UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION
ECE 410 SYSTEMS CONTROL I

December 12, 1995

Examiner: Professor Scott A. Bortoff

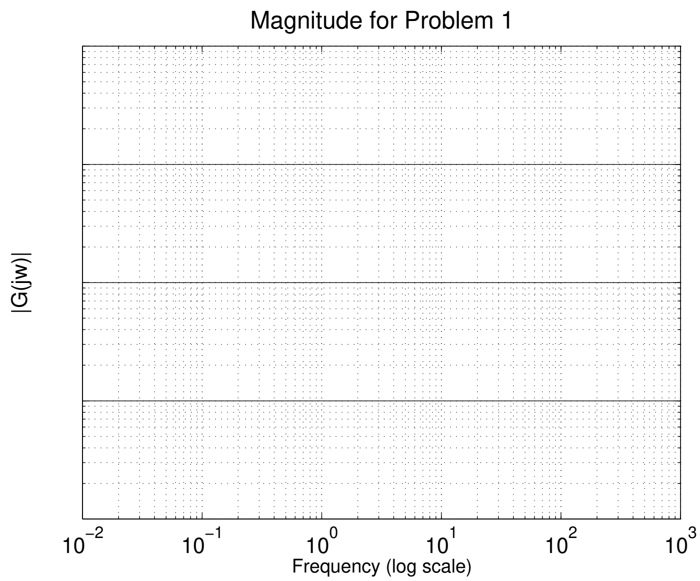
RULES OF EXAMINATION

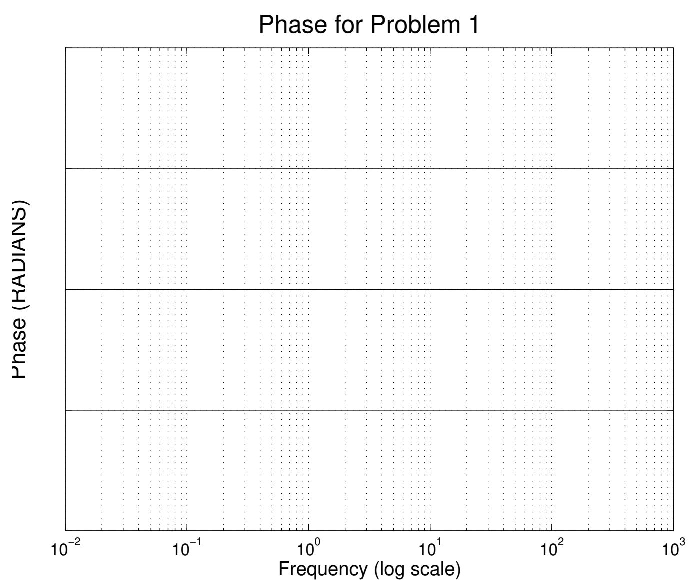
1. All answers are to be written **ON THIS EXAMINATION**, and **NOT** in the "TERM" booklet, which can be used as scratch paper.
2. This exam is type "C," a closed-book examination. You may use a single yellow aid sheet, supplied by the examiner or the Registrar's Office.
3. You may use pen, pencil, and drafting instruments such as a ruler.
4. You may use an electronic calculator.
5. Read through the examination before beginning. There are 11 pages, some of which have useful plots and formulae.

Problem 1. (10 points). On the two axes below, plot the magnitude and phase response of

$$G(s) = \frac{100s}{s^2 + 14s + 100}$$

for the frequency range $0.01 \leq \omega \leq 1000$ radians per second. If $G(s)$ were realized as a circuit, what mathematical operation would it approximate for low frequencies?

