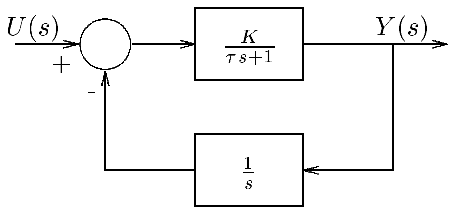
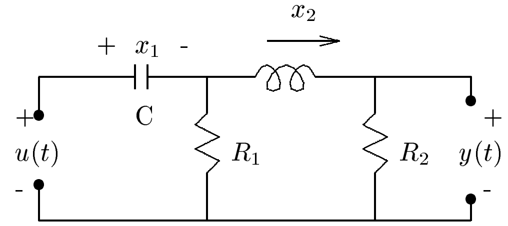
ECE 410 Homework # 2

Monday, September 28, 1998

Problem 9. Consider the block diagram below. Both K and τ are fixed constants. What is the system transfer function $Y(s)/U(s)$? In the limit, as $K \to \infty$ (τ is fixed), what operation does the circuit approximate?



Problem 10. Write the state equations for the circuit where the states are x_1 = voltage across the capacitor, and x_2 = current through the inductor. What are the A, B, C, and D matrices? Draw a block diagram. Next, write the state equations in controllable canonical form. (The states will now be different.) What are the new A, B, C, and D matrices? What is the transfer function from U to Y?



Problem 11. Given

$$G(s)\frac{Y(s)}{U(s)} = \frac{K\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2},$$

where $\omega_n > 0$, $0 < \delta < 1$ (the underdamped case), find the unit step response $y(t)$. What is the frequency of oscillation ω_d, in terms of ω_n and δ? What is the *phase*? (Don't spend too much time on this.) Answer...

- $y(t) = K\left[1 - \frac{1}{\sqrt{1-\delta^2}} \exp^{-\delta\omega_n t} \sin(\omega_d t + \phi)\right]$
- $\omega_d = \omega_n\sqrt{1-\delta^2}$
- $\phi = \tan^{-1}\left(\frac{\sqrt{1-\delta^2}}{\delta}\right)$

Problem 12. For the system

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2},$$

where $K > 0$, $\omega_n > 0$, and $0 < \delta < 1$, show the step response has a *peak value* of

$$y(t^*) = M_p = K(1 + \exp^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}}).$$

at time

$$t^* = \frac{\pi}{\omega_n\sqrt{1-\delta^2}}.$$

Problem 13. For the system in Problems 11 and 12, define the *percent overshoot* as

$$\text{percent overshoot} \;=\; \frac{M_p - y(\infty)}{y(\infty)} * 100$$

and show this satisfies

$$\text{percent overshoot} \;= 100\exp^{-\pi\frac{\delta}{\sqrt{1-\delta^2}}}.$$

Plot this expression as a function of δ for $0 < \delta < 1$. What is the maximum value of the percent overshoot as a function of δ?

Problem 14. The purpose of this problem is to understand the effect of *zeros* on the step response of a second-order system. Consider

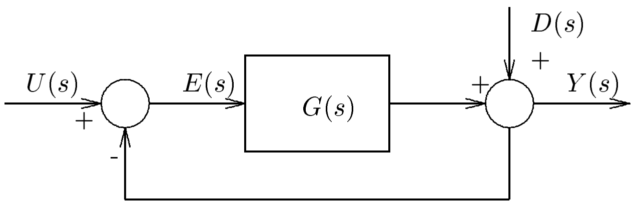
$$H(s) = \frac{\omega_n^2}{s^2 + s\delta\omega_n s + \omega_n^2},$$

with $\omega_n = 1$, $\delta = 0.8$. Is the system under, over, or critically damped? Using MATLAB, plot the step response. Now consider the new system $G(s) = (1 + s/a)H(s)$, i.e., add a zero at $-a$. On the same plot, draw the step response for various values of a. For example, $a = 0.1,\ 0.2,\ 0.5,\ 1.0,\ 2.0,\ 10.0,\ 100.0$. Include negative values (very important!). Notice what happens to $y(\infty)$, $\dot{y}(0)$, and $y(0)$. What happens to the system as the zero "crosses" the origin? What happens when the zero is *at* the origin?

