UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION
ECE 410 SYSTEMS CONTROL I
December 9, 1997
Examiner: Professor Scott A. Bortoff

☞ **NAME:** ______________________________

☞ **NUMBER:** __________________________

RULES OF EXAMINATION

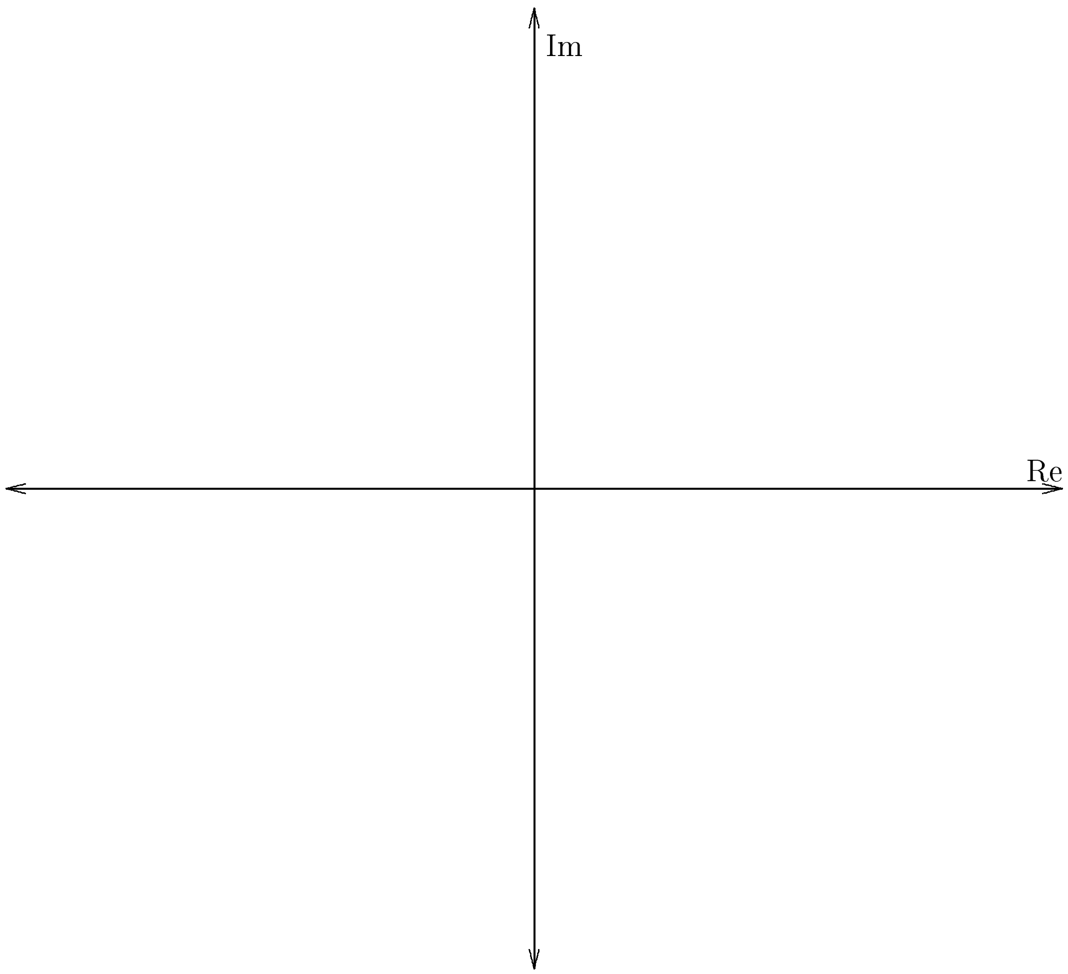
1. All answers are to be written **ON THIS EXAMINATION**, and **NOT** in the "TERM" booklet, which can be used as scratch paper.
2. This exam is type "C," a closed-book examination. You may use a single yellow aid sheet, supplied by the examiner or the Registrar's Office.
3. You may use pen, pencil, and drafting instruments such as a ruler.
4. You may use a non-programmable electronic calculator.
5. Read through the examination before beginning. There 5 problems and 16 pages. Page 16 contains useful information.

1	________	of 10
2	________	of 20
3	________	of 20
4	________	of 25
5	________	of 25
TOTAL	________	of 100

Problem 1. (10 marks) For the plant

$$P(s) = \frac{1+2s}{s^2(1+s/\alpha)},$$

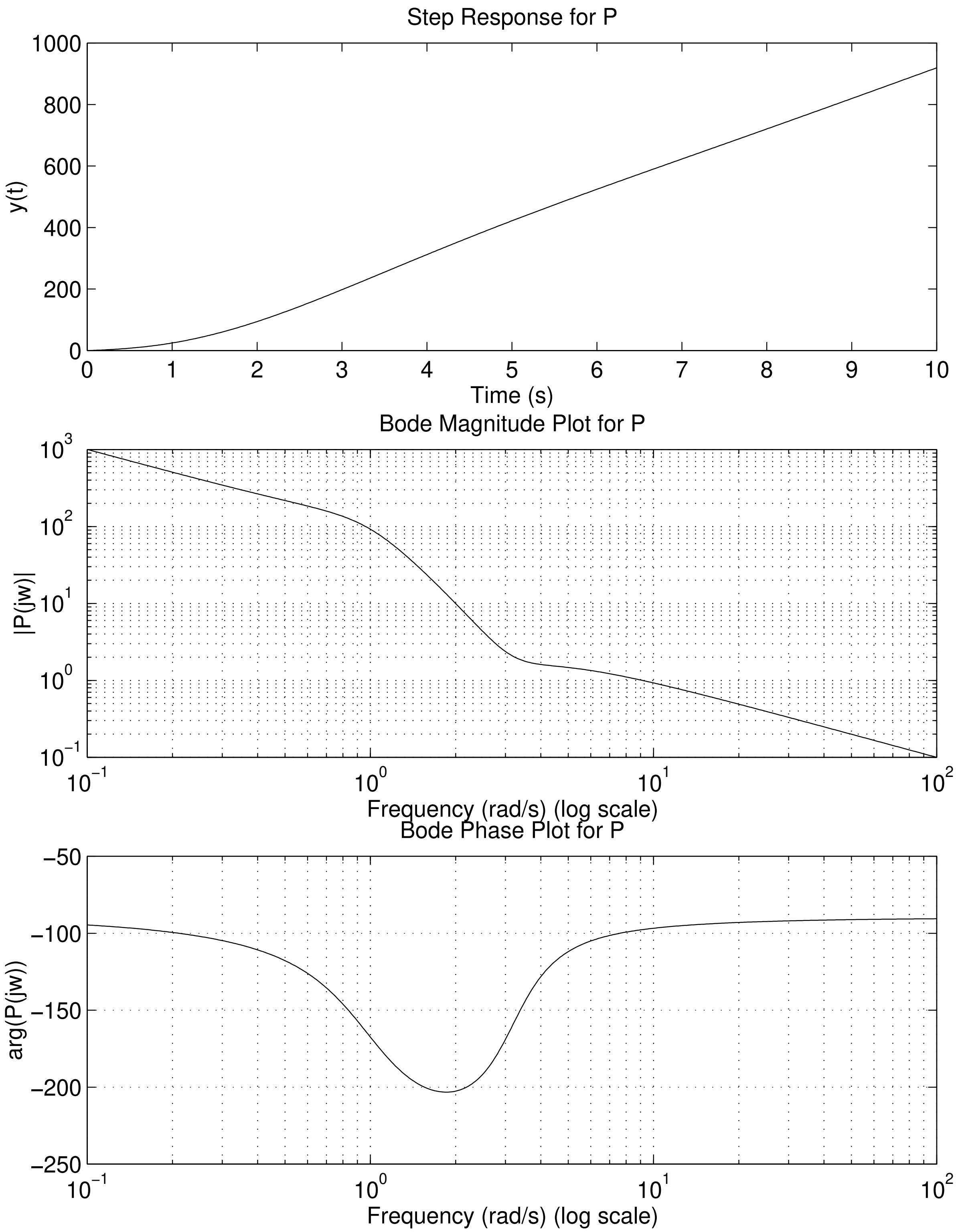
sketch the root-locus (pole-zero locations of $T(s) = \frac{P}{1+P}$) as a function of $\alpha > 0$. Label the origin and angles of the asymptotes. On the next page, use a Routh Array to determine the range of $\alpha > 0$ for which $T(s)$ is stable.



