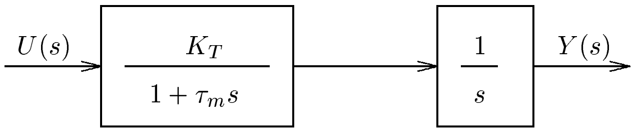
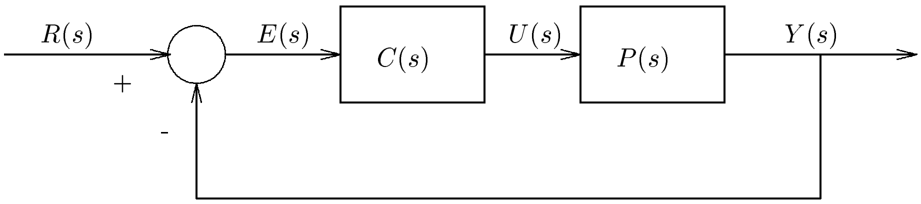
ECE 410 Homework # 3

Monday, October 5, 1998

Problem 22. Given the block diagram of a servo-system

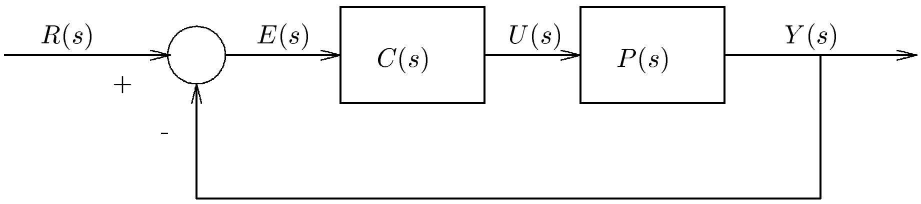


where $y(t)$ is the shaft position and $u(t)$ is the voltage applied to the winding. Let $P(s) = \frac{Y(s)}{U(s)}$ denote the servo transfer function from voltage input to position output. We wish to design a controller $C(s)$ for the closed-loop system



and ensure that $y(t)$ tracks $v(t) = t^2$ with zero steady-state error, i.e., such that $e(t) = v(t) - y(t) \to 0$ as $t \to \infty$. What does the final value theorem say about the structure of $C(s)$?

Problem 23. For the system



the controller transfer function is $C(s) = K$, and the plant transfer function is

$$P(s) = \frac{1}{s^3}.$$

Draw the root locus for $-\infty < K < \infty$. Is there an interval $a < K < b$ for which the closed-loop system is stable?

Problem 24. Repeat Problem 23 for the following plants:

$$P(s) = \frac{s-1}{s(s+1)} \qquad P(s) = \frac{s}{(s-1)(s+1)}$$

$$P(s) = \frac{(s+1)}{s(s-1)} \qquad P(s) = \frac{(s-1)}{s^2(s+2)}$$

$$P(s) = \frac{(s+1)}{s^2(s+2)} \qquad P(s) = \frac{(s+1)}{s^2}$$

$$P(s) = \frac{(s+1)}{s^3} \qquad P(s) = \frac{(s+1)(s+2)}{s^3}$$

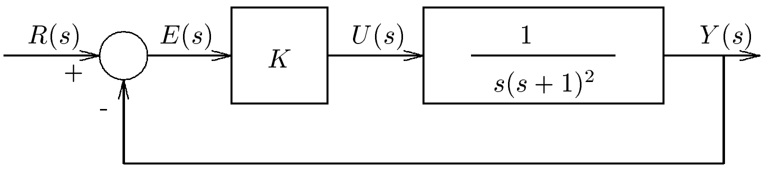
$$P(s) = \frac{1}{(s+1)(s+2)(s+3)(s+4)} \qquad P(s) = \frac{(s+5)}{(s+1)(s+2)(s+3)(s+4)}$$

Problem 25. Using MATLAB, plot the root locus for each of the plants in Problem 24, to check your work. Use the `rlocus` command. For example, for the first one, you would start MATLAB and type

```
num=[1, -1];
den=[1, 1, 0];
rlocus(num,den);
```

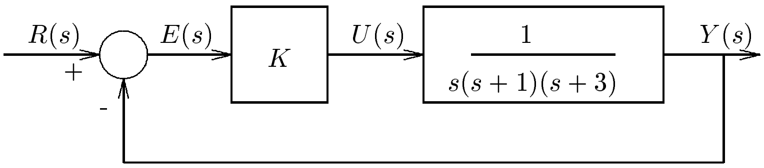
Note that you can get help on MATLAB commands such as `rlocus` by typing `help rlocus`. In fact, the `rlocus` command has several options that allow plots of the complementary root locus, or saving the roots in a matrix.

Problem 26. The closed-loop system below represents the height dynamics of a hot-air balloon, where $y(t)$ is the height, and $u(t)$ is the heat flowing from the flame (under the pilot's control) to the airmass within the balloon. The pilot has a great idea: Design an automatic controller to regulate height at a given reference. His idea is to use a constant-gain feedback controller, as shown below.



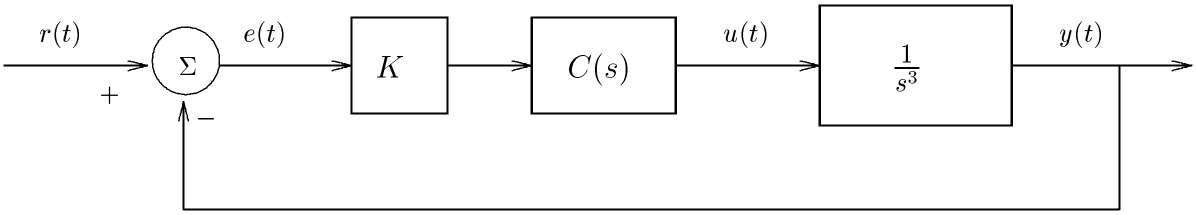
Draw the root locus for $-\infty < K < \infty$. Is there an interval $a < K < b$ for which the closed-loop system is stable? If so, find this range using a Routh array. Do you think his constant-gain controller is a good idea?

Problem 27. Given the closed-loop system



Draw the root locus for $-\infty < K < \infty$. Is there an interval $a < K < b$ for which the closed-loop system is stable? If so, find this range using a Routh array.

Problem 28. Consider the following closed-loop system.



1. Draw the root locus plot for the closed-loop system for $-\infty < K < \infty$, $C(s) = 1$. Is the system stable for any K?

2. Draw the root locus for $-\infty < K < \infty$ and $C(s) = \frac{1+s}{1+0.1s}$. Is the system stable for any K?

3. Prove that there exists *NO* first order controller of the form $C(s) = \frac{s+a}{s+b}$ to stabilize the closed-loop system for *any* K. Note: in this part, $C(s)$ could be non-minimum phase, i.e., you must consider the case $-\infty < a < \infty$ and $-\infty < b < \infty$. Hint: consider a Routh Array.

4. Draw the root locus for

$$C(s) = \frac{(1+s)^2}{(1+0.1s)^2}.$$

5. For what range of K is the closed-loop system from part (4) stable? Use Routh array here.