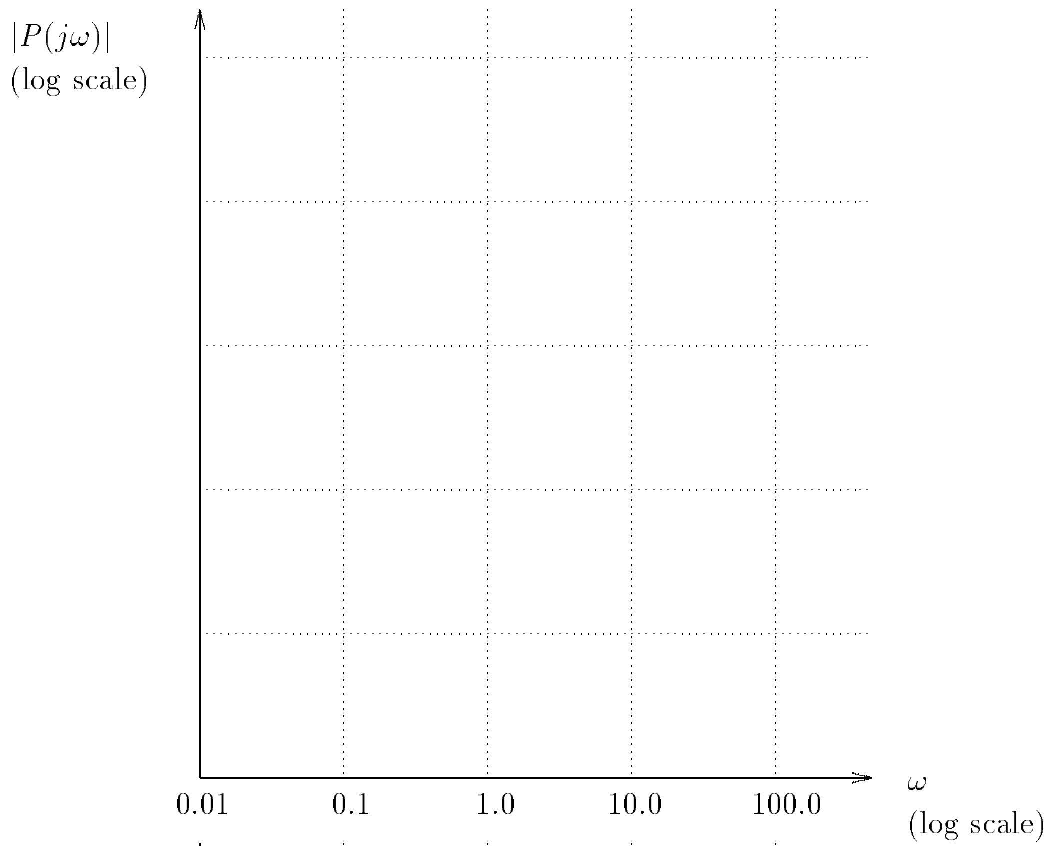
3. TRUE or FALSE: Assume $d(t) = A\cos(\omega t)$ and $v(t) = 0$, for all $t \geq 0$. Then $\lim_{t\to\infty} y(t) = 0$ *if and only if* either $C(s)$ or $P(s)$ has poles at $j\omega$ and $-j\omega$.