Problem 1. (continued)

Range of α is ____________________

Problem 2. (20 marks) The step response, and the magnitude and phase plots are drawn below for a plant $P(s)$.



Problem 2. (continued) Define $\boldsymbol{T}(s) = \frac{Y(s)}{R(s)} = \frac{P(s)}{1+P(s)}$.

- (4 marks) What is the phase margin θ_m? ____________

- (4 marks) What is the gain margin g_m? ____________

- (3 marks) What is the cut-off frequency ω_c? ____________

- (3 marks) What is the bandwidth ω_b **for** $\boldsymbol{T}(s)$? ____________

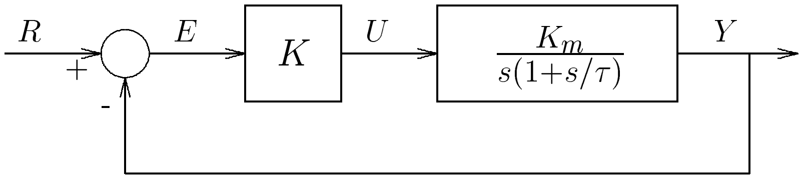
- (3 marks) What is the steady-state tracking error **for** $\boldsymbol{T}(s)$, when $r(t)$ is a unit step?

$\lim_{t\to\infty} e(t) =$ ____________

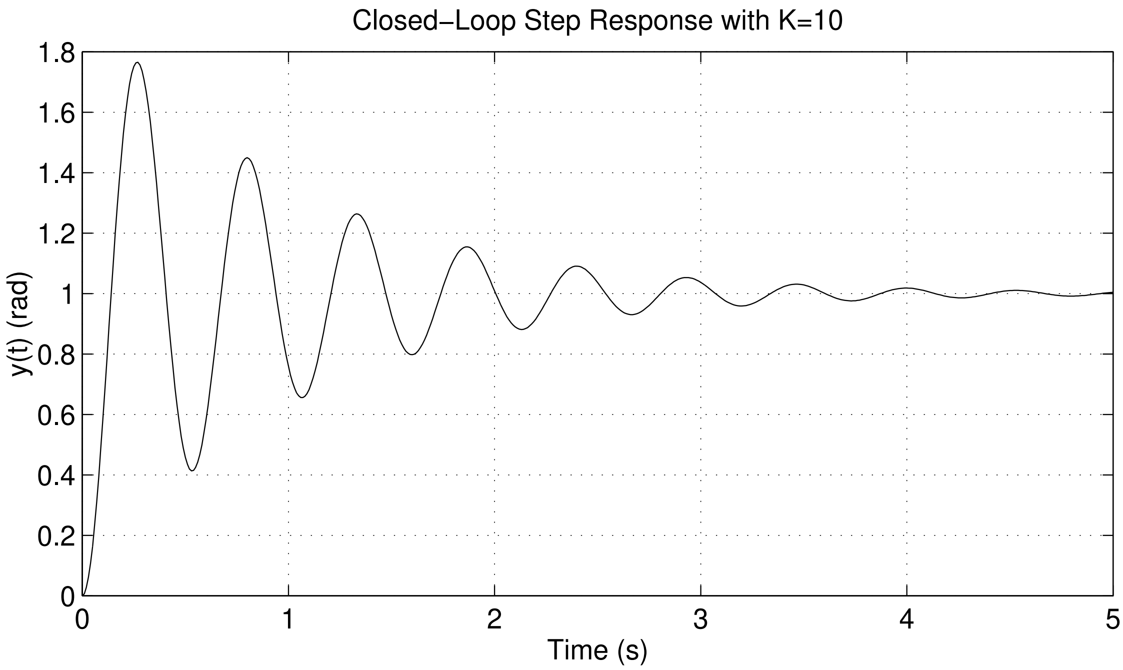
- (3 marks) What is the range of gain K for which $\frac{KP(s)}{1+KP(s)}$ is stable?

RANGE: ________________________

Problem 3. (20 marks) The following block diagram depicts a simplified servomotor system, such as in the ECE 410 lab.



