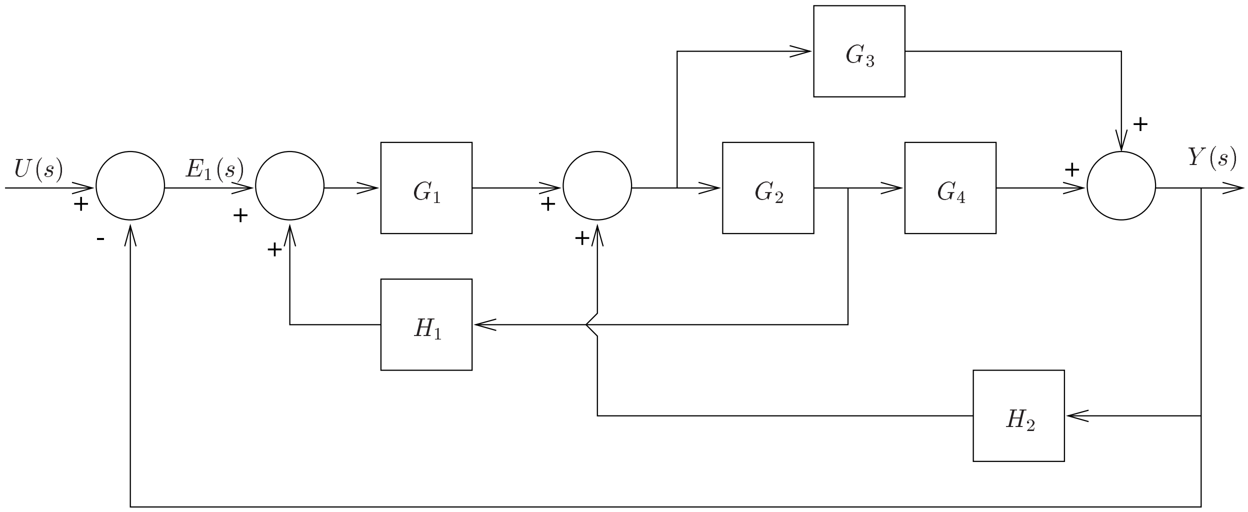
ECE 410 Homework # 1

Monday September 14, 1998

Problem 1. Find the closed-loop transfer functions $Y(s)/U(s)$ and $E_1(s)/U(s)$, in terms of H_1, H_2, and $G_1, \ldots, G_4$, for the following block diagram.



Problem 2. Write $Y(s)/U(s)$ and $E_1(s)/U(s)$ for Problems 1 and 2, in terms of s, assuming

$$G_1(s) = G_2(s) = \frac{1}{s} \quad G_3(s) = \frac{1}{s+1} \quad G_4(s) = \frac{s}{s+1} \quad H_1(s) = \frac{s+1}{s-1} \quad H_2(s) = 1.$$

Problem 3. Write the input/output differential equation for the system in Problems 1 and 2.

Problem 4. What is the **controllable canonical form** state variable realization for the system in Problem 3? That is, what are the A, B, C, and D matrices?

Problem 5. For the block diagram in Problems 1 and 2, assign state variables to be the outputs of the 5 integrators, and write a set of state equations by inspection from the block diagram. These will differ from Problem 4.

Problem 6. What is the characteristic equation of the A matrices in Problems 4 and 5? ($\Delta(s) =$ determinate$[sI - A]$). What is the denominator polynomial for $Y(s)/U(s)$ in Problem 2? Is the system stable?

Problem 7. Realize the transfer function

$$G(s) = \frac{s^2 + 2s + 1}{s^2 + s + 1}$$

as follows. First, draw a block diagram. Assign states as the integrator outputs. Write down the state equations. What is the A, B, C and D matrix here? How would you implement this transfer function using integrator, gain, and summer blocks?

Problem 8. In the state space realization below, the state vector is $x = [x_1\ x_2]^T$, the input signal is u, and the output is y. Find the transfer function $Y(s)/U(s)$.

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 5 \\ 0 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t). \end{aligned}$$