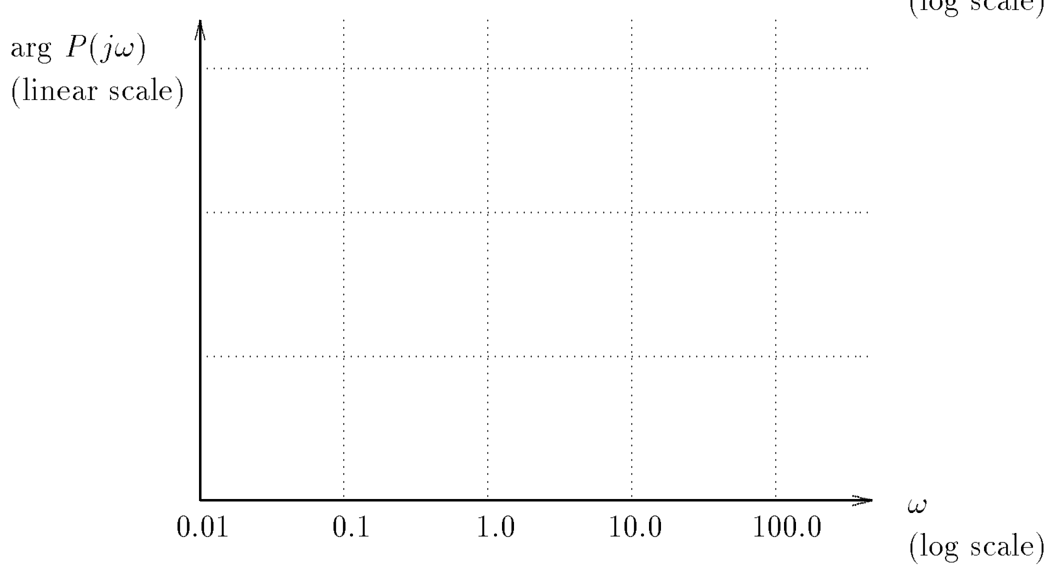
4. TRUE or FALSE: Assume $v(t) = A\cos(\omega t)$ and $d(t) = 0$, for all $t \geq 0$. Then $\lim_{t\to\infty} y(t) = 0$ *if and only if* either $C(s)$ or $P(s)$ has poles at $j\omega$ and $-j\omega$.

Problem 2. (20 points). On the axes below, draw the asymptotic approximations to the Bode magnitude and phase for the system

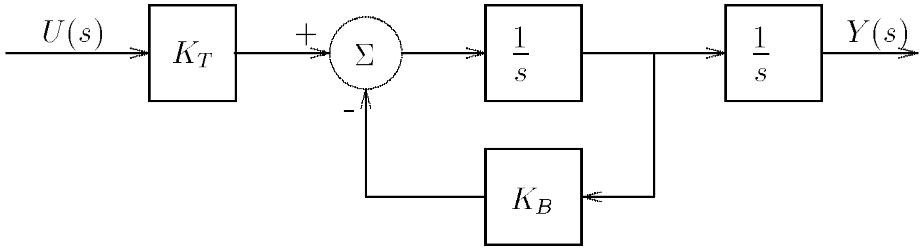
$$P(s) = \frac{s - 10}{s(s + 10)}.$$

Note: You may wish to practice in your exam booklet first. Neatness counts!

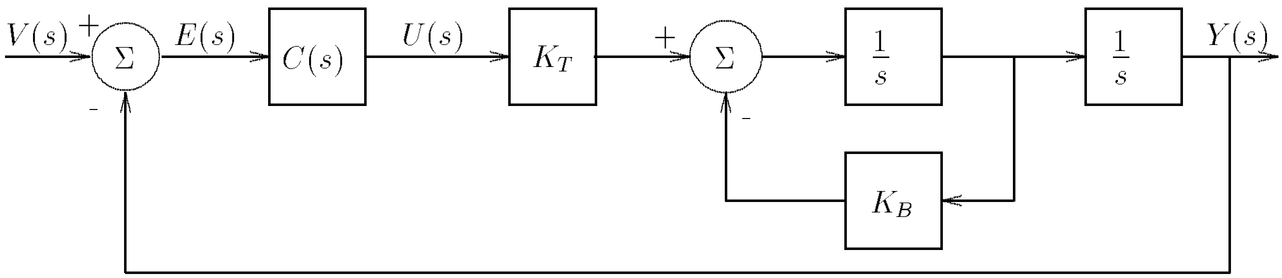




Problem 3. (40 points). A servomotor that is to be used as a position control system is diagrammed below.



