UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION
ECE 410 SYSTEMS CONTROL I
December 12, 1996
Examiner: Professor Scott A. Bortoff

Name: ______________________________

Student Number: ______________________________

RULES OF EXAMINATION

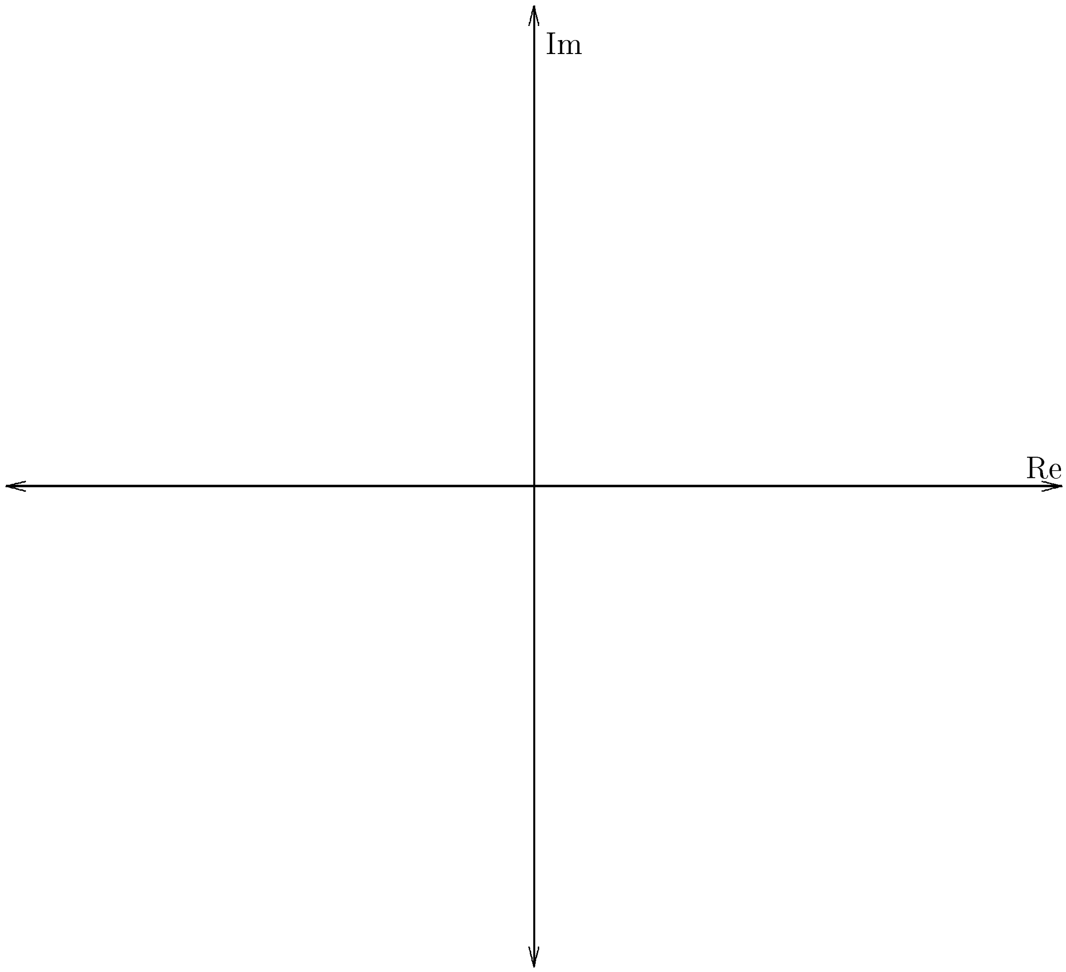
1. All answers are to be written **ON THIS EXAMINATION**, and **NOT** in the "TERM" booklet, which can be used as scratch paper.
2. This exam is type "C," a closed-book examination. You may use a single yellow aid sheet, supplied by the examiner or the Registrar's Office.
3. You may use pen, pencil, and drafting instruments such as a ruler.
4. You may use a non-programmable electronic calculator.
5. Read through the examination before beginning. There are 15 pages, **some of which have useful plots and formulae.** You may detatch the last two pages.

1	______	of 10
2	______	of 20
3	______	of 20
4	______	of 30
5	______	of 20
TOTAL	______	of 100

Problem 1. (10 points) Sketch the root-locus for $K > 0$ for the plant

$$P(s) = \frac{(s+2)^2}{s^3(s+1)^3}.$$

Label the origin and angles of the asymptotes. Is the system $\frac{KP(s)}{1+KP(s)}$ stable for *any* $K > 0$?



