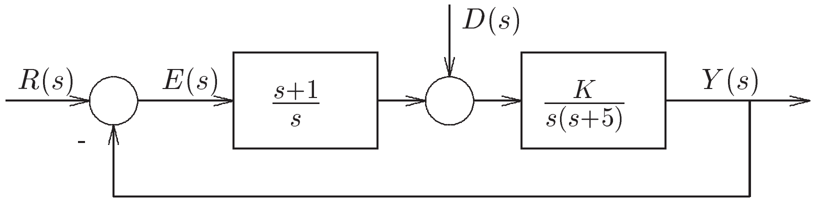
ECE 410 Homework #4

Monday, October 24, 1998

Problem 29. In the following diagram, the constant K satisfies $4 \le K \le 6$, but its precise value is unknown. Assume the closed-loop system is stable.



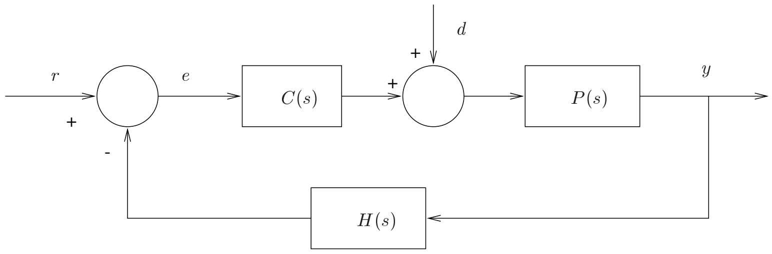
- Assuming $d(t) = 0$ and $r(t) \neq 0$, write $\frac{Y(s)}{R(s)}$ and $\frac{E(s)}{R(s)}$, as a function of K.
- Assuming $r(t) = 0$ and $d(t) \neq 0$, write $\frac{E(s)}{D(s)}$, as a function of K.
- Assuming $d(t) = 0$, can the closed-loop system track a step input (applied at $r(t)$), with 0 steady-state error?
- Assuming $d(t) = 0$, can the closed-loop system track a ramp input ($r(t) = t$), with 0 steady-state error?
- What is the stady-state error, $\lim_{t\to\infty} e(t)$, if $r(t) = t^2$ and $d(t) = 0$?
- What is the stady-state error, $\lim_{t\to\infty} e(t)$, if both $r(t)$ and $d(t)$ are unit steps?
- What is the stady-state error, $\lim_{t\to\infty} e(t)$, if $r(t) = t$ and $d(t)$ is a unit step?

Problem 30. Compute the sensitivity function S for the system in Problem 29. (Recall that $S = \frac{\partial T}{\partial P}\frac{P}{T}$, where $T = \frac{Y}{R}$ is the closed-loop transfer function, and $P = \frac{K}{s(s+5)}$.) The answer is a rational function of complex variable s and real variable K.

Problem 31. Assuming $K = 5$, plot the magnitude of $S(j\omega)$ versus ω on log-log axes, using MATLAB. To do this, start MATLAB, enter the numerator and denominator coefficients into two vectors, e.g., `num = [1 2 3]` would represent s^2+2s+3, and type `bode(num,den)`. For what range of frequencies is the sensitivity below 0.1 (i.e., 10%)? If $r(t) = \sin(0.1t)$, by how much would the magnitude of $e(t)$ change if K varied 20% from its nominal value of $K = 5$? (The answer is 0.2%)

Problem 32. Refer to the block diagram in Problem 33 (next page). Suppose P is strictly proper, and H is proper. A typical design specification is: The closed-loop system should track a sinusoidal input signal $r(t) = \sin(\omega t)$ **with less than 1% tracking error**, for all $\omega < \omega_0$, where ω_0 is a given constant which is smaller than the bandwidth of $T = Y/R$. In other words, $\max |e(t)| < 0.01$ for this $r(t)$. Translate this specification into a constraint on $|T(j\omega)|$.

Problem 33. For the general feedback system below, compute the sensitivity of the closed-loop transfer function $T(s) = Y(s)/R(s)$ with respect to the sensor $H(s)$. This is denoted S_H. If other design specifications require the loop-gain $|CPH(j\omega)|$ to be large (for example, Problem 32), can there be significant uncertainty in H? It is important to understand the design trade-off between uncertainty in H and large loop gain.



Problem 34. Sketch the magnitude and phase plots (Bode plots) for the following transfer functions. Check your work using MATLAB. (Use the `bode(num,den)` command, e.g., for (g), `num=[1000]; den=[1 110 1000 0]`.)

a. $G(s) = \frac{1000}{(s+1)(s+10)(s+1000)}$

b. $G(s) = \frac{900(s+10)}{s(s+3)(s+300)}$

c. $G(s) = \frac{100^2}{s^2+20s+100^2}$

d. $G(s) = \frac{100^2(s^2+s+100)}{s^2(s+100)^2}$

e. $G(s) = \frac{-1000}{(s+1)(s+300)}$

f. $G(s) = \frac{s+3}{s^2(s-10)}$

g. $G(s) = \frac{1000}{s(s+10)(s+100)}$

h. $G(s) = \frac{1000(s+1)}{s^2(s+10)}$

i. $G(s) = \frac{100^2(s+10)}{s(s^2+2s+100^2)}$

j. $G(s) = \frac{10(s^2+6s+9)}{9(s+1)(s+100^2)}$

k. $G(s) = \frac{(s-1)}{s(s+10)}$

l. $G(s) = \frac{1000(s^2-s+100)}{s(s^2-2s+10000)}$

Problem 35. Sketch the Nyquist plots for the 12 transfer functions in Problem 34. Use the MATLAB command `nyquist(num,den)` to check your work. (The `axis` command is helpful. Type `help axis`.)

