Laboratory 2: Root Locus Design

In this experiment, you will design a dynamic compensator for the servomotor system, using the root locus and MATLAB as design tools. The following step-response performance specifications must be satisfied by the closed-loop system:

- The rise time t_r must satisfy $0.75\text{s} < t_r < 1.0\text{s}$;
- The percent overshoot must be less than 10%.

The controller design must be based upon the model you identified in Lab 1.

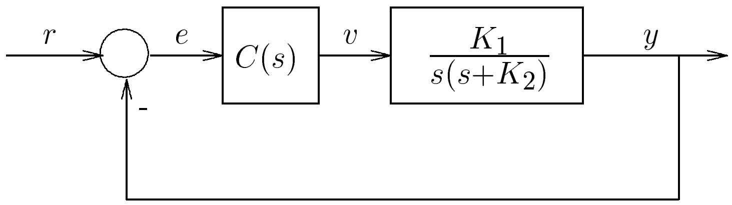

Figure 1: Simplified servomotor block diagram.

1 Preparation

The laboratory preparation involves designing the controller $C(s)$ so the closed-loop system in Figure 1 satisfies the above specifications, and then implementing it in discrete-time.

1. Given a second-order system with no zeros,

$$H(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2},$$

   compute a pair $(\delta,\, \omega_n)$ so $H(s)$ satisfies the design specifications. What are the poles? (The answer is not unique.)

2. Using MATLAB, plot the step response of the second-order system from Step 1. Check that it satisfies the design specifications.

3. Using the MATLAB root locus plotting function `rlocus`, show that a constant-gain control $C(s) = K$ can *not* satisfy the design specifications for any K for the closed-loop servomotor system. Use the values for K_1 and K_2 that you identified in Lab 1.

4. Using MATLAB's `rlocus` function, design a first-order controller

$$C(s) = K\frac{s + b_0}{s + a_0} \tag{1}$$

such that the closed-loop system $G(s) = \frac{Y(s)}{R(s)}$ does satisfy the design specifications. Do this by fixing the locations of the pole $s = -a_0$ and the zero $s = -b_0$, and plotting the root locus as a function of $K > 0$. Change a_0 and b_0 until the dominant branches of the root locus pass through the desired values computed in Step 1. (Recall that the origin-of-the-asymptotes is a function of a_0 and b_0.)

5. In MATLAB, plot the step response of the continuous-time closed-loop system $G(s)$, using $P_1(s)$ identified in Lab 1. Have you satisfied the design specifications? If not, return to Step 1.

6. Using the bilinear transformation, compute a discrete-time approximation $\hat{C}(z)$ to the $C(s)$ computed in Step 3. $\hat{C}(z)$ should be first-order. You may wish to use the MATLAB command `bilinear` to do this. Assume the controller sample rate is 250Hz.

7. Modify the source `lab.c` to implement the dynamic controller. To implement the filter's state, use `static float` variables in the `Control` subroutine.

2 Laboratory Procedure

1. Make sure the servomotor system is wired correctly. Your job is to connect the servomotor box, the power amplifier, and the bench-top interface board. **Again, PLEASE DO NOT ATTEMPT TO CONNECT THE RIBBON CABLE FROM THE BENCH-TOP INTERFACE BOARD TO THE COMPUTER. LET THE TA DO THIS.**

2. Compile and run your version of the program `lab.c`. Use the same step input $r(t)$ as in Lab 1. Save the actual angle $y(t)$, the reference $r(t)$ to a data file.

3. Load the experimental data into MATLAB, and plot $r(t)$ and $y(t)$ versus time t. Does your controller satisfy the design specifications? If so, demonstrate the step response to the TA. Show the TA the MATLAB plot too.

4. As you did for Lab 1, also plot the simulated step response computed in Step 5 of the Pre-lab. Do the experimental and simulated step responses agree?

5. Observe what happens to the closed-loop system when the sample rate is reduced to 100Hz and 50Hz and 25Hz, but the controller $\hat{C}(z)$ is unchanged. (See Question 2 at the end of this lab.)

6. Observe what happens to the closed-loop system when the gain K is increased beyond its designed value. (See Question 5 at the end of this lab.)

3 Before Leaving Lab

Before leaving lab, make sure to do the following.

- Show the TA your pre-lab. The TA will check that it is done, and he/she will check for correctness. He/She may also ask questions to ensure that you know the lab procedure, so make sure you are prepared before coming to lab. (That is for your own good - if you aren't prepared, you won't finish.)
- Demonstrate to the TA the servomotor system closed-loop step response.
- Show the TA the MATLAB step response plots.
- Show the TA the step response when the sample rate is 25Hz.

4 Laboratory Notebook Write-Up

Your laboratory report should include a description of the procedure, the work done in the pre-lab, including answers to the questions, a listing of the subroutines implementing the controller, and the step response plots. The following questions should be answered.

1. Is the controller a *lead* or a *lag* controller?
2. Is your design unique? Are there other combinations of K, a_0 and b_0 that satisfy the design specifications?
3. Theoretically, what happens to the closed-loop poles as $K \to \infty$, assuming the plant is $P_1(s)$ and the controller is the continuous-time $C(s)$? (A root-locus plot is sufficient here.)
4. Theoretically, what happens to the closed-loop poles as $K \to \infty$, assuming the plant is $P_2(s)$, (defined in the *Introduction to the Control Systems Laboratory*, equation (4)) and the controller is the continuous-time $C(s)$? (A sketch of the root-locus is sufficient here.)
5. Experimentally, what happens to the closed-loop system as $K \to \infty$?