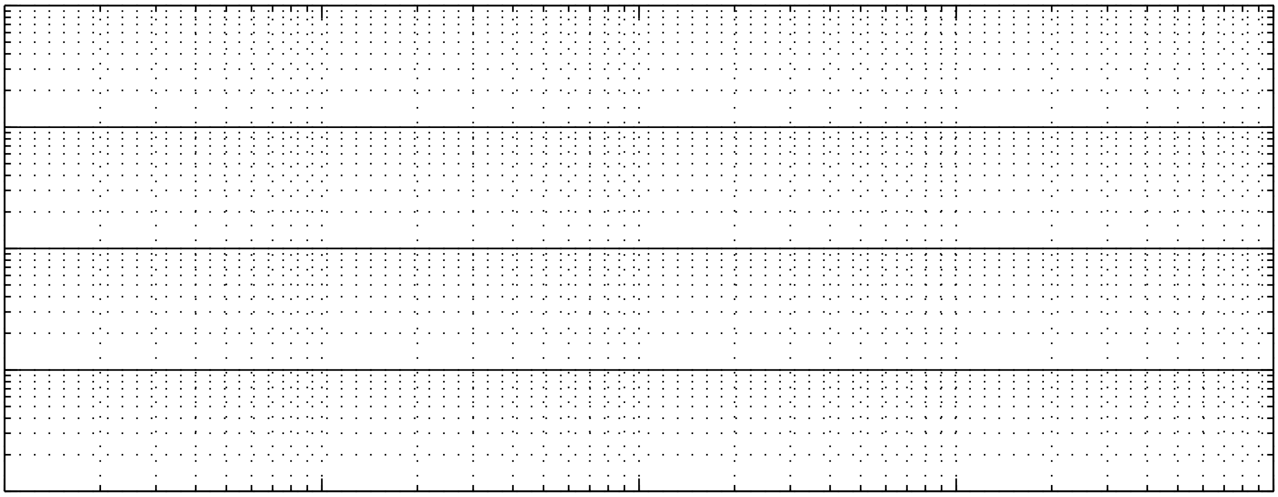
Problem 2. (20 points) For the system

$$P(s) = \frac{(s-1)(s-10)}{s(s+10)^2}$$

- (10 points) sketch the Bode magnitude and phase (on the axes below); and
- (5 points) sketch the Nyquist plot (on the next page).

Magnitude Plot for Problem 2

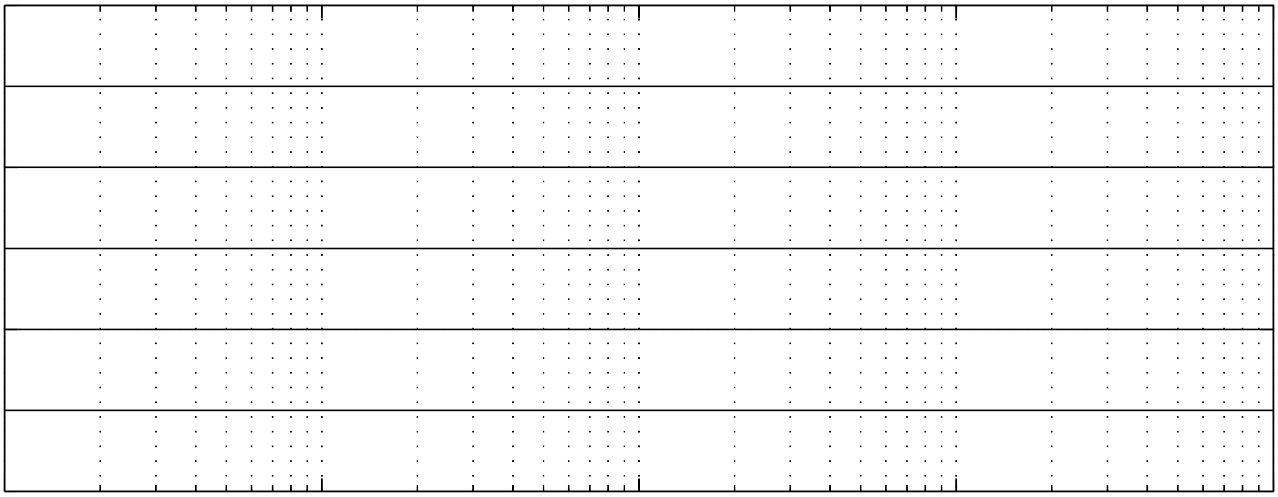
|P(jw)|



Frequency (log scale)