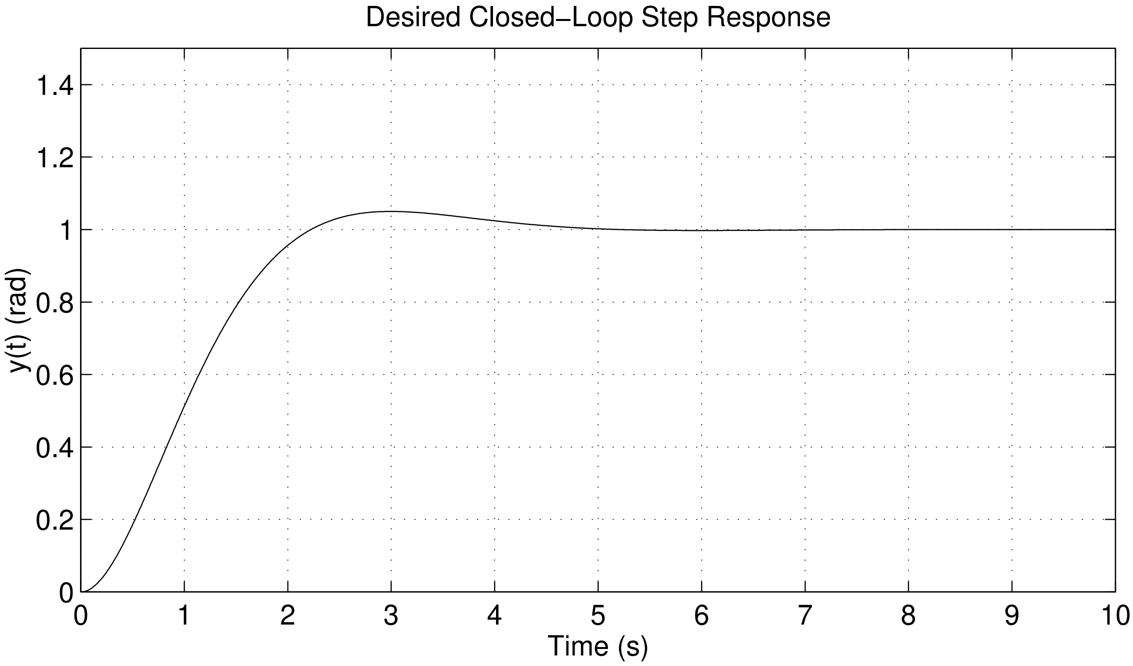
• (10 marks) With the controller $K = 10$, a unit step is applied at R, and the resulting $y(t)$ is shown below. Determine K_m and τ.



$$K_m = \underline{\hspace{3cm}}$$

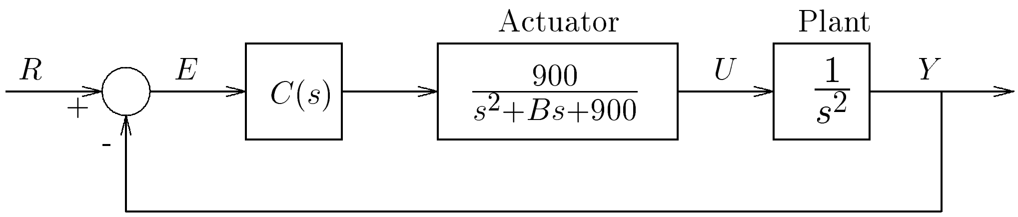
$$\tau = \underline{\hspace{3cm}}$$

Problem 3. (continued) (10 marks) Compute the gain K such that the closed-loop system has a step response shown below. (Note the scale on the axes is different.)



$K =$ __________

Problem 4. (25 marks) In the following control system,



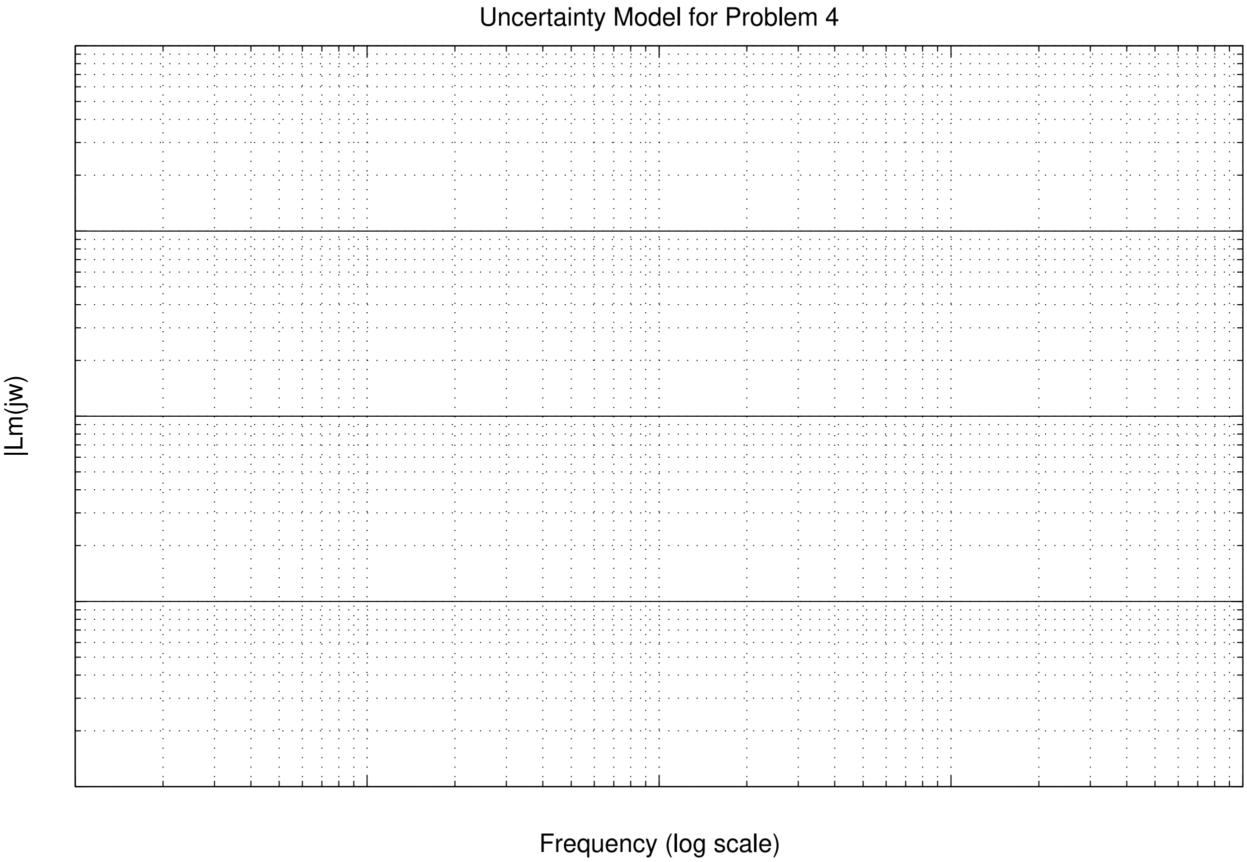
a precise numerical value for B is unknown. However, it does satisfy

$$10 \leq B \leq 30.$$

Define $P(s) = \frac{1}{s^2}$, and $\tilde{P}(s) = P(s) \cdot \frac{900}{s^2+Bs+900}$.