Problem 15. Which system has the *largest* percent overshoot?

$$H(s) = \frac{1}{s^2 + 1.4s + 1} \quad \text{or} \quad G(s) = \frac{4}{s^2 + 4s + 4}?$$

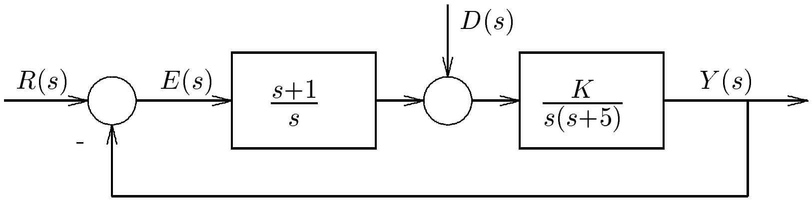
Problem 16. Integrators are often added to a control system design to reject constant disturbances in steady-state operation. For the system



what is the steady state error, $\lim_{t\to\infty} e(t)$, when $u(t) = 0$, the disturbance $d(t)$ is a step of height 2, and $G(s)$ is

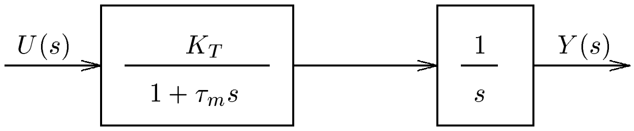
$$1.\quad G(s) = \frac{10(s+1)}{(s+2)(s+5)} \qquad 2.\quad G(s) = \frac{10(s+1)}{s(s+2)(s+5)}.$$

Problem 17. In the following diagram, the inputs entering the summers are assumed to be positive, unless otherwise indicated, and the constant K satisfies $4 \leq K \leq 6$, but its precise value is unknown. Assume the closed-loop system is stable.

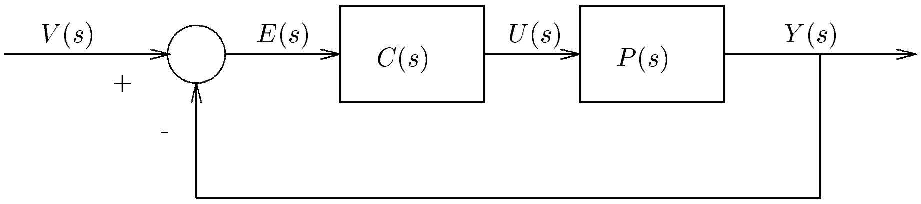


- Assuming $d(t) = 0$ and $r(t) \neq 0$, write $\frac{Y(s)}{R(s)}$ and $\frac{E(s)}{R(s)}$, as a function of K.
- Assuming $r(t) = 0$ and $d(t) \neq 0$, write $\frac{E(s)}{D(s)}$, as a function of K.
- Assuming $d(t) = 0$, can the closed-loop system track a step input (applied at $r(t)$), with 0 steady-state error?
- Assuming $d(t) = 0$, can the closed-loop system track a ramp input ($r(t) = t$), with 0 steady-state error?
- What is the stady-state error, $\lim_{t\to\infty} e(t)$, if $r(t) = t^2$ and $d(t) = 0$?
- What is the stady-state error, $\lim_{t\to\infty} e(t)$, if both $r(t)$ and $d(t)$ are unit steps?
- What is the stady-state error, $\lim_{t\to\infty} e(t)$, if $r(t) = t$ and $d(t)$ is a unit step?