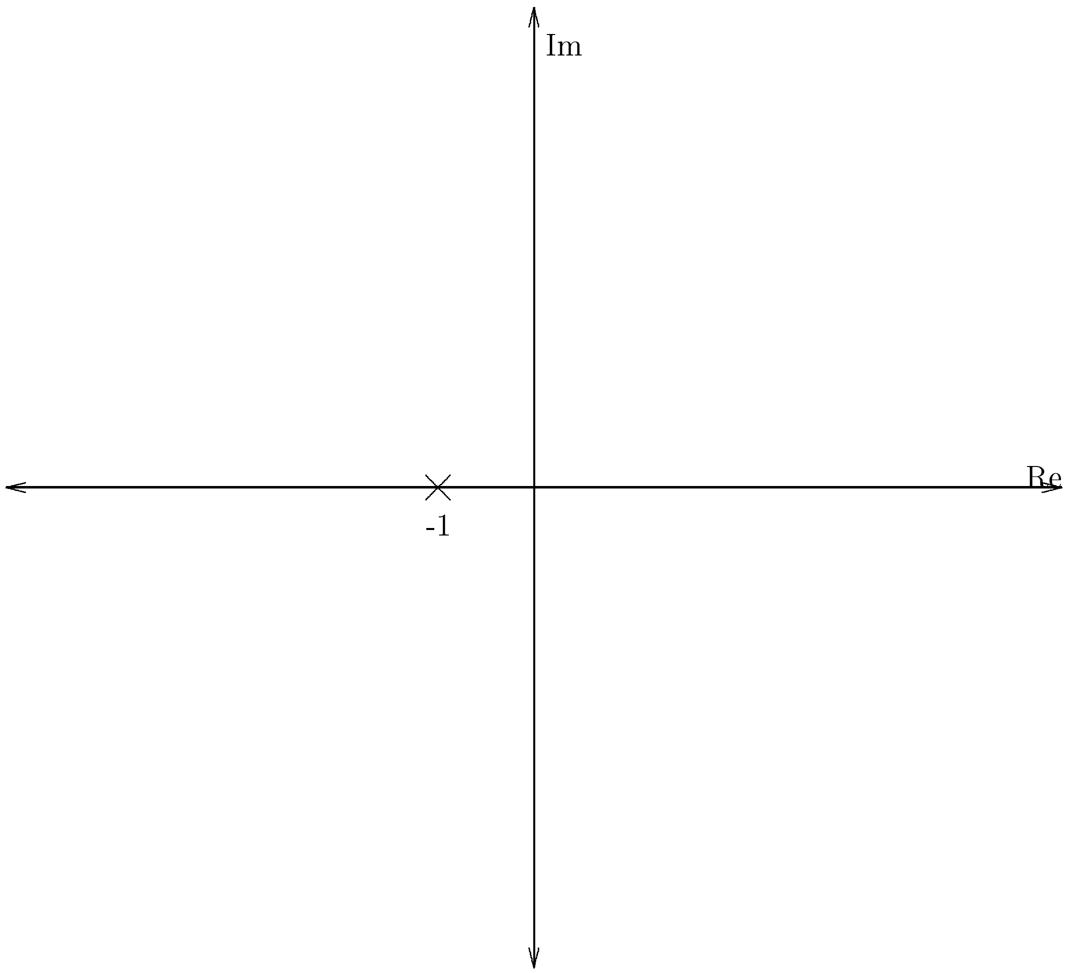
Phase Plot for Problem 2

Phase (degrees)



Frequency (log scale)

Nyquist Plot for Problem 2



- (5 points) What is the range of gain K for which $\frac{KP(s)}{1+KP(s)}$ is stable?

Range of K is ____________________

Problem 3. (20 points) A laboratory system is described by the following set of ordinary differential equations

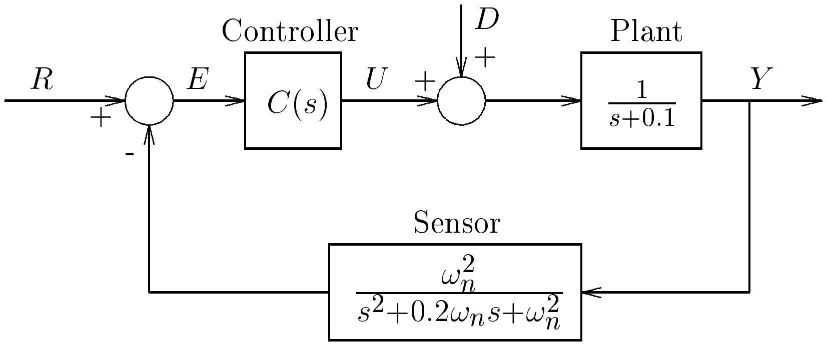
$$\begin{aligned} \dot{x}_1 &= x_2 + u \\ \dot{x}_2 &= -x_1 + u \\ y &= x_1 + 2x_2 \end{aligned}$$

where x_1 and x_2 are the states, u is an input, and y is an output.

