Laboratory 4: Robust Design

In this lab, we will design two controllers for a system with unmodeled dynamics. The designs will satisfy a set of performance specifications in spite of these dynamics. MATLAB is used to design and analyze the controller. There is no hardware component to this lab, and it may be done outside GB347 at any time.

Consider a servomotor connected to an inertial load via a flexible shaft, as illustrated in Figure 1. Such systems arise in robotic applications, for example, where flexibility in the gears of the transmission can be modeled with a linear spring. This system can be modeled

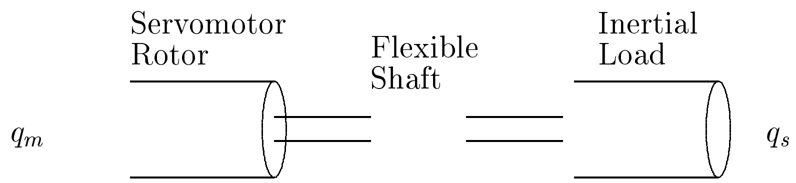

Figure 1: Diagram of a Robot Joint with Flexible Transmission.

with two second-order linear differential equations

$$J_m \ddot{q}_m + b_m \dot{q}_m = k \cdot (q_m - q_s) \tag{1}$$

$$k \cdot (q_m - q_s) = J_s \ddot{q}_s + b_s \dot{q}_s, \tag{2}$$

where u is the motor torque, J_m is the rotor inertia, b_m is the rotor damping, k is the spring constant of the flexible shaft, J_s is the load inertia, b_s is the damping of the load, q_m is the motor angle, and q_s is the load angle. In this system, the output is the load angle, so $y = q_s$, and the input is the motor torque u. In most practical systems, the spring is very stiff, meaning k is very large. In this case, $q_m - q_s$ will be small, and this means that the damping terms will be difficult to identify. (Recall in Lab 1 that your ability to identify K_1 and K_2 were affected by the controller gain K.) Thus, a controller must be designed that will satisfy a set of performance specifications for a range of possible damping values.

Assume the parameters have numeric values that give the following transfer function between the motor torque u and load angle y:

$$\frac{Y(s)}{U(s)} = P(s) = \underbrace{\frac{1}{s(s+1)}}_{P_1(s)} \underbrace{\frac{100}{s^2 + 20\delta s + 100}}_{P_2(s)}. \tag{3}$$

In this model, $P_1(s)$ represents the rigid part of the system, and $P_2(s)$ models the flexible part. In $P_2(s)$, the precise value for the damping term δ is unknown, but it satisfies the bounds

$$0.5 < \delta < 1. \tag{4}$$

Note that $P_2(s)$ has complex-conjugate poles with a high natural frequency ($\omega_N = 10$) relative to that of $P_1(s)$.

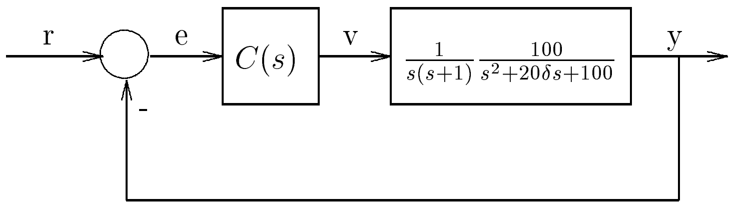

Figure 2: The full-order closed-loop system for Lab 4.

1 Laboratory Procedure

We shall design two controllers for the closed-loop system shown in Figure 2. Both controllers will use the same bound on the model uncertainty due to δ.

1. Express the plant $P(s)$ in the multiplicative perturbation form

$$P(s) = P_1(s)(1 + L_m(s)) \tag{5}$$

for some $L_m(s)$, which will be a function of δ. Thus, P_1, models the rigid part of the system, and is the *nomimal* plant.

2. Using the MATLAB command `bode`, plot on the same axes $|L_m(j\omega)|$ for the two extreme values of δ.

3. Suggest a function $l_m(\omega)$ that satisfies

$$|L_m(j\omega)| < l_m(\omega)$$

for all ω and all admissible δ. Using MATLAB, plot l_m on the same axes as in Step 2. Print-out the plot.

4. Using the appropriate Bode magnitude plot (different from Step 3), design a first-order controller $C_1(s)$ so the system satisfies the following performance specifications:

   - The phase margin θ_m must satisfy $50° < \theta_m$;
   - For sinusoidal inputs $r(t) = \sin(\omega t)$, the maximum tracking error $e_{\max} = \max|e(t)|$ must satisfy

$$e_{\max} < \frac{1}{316},$$

   for $\omega < 0.001$;
   - The bandwidth of the closed-loop system ω_b must satisfy $0.2 < \omega_b < 1.2$;
   - The controller $C_1(s)$ must satisfy all performance specifications robustly, using the uncertainty bound $l_m(\omega)$.

   The plot should include $l_m^{-1}(\omega)$ to represent uncertainty, a function $\rho(\omega)$ to represent the tracking error specification, and the designed $|C_1(j\omega) \cdot P_1(j\omega)|$.

5. Plot the $\rho(\omega)|S(j\omega)|+l_m(\omega)|T(j\omega)|$, where S is the sensitivity function (for the nominal system with P_1) and T is the complementary sensitivity function (again, for $P_1(s)$). Is it less than 1 for all ω? If not, re-design your $C_1(s)$.

6. Using the MATLAB command `nyquist`, and on the same set of axes, plot the Nyquist plots of $C_1(s)P_1(s)$ (the nominal plant), and the Nyquist plot of $C_1(s)P(s)$ for extreme values of δ. Is the phase margin specification satisfied for all possible values of δ?

7. Using the same steps, design a second controller $C_2(s)$ to satisfy the following specifications:

   - $C_2(s)$ must be no more than third order;
   - The phase margin θ_m must satisfy $55^\circ < \theta_m$;
   - For sinusoidal inputs $r(t) = \sin(\omega t)$, the maximum tracking error $e_{\max} = \max|e(t)|$ must satisfy
   $$e_{\max} < \frac{1}{316}$$
   for $\omega < 0.001$;
   - The bandwidth of the closed-loop system ω_b must satisfy $2 < \omega_b < 5$;
   - The controller $C_2(s)$ must satisfy all performance specifications robustly, using the uncertainty bound $l_m(\omega)$.

   These are the same specifications, except the bandwidth now satisfies $2 < \omega_b < 5$, and the controller can have two additional poles & zeros. The design should be shown on one Bode magnitude plot.

8. For the second design, plot $\rho(\omega)|S(j\omega)| + l_m(\omega)|T(j\omega)|$, where S is the new sensitivity function and T is the new complementary sensitivity function.

9. For the second design, plot the Nyquist plots of $C_2(s)P_1(s)$ (the nominal plant), and the Nyquist plot of $C_2(s)P(s)$ for extreme values of δ. Is the phase margin specification satisfied robustly for these two values?

2 Laboratory Report

The laboratory report should include all of the plots used for the two designs, and a description of the steps taken in doing the designs. The report is due **FRIDAY December 4, 1998** (the last day of classes). The lab can be done in groups, and the report can be submitted by groups of up to **two** persons.