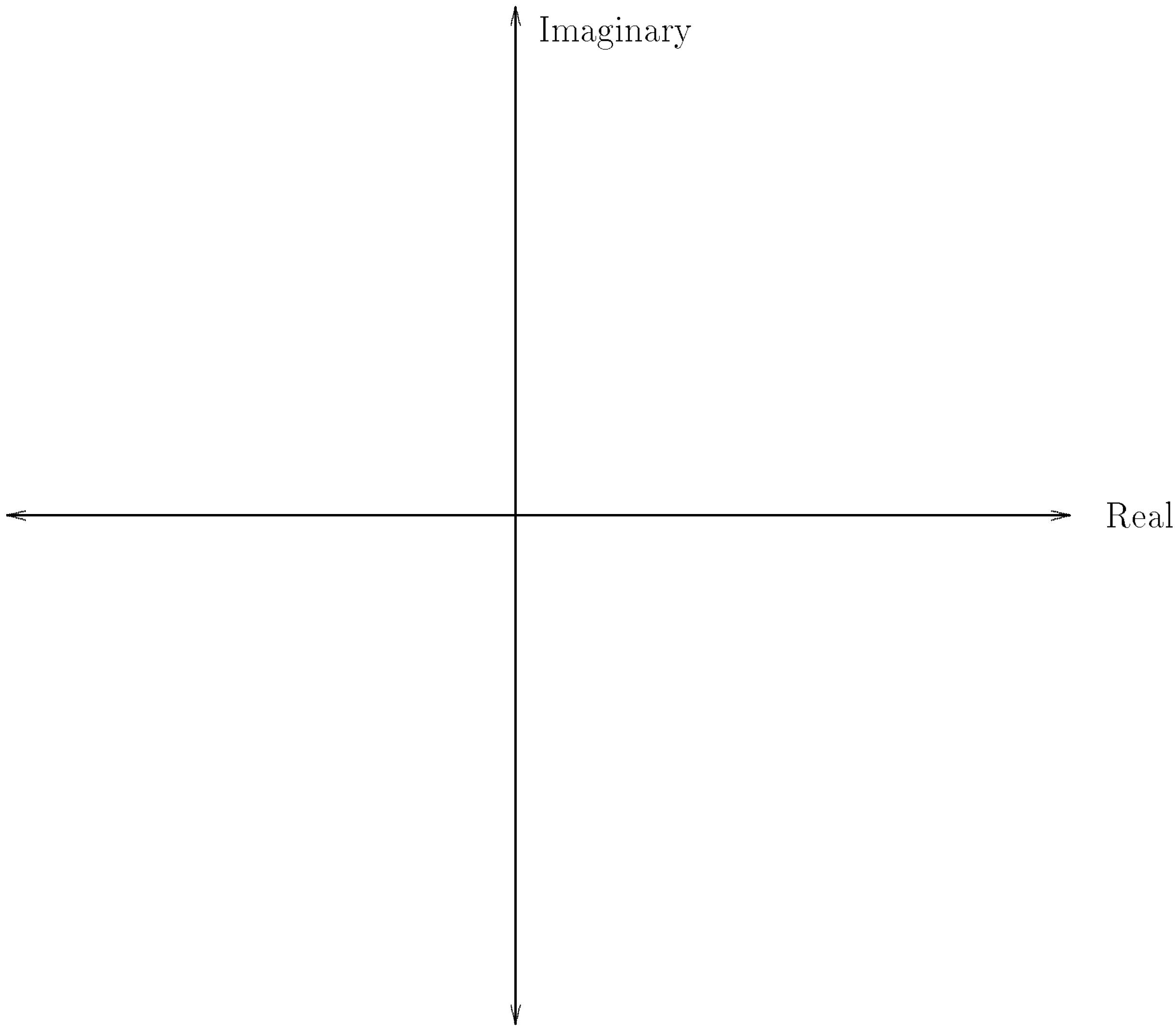




Operation?

Problem 2. (10 points). On the axes below, sketch the root-locus ($K > 0$) for

$$P(s) = \frac{(s^2 + 1)}{(s^2 + 3s + 2)(s^2 - 3s + 2)}$$



Is the closed-loop system

$$T(s) = \frac{K \cdot P(s)}{1 + K \cdot P(s)}$$

stable for any $K > 0$? If yes, what are the values of $K > 0$ that make it stable?