Problem 18. Given the block diagram of a servo-system

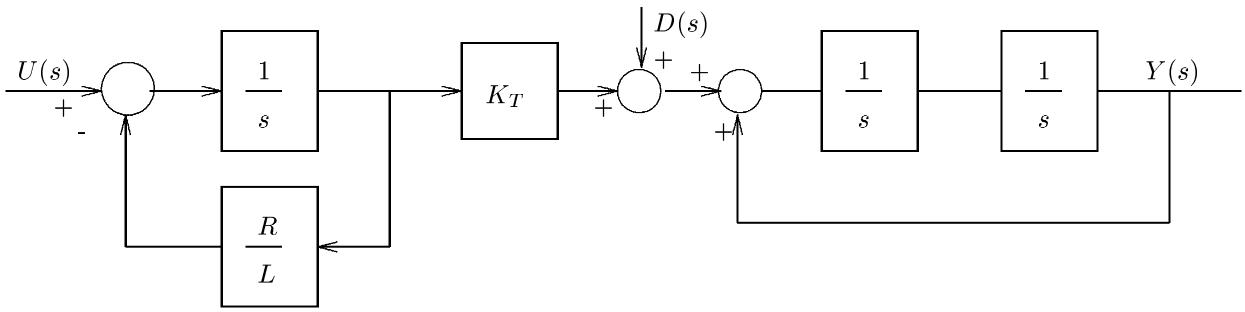


where $y(t)$ is the shaft position and $u(t)$ is the voltage applied to the winding. Let $P(s) = \frac{Y(s)}{U(s)}$ denote the servo transfer function from voltage input to position output. We wish to design a controller $C(s)$ for the closed-loop system



and ensure that $y(t)$ tracks $v(t) = t^2$ with zero steady-state error, i.e., such that $e(t) = v(t) - y(t) \to 0$ as $t \to \infty$. What does the final value theorem say about the structure of $C(s)$?

Problem 19. Given the following block diagram for the magnetically suspended ball



where $\frac{R}{L} = 1$, $K_T = 10$, $u(t)$ is the voltage input, $y(t)$ is the ball height, which is the output to be controlled, and $d(t)$ is a force disturbance on the ball. We must design a closed-loop controller $C(s) = \frac{U(s)}{E(s)}$, where $E(s) = -Y(s)$ for this system.

- What is $\frac{Y(s)}{U(s)}$, assuming $d(t) = 0$?
- What is $\frac{Y(s)}{D(s)}$, assuming $u(t) = 0$?
- What is $\frac{Y(s)}{D(s)}$, assuming the controller is in the loop, i.e., $U(s) = -C(s)Y(s)$.
- If $d(t)$ is a unit step input, at least how many poles at the origin must $C(s)$ have to ensure that $\lim_{t\to\infty} e(t) = 0$? Ignore the stability question here.

Problem 20. Assume that for $d(t) = \sin(\omega_0 t)$ in Problem 19, it is specified that $\lim_{t\to\infty} e(t) = 0$ in the steady state. What constraint does this place on $C(s)$?

Problem 21. What is a good transfer function model for the system with step response illustrated in the Figure 1? Assume all initial conditions are zero.

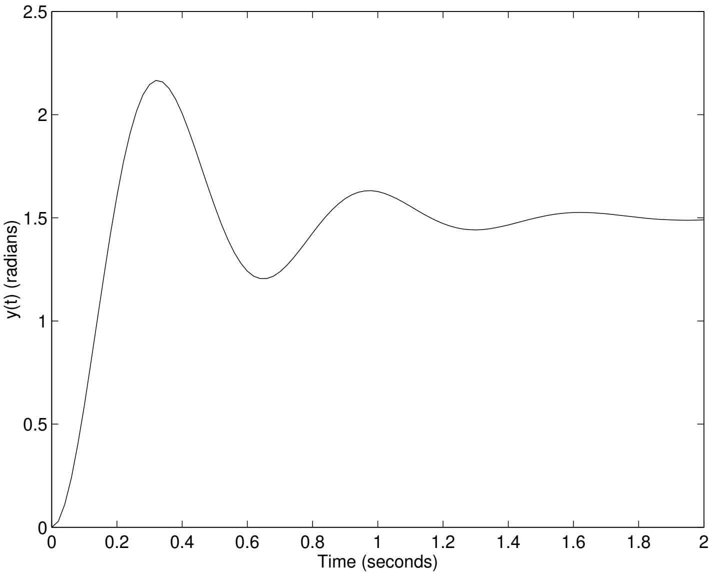


Figure 1: Step response for Problem 21.