1. (1 mark) Using the multiplicative uncertainty model, $\tilde{P}(s) = P(s)(1 + L_m(s))$, determine $L_m(s)$.

2. (2 marks) Using your calculator, compute precise values for $|L_m(j30)|$, for both $B = 10$ and $B = 30$. Using these numbers, sketch $|L_m(j\omega)|$ for both $B = 10$ and $B = 30$ on the next page.



3. (2 marks) On the plot above, draw a continuous, **piecewise linear** function $l_m(\omega)$ that satisfies $|L_m(j\omega)| < l_m(\omega)$ for all $\omega \geq 0$. You may use only **TWO LINEAR** (straight-line) segments.

4. (20 marks) Design a **first-order** controller $C(s)$ so that the system satisfies the following specifications.

   (a) $\frac{Y(s)}{R(s)}$ must be stable, for any $10 \leq B \leq 30$.

   (b) $\frac{Y(s)}{R(s)}$ must track a the input $r(t) = t$ with 0 steady-state error, for any $10 \leq B \leq 30$.

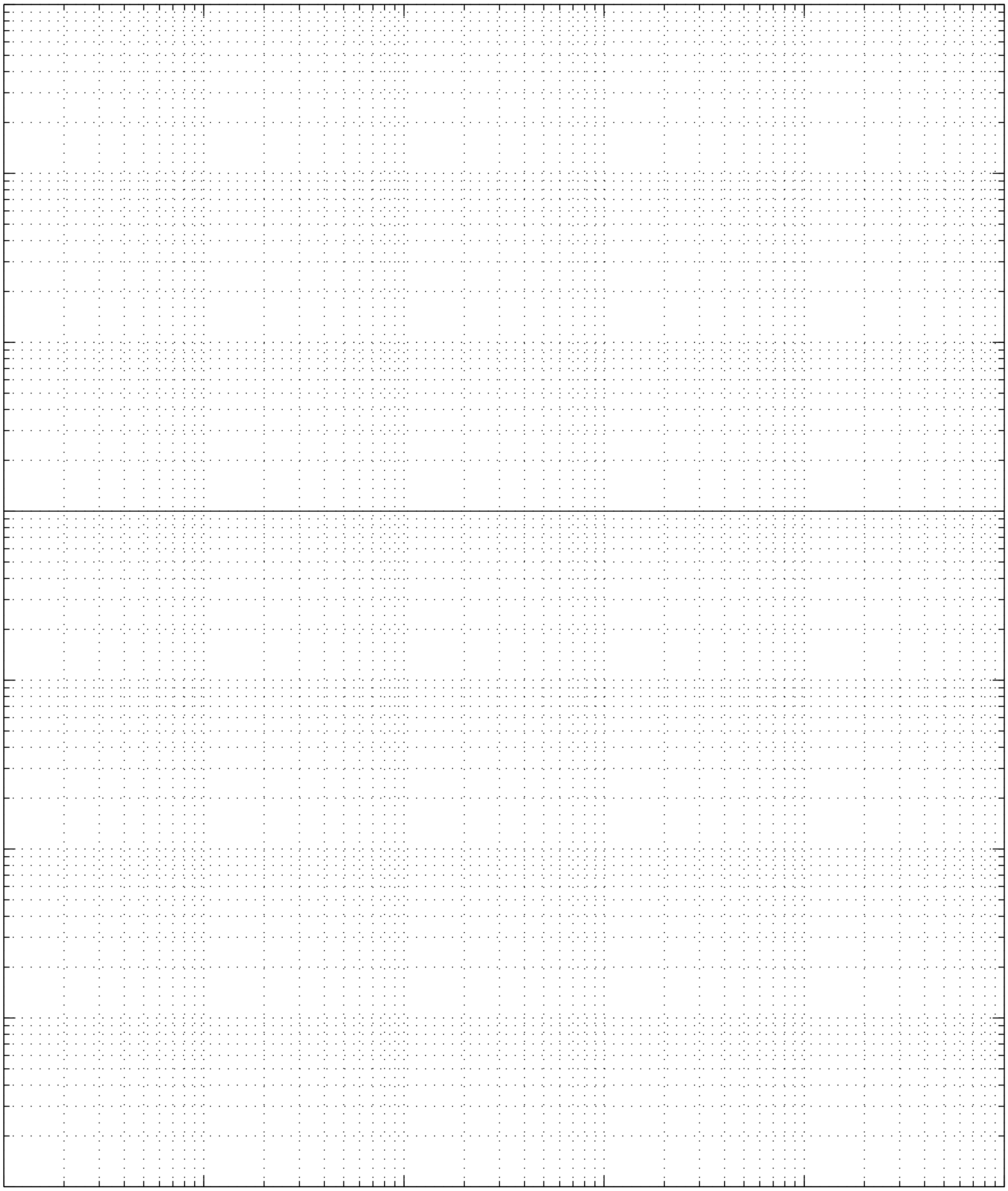
   (c) In the **steady-state**, the tracking error $e(t)$ must satisfy $\max_{t>0} |e(t)| < 0.005$ when $r(t) = \sin(\omega t)$, for all $\omega < 0.1$, for any $10 \leq B \leq 30$.

   (d) The phase margin θ_m for $C(s) \cdot P(s)$ (the *nominal* system) must satisfy $70° < \theta_m$.

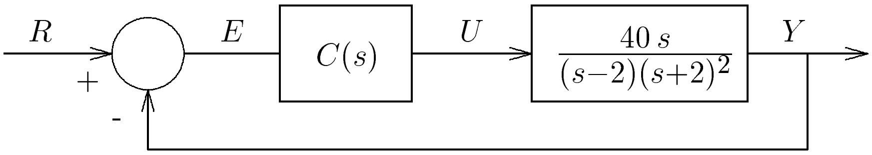
   (e) The closed-loop bandwidth ω_b for the nominal system must satisfy $\omega_b > 2$.

   (f) In the steady-state, the closed-loop system must *attenuate* an input signal $r(t) = \sin(\omega t)$ by a factor of 100 for $\omega > 100$, for any $10 \leq B \leq 30$, i.e., $\max_{t>0} |y(t)| < 0.01$ when $r(t) = \sin(\omega t)$ for $\omega > 100$.