Here, $u(t)$ is the applied motor voltage input, $y(t)$ is the shaft position output, and $K_T > 0$ and $K_B > 0$ are motor constants, which are known to be positive. Because $\frac{Y(s)}{U(s)}$ is unstable, we must design a stabilizing feedback controller $C(s)$, giving the closed-loop system



However, before we can design $C(s)$, we need to know K_T and K_B.

- (5 points.) Assuming $C(s) = K$, where K is a fixed constant, what is the closed-loop transfer function $\frac{Y(s)}{V(s)}$, as a function of K, K_B, K_T and s?

- (10 points.) Assuming $C(s) = K$, and assuming $K_B > 0$ and $K_T > 0$ are fixed constants, plot the root locus for the system as a function of K, for $-\infty < K < \infty$. That is, plot the closed-loop pole locations for fixed K_T and K_B, as K varies.

- (5 points.) As a function of K_T and K_B, what is the range of K for which the closed-loop system is stable? (Again, it is known that $K_T > 0$ and $K_B > 0$.)

- (20 points.) With $C(s) = K = 10$, a unit step is applied to $v(t)$ of the closed-loop system. The resulting $y(t)$ is plotted on the following page. What are K_T and K_B, within 10%?

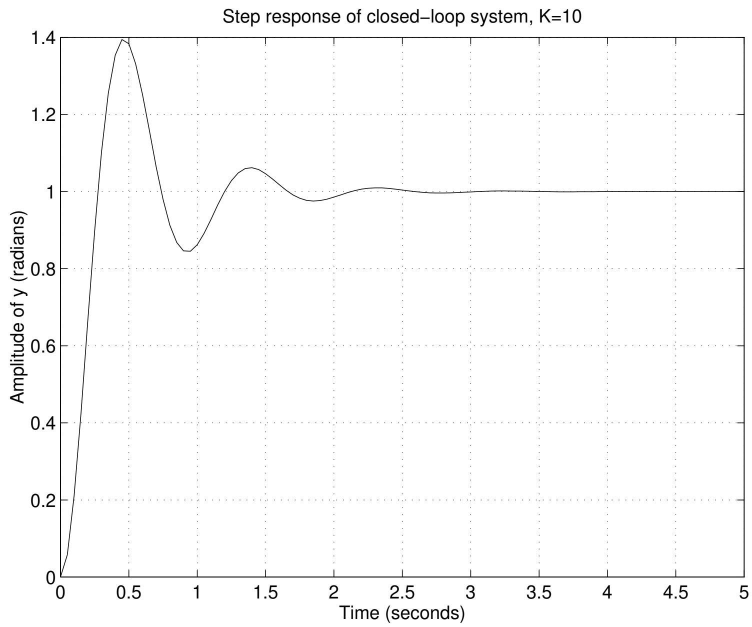


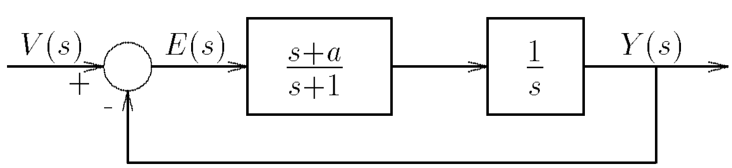
Figure 1: Step response for Problem 2.

You may find the following formulae useful. For the second order system

$$H(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2},$$

$$\begin{aligned} \text{percent overshoot} &= 100e^{-\frac{\delta\pi}{\sqrt{1-\delta^2}}} && (1)\\ \text{damped frequency} &= \omega_d = \omega_n \cdot \sqrt{1-\delta^2} && (2)\\ \text{settling time} &= t_s = \frac{3}{\delta\omega_n} && (3) \end{aligned}$$

Problem 4. (20 points.) For the system diagrammed below,



- (10 points.) Plot the root locus as a function of a, for all $-\infty < a < \infty$.
- (10 points.) For what range of a is the closed-loop system stable?