1. (5 points) What is $P(s) = \frac{Y(s)}{U(s)}$?

2. (15 points) Compute the state feedback $u = -k_1 x_1 - k_2 x_2$ that places both of the poles at $s = -1$.

Problem 4. (30 points) In the following control system,



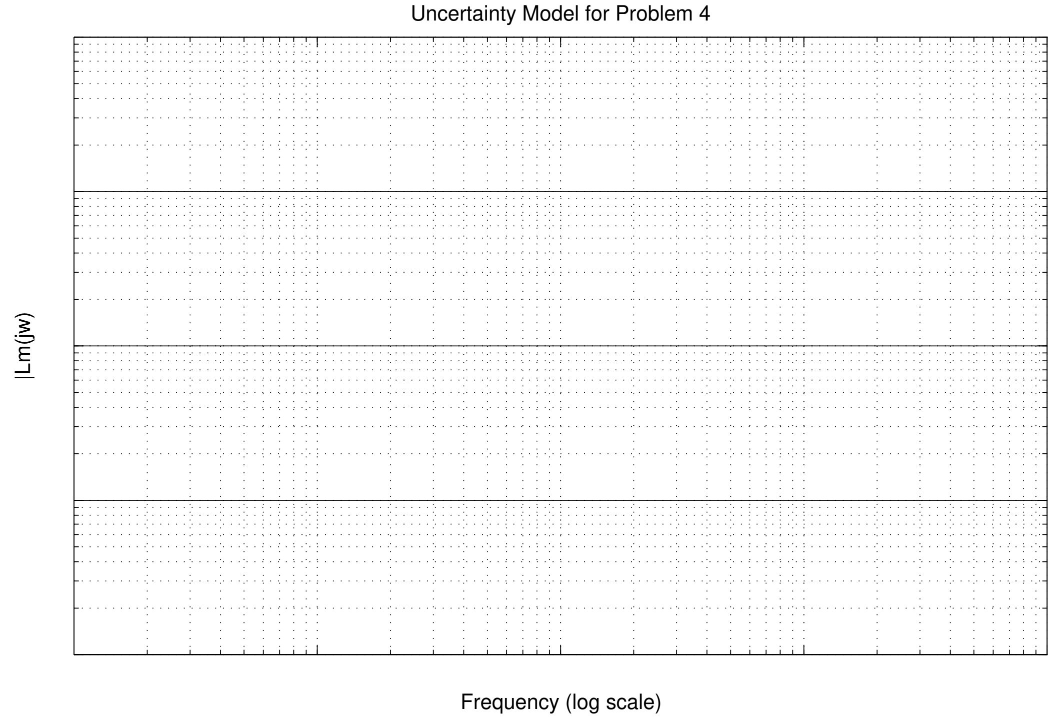
a precise numerical value for ω_n is unknown. However, it does satisfy

$$10 \leq \omega_n \leq 30.$$

Define $P(s) = \frac{1}{s+0.1}$ and $\tilde{P}(s) = P(s) \cdot \frac{\omega_n^2}{s^2+0.2\omega_n s+\omega_n^2}$.

1. (2 points) Using the multiplicative uncertainty model, $\tilde{P}(s) = P(s)(1 + L_m(s))$, determine $L_m(s)$ as a function of ω_n.