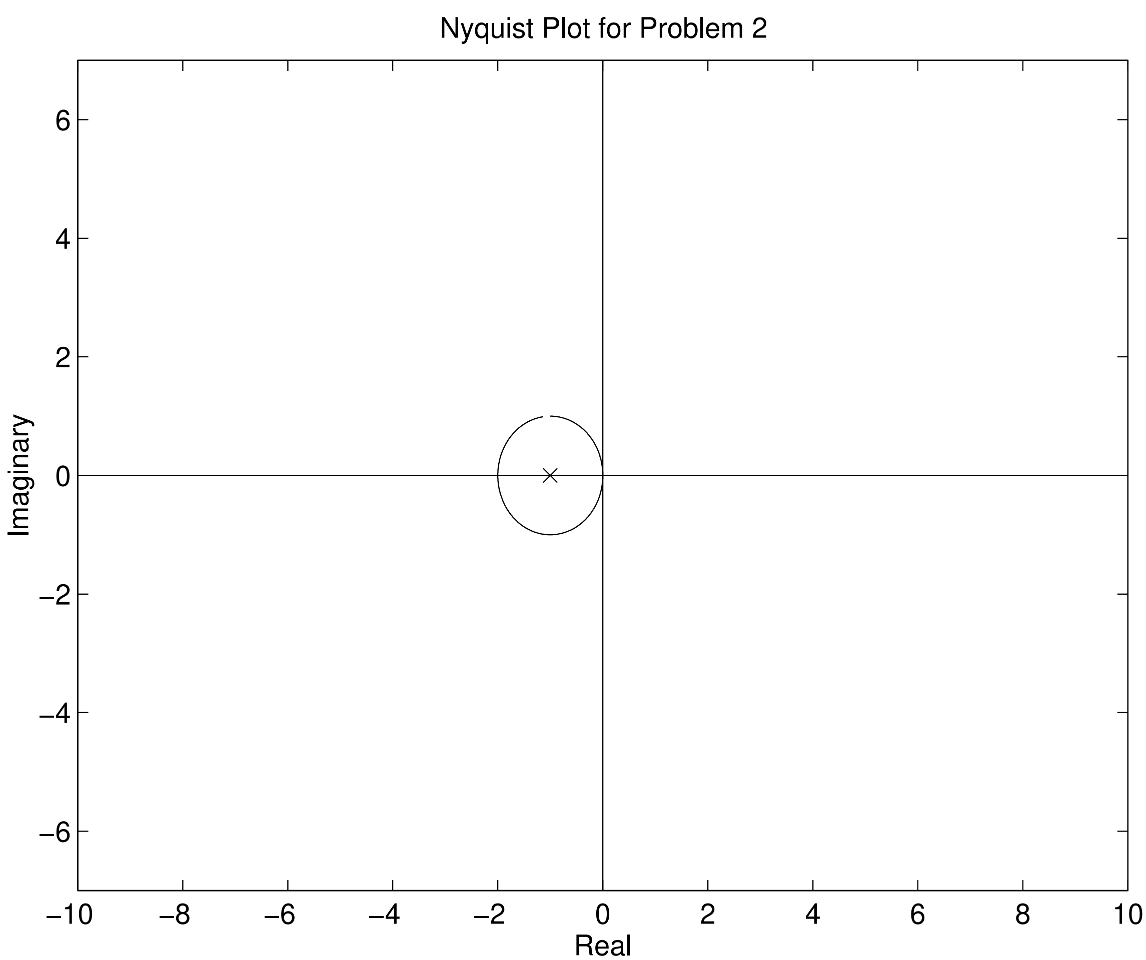
Problem 3. (20 points). For the transfer function

$$G(s) = \frac{4(s+1)}{s^2(s^2+s+10)},$$

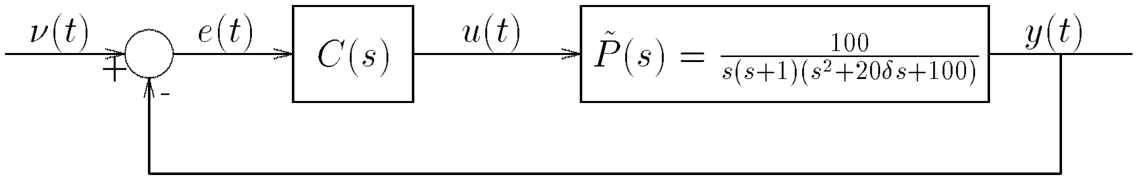
1. (10 points). Sketch the Nyquist plot, including the behavior at infinity. How many net encirclements of $s = -1$ are there? What interpretation does this interger have in terms of closed-loop stability?

2. (10 points). For what values of K is the following system stable:

$$\frac{K \cdot G(s)}{1 + K \cdot G(s)}?$$



Range of K?



Problem 4. (30 points) The figure above is a block diagram of a servomotor connected to a flexible shaft. The actual plant $\tilde{P}(s)$ is known to be of the form

$$\tilde{P}(s) = \frac{1}{s(s+1)} \cdot \frac{100}{s^2 + 20\delta s + 100}.$$

Where the damping term δ is known to satisfy

$$\frac{1}{2} \leq \delta \leq 1,$$

but is not known precisely.