The complete design must be done on the Bode magnitude plot on the next page.



Problem 5. (25 marks) Consider the following closed-loop system.



- (7 marks) On the following pages, draw the Bode magnitude, Bode phase, Nyquist, and root locus plots for the plant

$$P(s) = \frac{40\ s}{(s-2)(s+2)^2}.$$

- (18 marks) Design a second-order controller of the form

$$C(s) = K\ \frac{a_0 a_1}{b_0^2}\ \frac{(s+b_0)^2}{(s+a_0)(s+a_1)}$$

to stabilize the closed-loop system $T = \frac{CP}{1+CP}$, with gain and phase margins satisfying

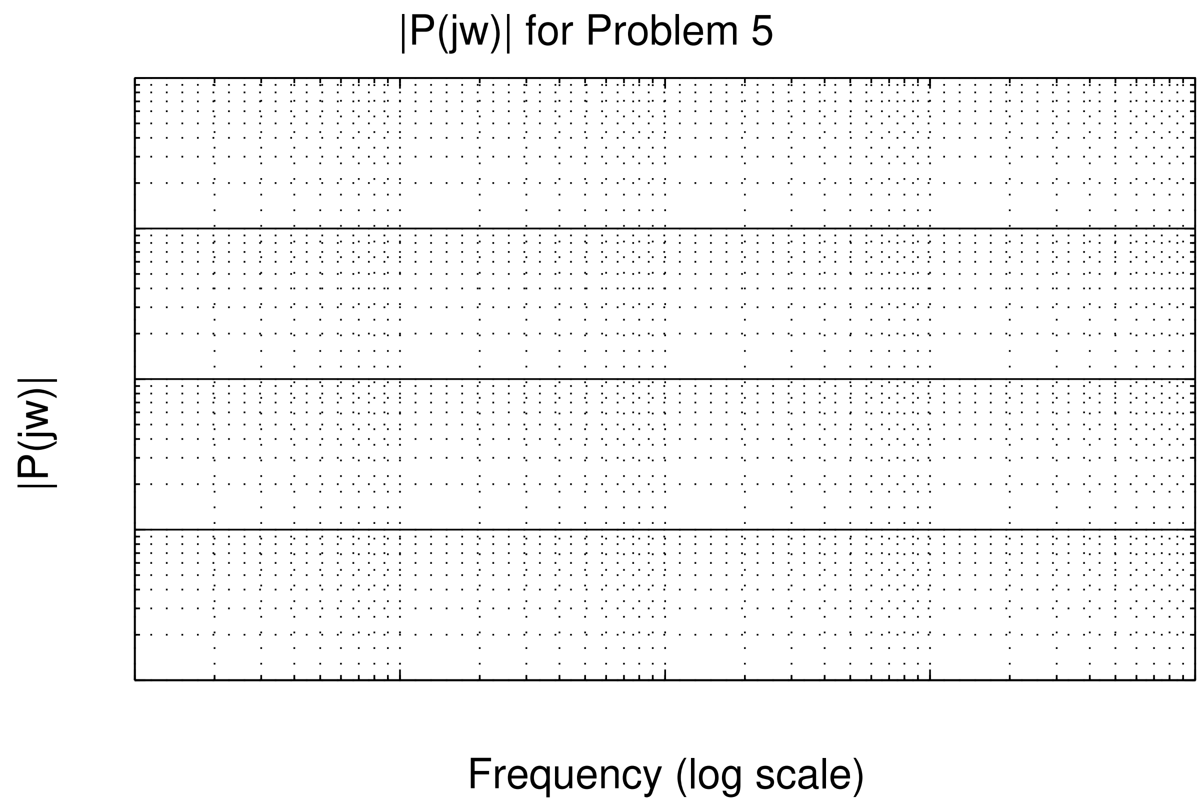
$$\begin{aligned} g_m &< 0.5 \\ \theta_m &> 20^\circ. \end{aligned}$$

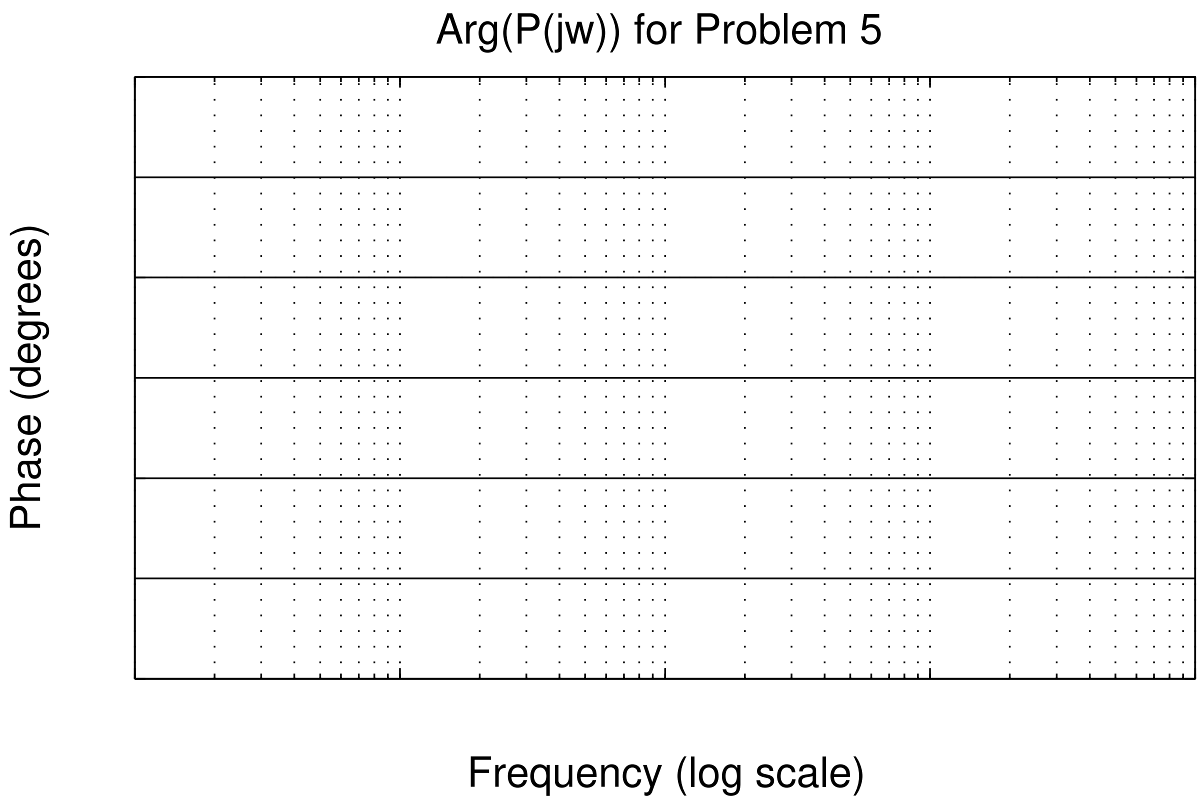
The controller parameters a_0, a_1, b_0, and K are real numbers, and must satisfy the following constraints.