Problem 36. Construct the Nyquist Plot for the open-loop system

$$G(s) = \frac{1}{s(s+1)}.$$

Using this plot, which of the following systems are stable? If stable, what are the gain and phase margins?

$$\frac{0.5 \cdot G(s)}{1 + 0.5 \cdot G(s)}, \quad \frac{5.0 \cdot G(s)}{1 + 5.0 \cdot G(s)}, \quad \frac{50.0 \cdot G(s)}{1 + 50.0 \cdot G(s)}.$$

Check your answers using the MATLAB command `margin`. For $k = 0.5$, this is done as follows.

```
> num=[0.5];
> den=[1 1 0];
> margin(num,den);
```

Problem 37. For the open-loop system

$$G(s) = \frac{K(s+10)^2}{10s^2(s+1)}$$

sketch the Nyquist and Bode plots (both magnitude and phase). For what range of K is the closed-loop system $G(s)/(1+G(s))$ stable? Answer: $K > 40.0$.

Problem 38. For Problem 37, what is the phase margin when $K = 41$? Answer: $0.6°$. Using MATLAB, plot the step response of the closed-loop system when $K = 41$. Answer:

```
> num=41*[0 1 20 100];
> den=10*[1 1 0 0];
> margin(num,den);       % plots magnitude, phase, computes phase margin
> dencl=num+den;         % denominator of closed loop
> numcl=num;             % numerator of closed loop
> step(numcl,dencl);     % step response
```

Problem 39. Sketch the Nyquist and Bode plots for

$$G(s) = \frac{(s+1)(s-1)}{(s+10)(s-10)}.$$

Trick question: How many clockwise encirclements of $s = -1$ are required for stability of $G(s)/(1+G(s))$ Answer: $G(s)$ is not strictly proper, so $G/(1+G)$ makes no sense. We need to add a strictly proper controller in the loop. Design a $C(s)$ to ensure stability of $C(s)G(s)/(1+C(s)G(s))$.

Problem 40. Using the Nyquist criterion, and the `margin` command in MATLAB, determine the range of gain K for which the closed-loop systems $KG(s)/(1+KG(s))$ are stable, when

a. $G(s) = \frac{10000}{s(s+10)(s+100)}$

b. $G(s) = \frac{10(s+1)}{s^2(s+10)}$

c. $G(s) = \frac{1000(s+1)(s+10)}{s^3(s+100)}$

d. $G(s) = \frac{10^6(s+10)}{(s+1)^2(s^2+20s+100^2)}$

e. $G(s) = \frac{0.1(s+10)}{s^2(s+10)}$

Answer: For (b), type the following into `Matlab`.

```
> num=[10 10];
> den=[1 10 0 0];
> nyquist(num,den);   % to make the nyquist plot
> axis([-5 5 -3 3]);  % to see the nyquist plot near 0
```

Since (b) has phase between $-180°$ and $-90°$, as $\omega \to +\infty$, there are no encirclements, and the range of K for which the closed-loop system is stable is $0 < K < \infty$. (Note the $<$ signs are not $\leq$ signs!) The gain margin is therefore ∞, as reported by `margin(num,den)`.

Problem 41. (Problem 2, 1996 final exam.) For the system

$$P(s) = \frac{(s-1)(s-10)}{s(s+10)^2}$$

- (10 points) sketch the Bode magnitude and phase plots; and
- (5 points) sketch the Nyquist plot.
- (5 points) What is the range of gain K for which $\frac{KP(s)}{1+KP(s)}$ is stable? (Use Routh array.)

Answer: Type the following into MATLAB:

```
> num=[0 1 -11 10];               % The numerator as given, and
> den=[1 20 100 0];               % the denominator as given.
> nyquist(num,den);               % This doesn't look good, so
> w=logspace(-1,3,200);           % specify a set of frequencies,
> [re,im]=nyquist(num,den,w);     % compute real and imag parts of P,
> plot(re,im,'c',re,-im,'r');     % plot the Nyquist plot
> [mag,phase]=bode(num,den,w);    % and the Bode plots.
> loglog(w,mag);                  % Note the log-log for magnitude,
> semilogx(w,phase);              % and the semilog for phase!
```

From the Nuquist plot, and Root Locus, we expect a range $0 < K < K_{MAX}$. Get K_{MAX} from the Routh Array. Answer is $K_{MAX} \approx 8.8$.