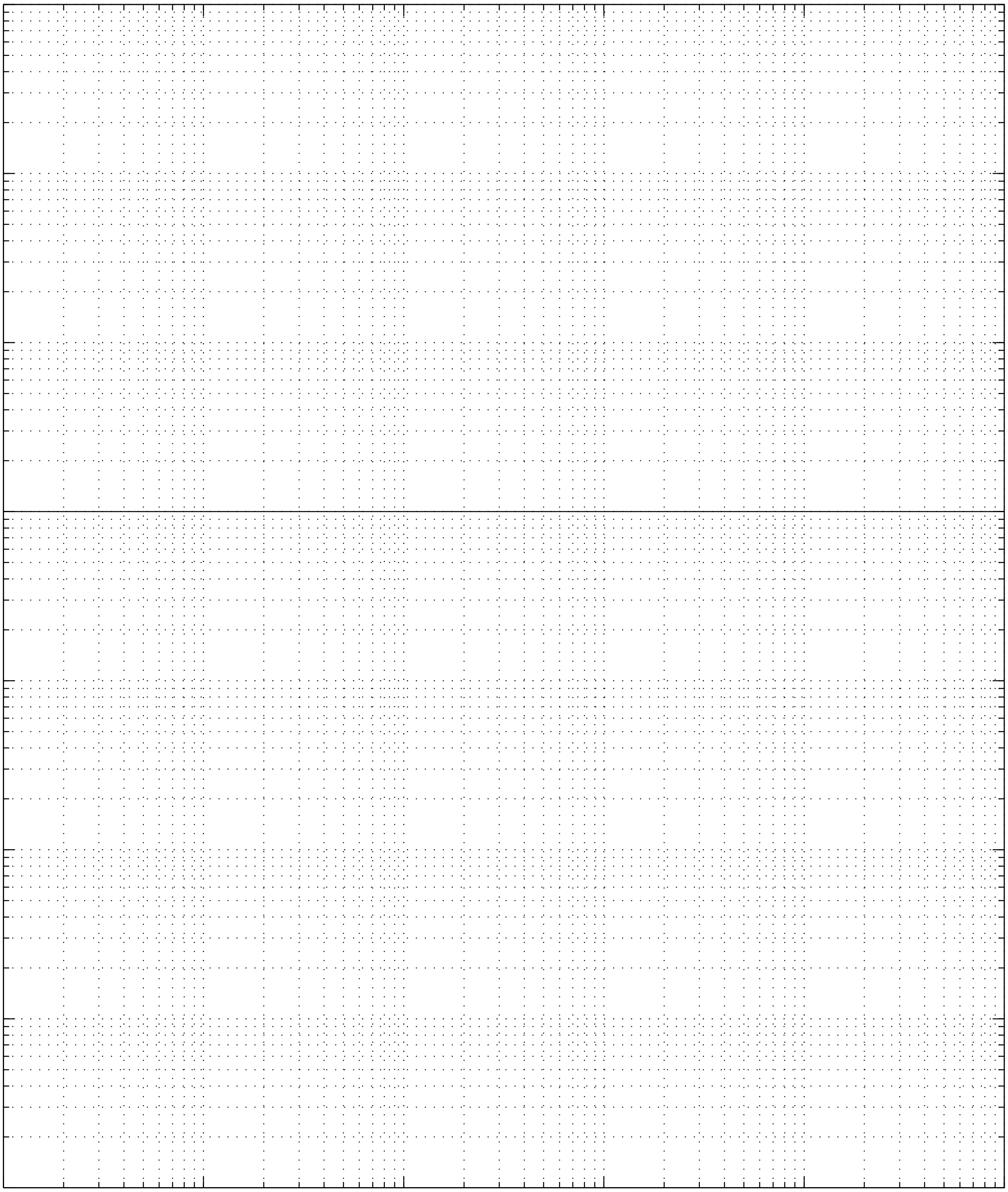
2. (5 points) On the next page, plot $|L_m(j\omega)|$ for $\omega_n = 10$ and $\omega_n = 30$.



3. (3 points) Determine a continuous, **piecewise linear** function $l_m(\omega)$ that satisfies $|L_m(j\omega)| < l_m(\omega)$ for all $\omega \geq 0$. You may use only **two** linear (straight-line) segments. **Draw $l_m(w)$ on the plot above**.

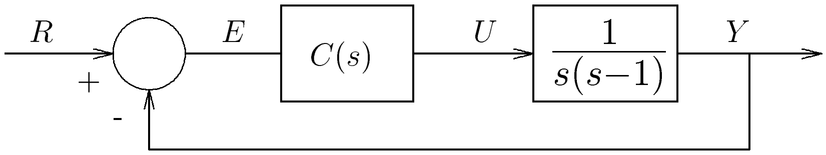
4. (20 points) Design a **second-order** controller $C(s)$ so that the system satisfies the following specifications.
   (a) $\frac{Y(s)}{R(s)}$ must be stable, for any $10 < \omega_n < 30$.
   (b) $\frac{Y(s)}{R(s)}$ must track a ramp input $r(t) = t$ with 0 steady-state error, (when $d(t) = 0$).
   (c) In the **steady-state**, the tracking error $e(t)$ must satisfy $\max_{t>0} |e(t)| < 0.316$ when $r(t) =$ unit step and $d(t) = \sin(\omega t)$, for all $\omega < 0.06$.
   (d) The phase margin θ_m for $C(s) \cdot P(s)$, the *nominal* open-loop system, must satisfy $50° < \theta_m < 70°$.
   (e) The closed-loop bandwidth must be as large as is reasonably possible.