$$\begin{aligned} |K| &< 2 \\ |a_0| &< 10 \\ |a_1| &< 10 \\ |b_0| &< 5 \end{aligned}$$

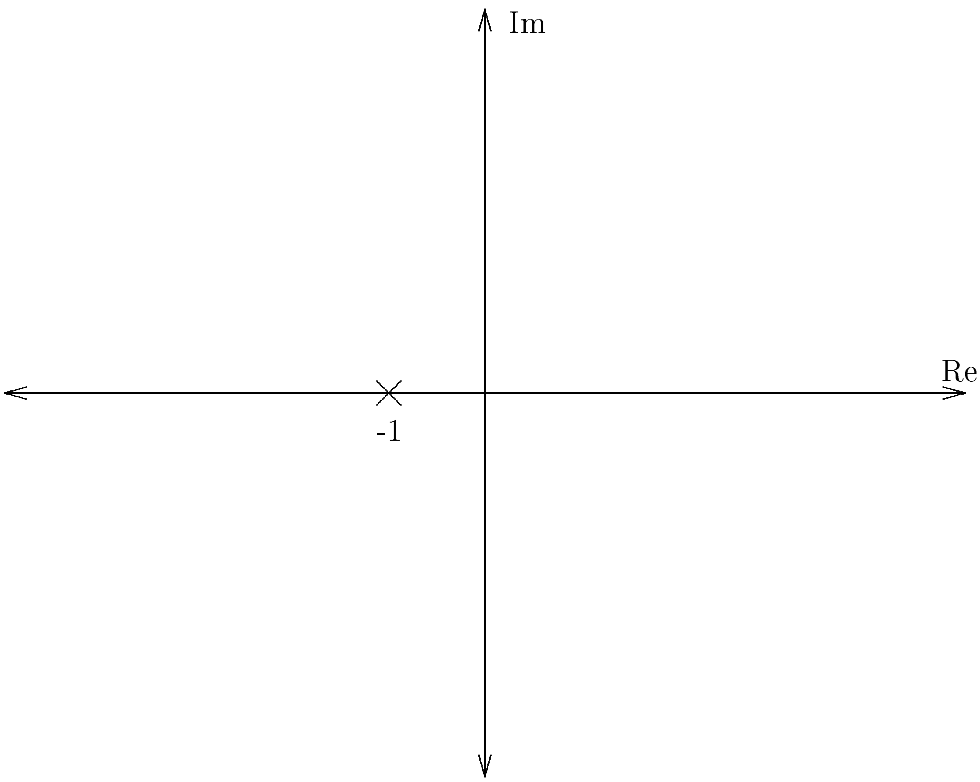
The complete design must include

1. (5 marks) A root locus plot for $C(s) \cdot P(s)$.
2. (8 marks) Asymptotic Bode magnitude and phase plots of $C(s) \cdot P(s)$, showing g_m and θ_m.
3. (5 marks) A Nyquist plot of $C(s) \cdot P(s)$.

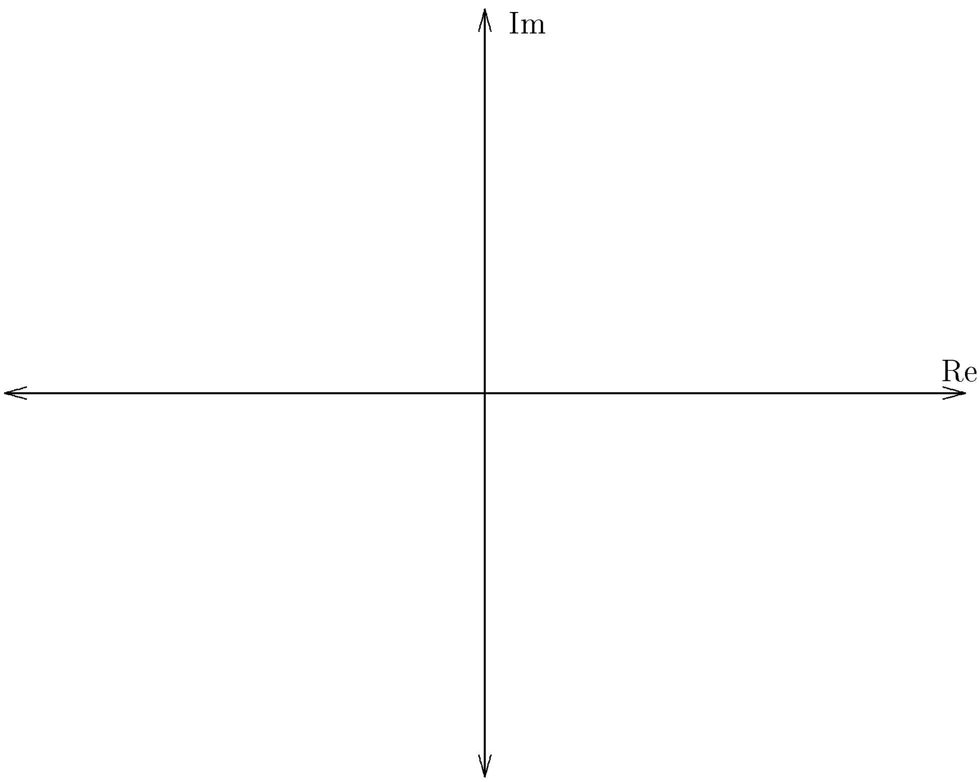

|P(jw)| for Problem 5
|P(jw)|
Frequency (log scale)


Arg(P(jw)) for Problem 5
Phase (degrees)
Frequency (log scale)