The complete design must be done on the Bode magnitude plot on the next page.



Work for Problem 4

Problem 5. (20 points) For the control system



where $P(s) = \frac{1}{s(s-1)}$, design a first-order controller

$$C(s) = K\,\frac{1 + s/a}{1 + s/b},$$

i.e., determine numerical values for K, a and b, to satisfy the following.

1. The closed-loop system must be stable.
2. The phase margin θ_m must satisfy $50° < \theta_m < 70°$.
3. The phase of $C(s) \cdot P(s)$ must be maximized at unity cross-over.
4. The closed-loop bandwidth ω_b must satisfy $5 < \omega_b < 15$.
5. The controller parameters must satisfy:

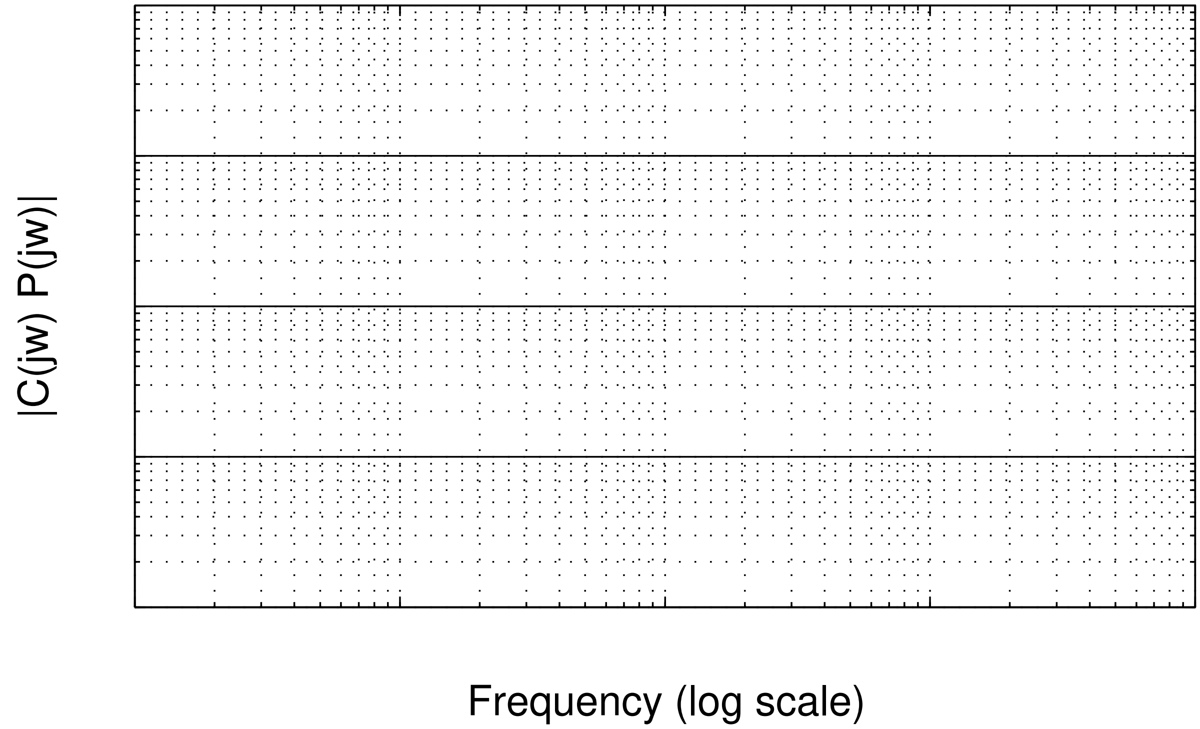
$$\begin{gathered} 0 < K < 10 \\ 0 < a < 50 \\ 0 < b < 50. \end{gathered}$$

To receive full marks:

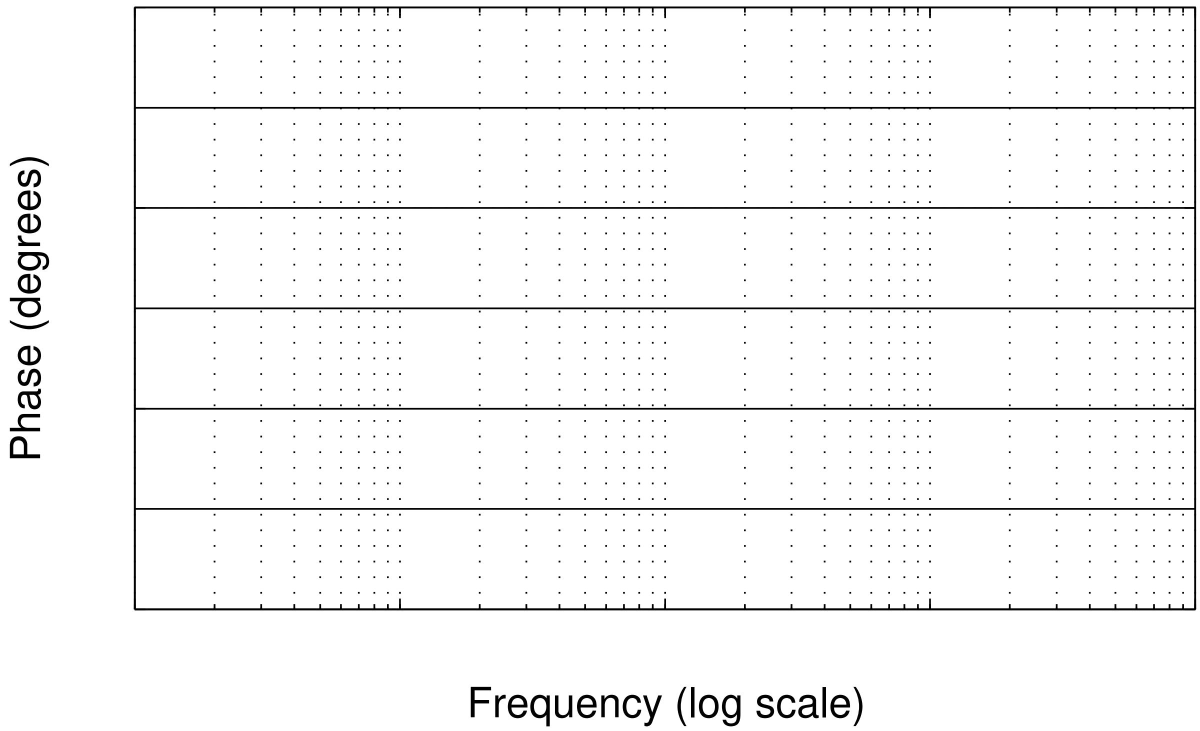
- (5 points) Sketch the magnitude and phase of $|P(j\omega)|$ and $|C(j\omega)P(j\omega)|$ on the next page (correct asymptotic plots will receive full marks); and
- (5 points) Sketch the Nyquist plot of $C(j\omega)\,P(j\omega)$ on page 12; and
- (10 points) Compute values for K, a and b.

Hint: You may find a root-locus helpful.