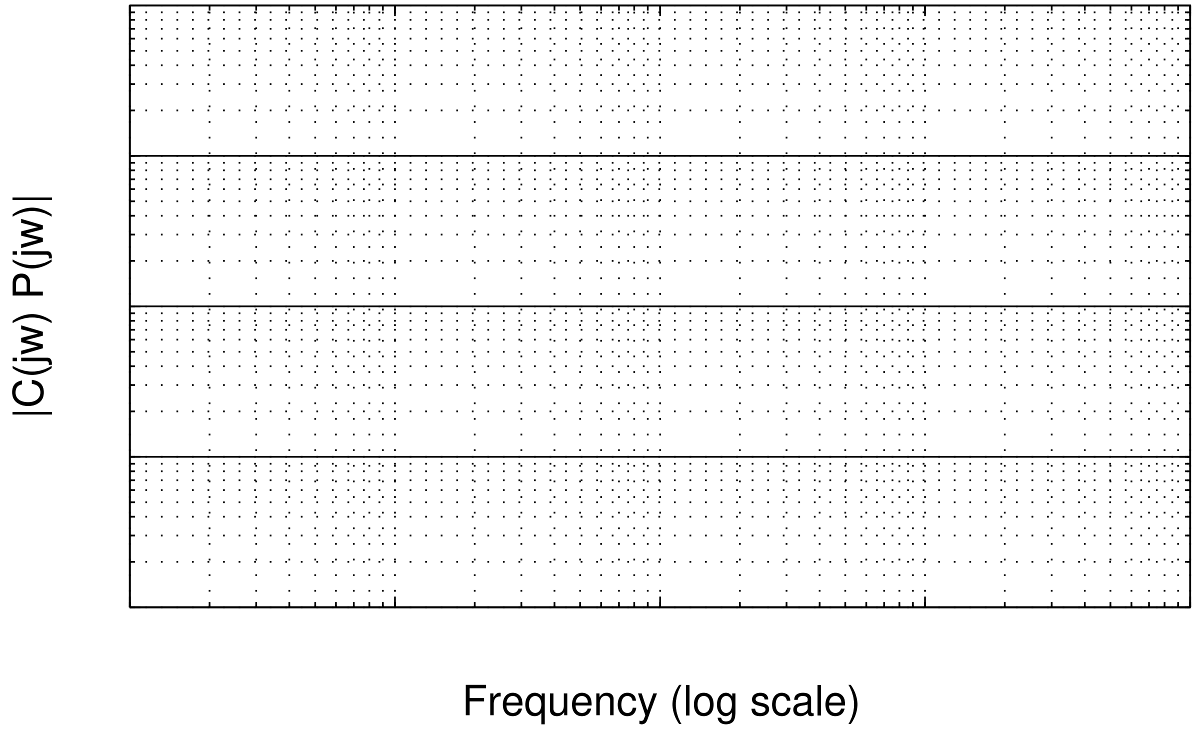
Nyquist Plot for $P(s)$



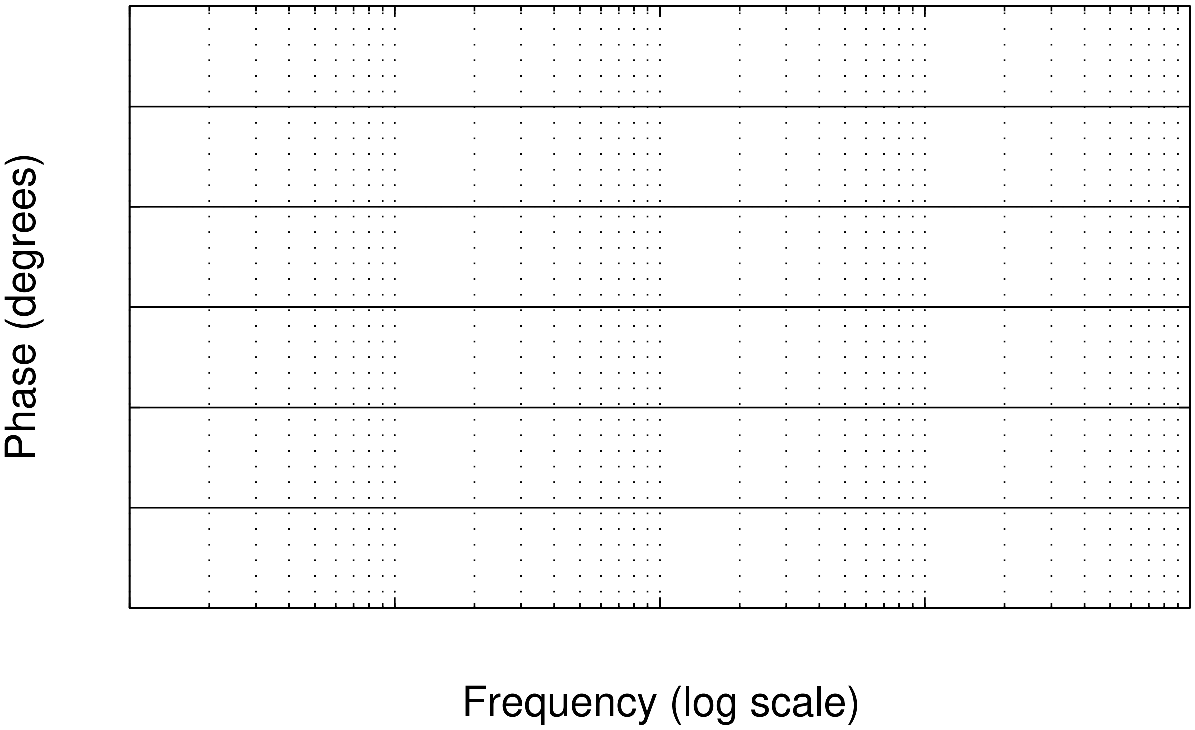
Root Locus for $P(s)$



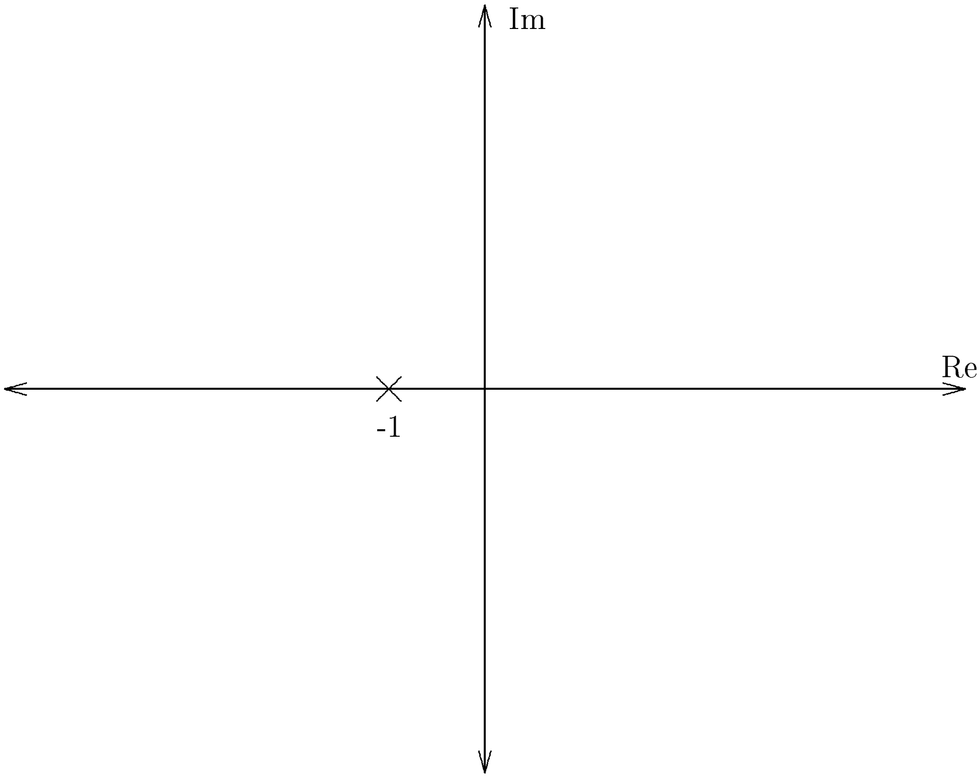
|C(jw) P(jw)| for Problem 5



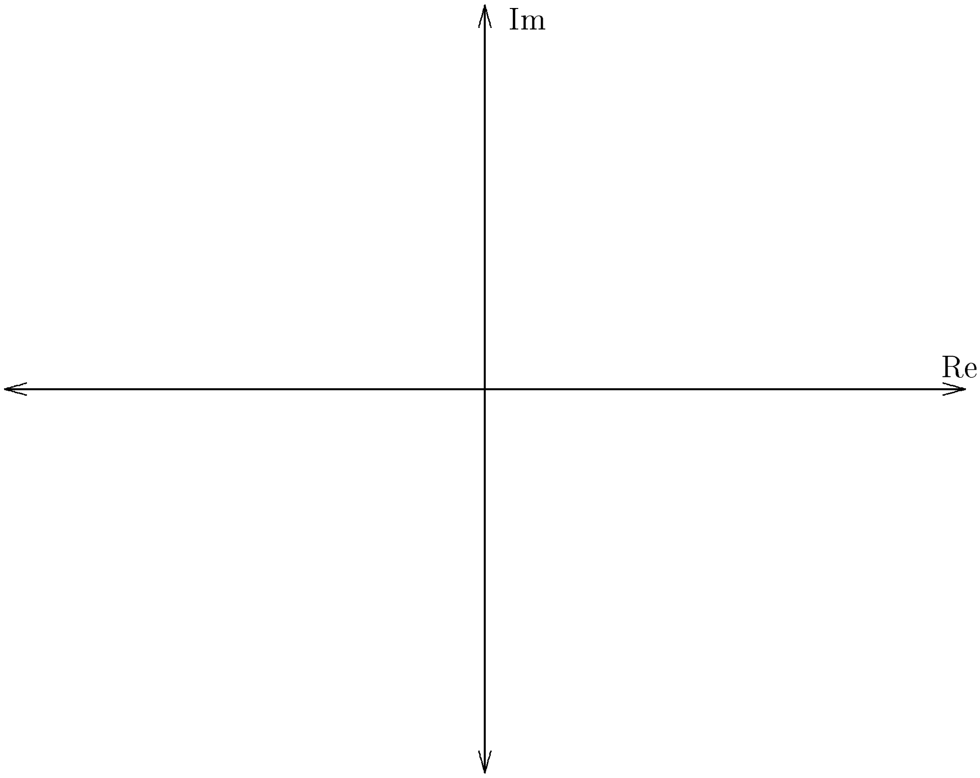
Arg(C(jw) P(jw)) for Problem 5



Nyquist Plot for $C(s) \cdot P(s)$



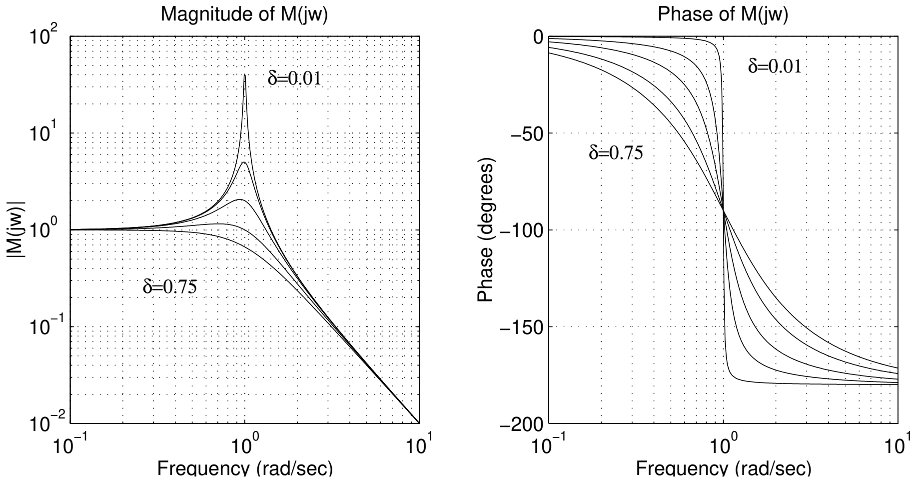
Root Locus for $C(s) \cdot P(s)$



- The following are for the step response for the system $M(s) = \frac{\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2}$,

$$\begin{aligned} \text{percent overshoot} &= 100e^{-\frac{\delta\pi}{\sqrt{1-\delta^2}}} & \text{damped frequency} &= \omega_d = \omega_n \cdot \sqrt{1-\delta^2} \\ \text{peak time} &= t^* = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} & \text{settling time} &= t_s = \frac{3}{\delta\omega_n} \\ \text{peak magnitude} &= y(t^*) = 1 + \exp^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} \end{aligned}$$

- Magnitude and phase are shown below for $\omega_n = 1.0$ and $\delta = 0.01,\ 0.1,\ 0.25,\ 0.5,\ 0.75$.



- Solutions to $as^2 + bs + c = 0$ are $s = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$.
- For a first-order lead controller $C(s) = \frac{1+s/a}{1+s/b}$, with $b > a$, the maximum phase shift occurs at $\omega = \sqrt{ab}$. Below is a plot of this maximum versus the ratio b/a.