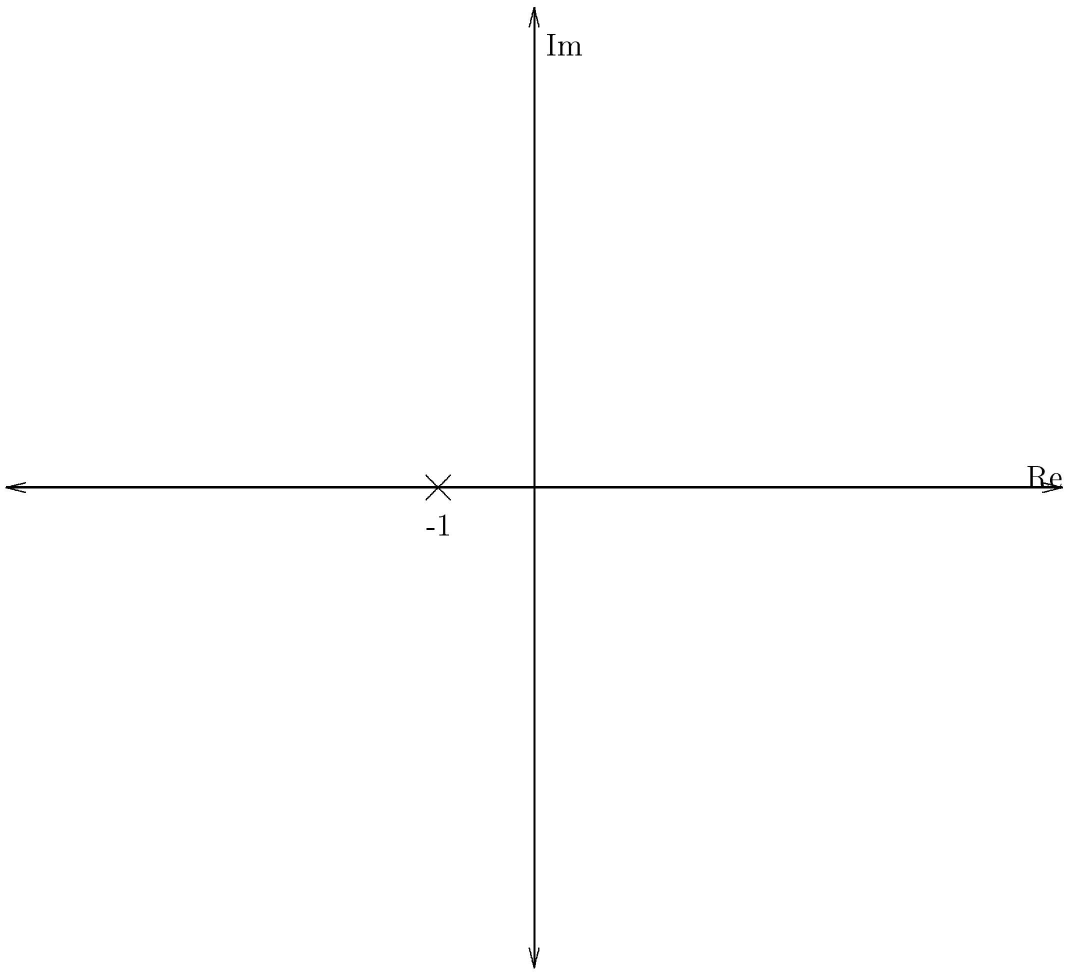
Magnitude Plot for Problem 5



Phase Plot for Problem 5



Nyquist Plot for Problem 5



Show design work here and on next page:

Design work for Problem 5

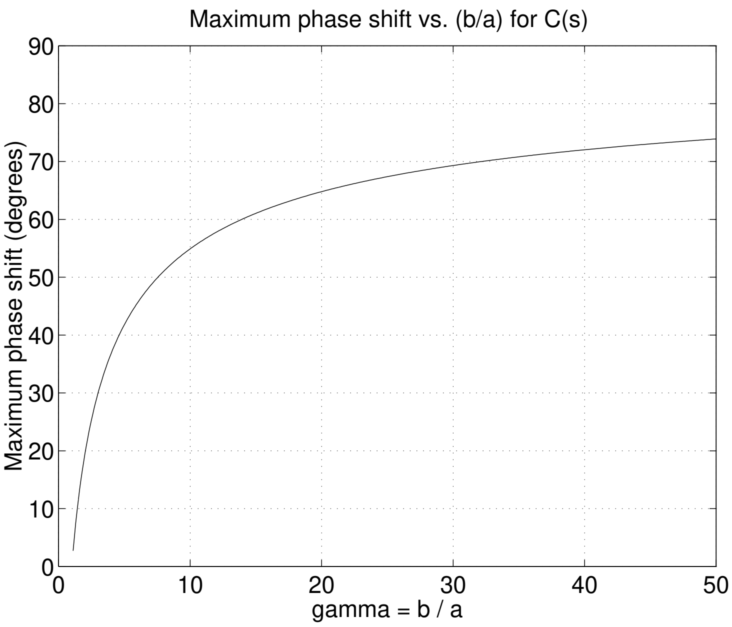
The following formulae are for the second order system

$$M(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

$$\begin{aligned}
\text{percent overshoot} &= 100e^{-\frac{\delta\pi}{\sqrt{1-\delta^2}}} && (1)\\
\text{damped frequency} &= \omega_d = \omega_n \cdot \sqrt{1-\delta^2} && (2)\\
\text{settling time} &= t_s = \frac{3}{\delta\omega_n} && (3)\\
\text{peak magnitude} &= \mathrm{M}_p = \frac{1}{2\delta\sqrt{1-\delta^2}} && (4)\\
\text{peak frequency} &= \omega_p = \omega_n\sqrt{1-2\delta^2} && (5)\\
\text{rule of thumb... } \mathrm{M}_p &\simeq \frac{1}{\sin(\theta_m)} && (6)
\end{aligned}$$

Miscellaneous information...

- Solutions to $as^2 + bs + c = 0$ are $s = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.
- For a first-order lead controller $C(s) = \frac{1+s/a}{1+s/b}$, with $b > a$, the maximum phase shift occurs at $\omega = \sqrt{ab}$. Below is a plot of this maximum versus the ratio b/a.



Magnitude and phase for the second-order system

$$M(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

are shown below, for $\omega_n = 1.0$ and $\delta = 0.01,\ 0.1,\ 0.25,\ 0.5,\ 0.75.$