1. (10 points) In order to design a robust controller, we must express the plant uncertainty in the multiplicative perturbation form $\tilde{P}(s) = P(s)(1+L_m(s))$ for some $L_m(s)$. If

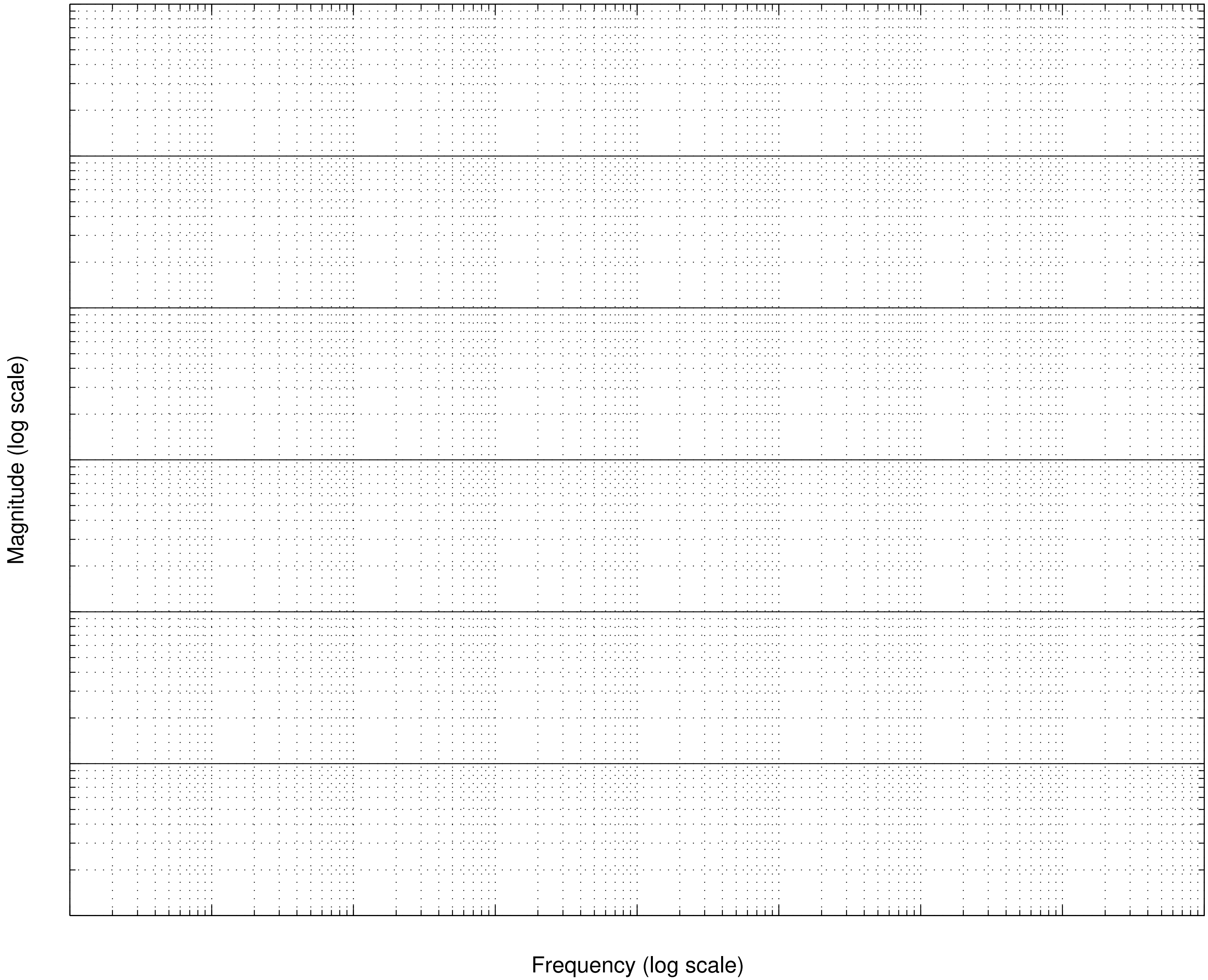
$$P(s) = \frac{1}{s(s+1)},$$

then what is $L_m(s)$?

$$L_m(s) = \underline{\hspace{6cm}}$$

For the two extreme values of δ, and on the same axis on the next page, sketch the Bode magnitude plots of $L_m(s)$.

Magnitude Plot of |Lm(jw)| for Problem 4



2. (10 points) Suggest a first-order, rational function $l_m(\omega)$ which satisfies

$$|L_m(j\omega)| < l_m(w)$$

for all ω and all δ satisfying $0.5 < \delta < 1$. Draw your proposed $l_m(w)$ on the same axes as the plots of $|L_m(j\omega)|$ from item 1.

$$l_m(\omega) = \underline{\hspace{4cm}}$$

3. (10 points) Using a Bode magnitude plot, design a first-order Lag-type control $C(s)$ for the system such that all of the following specifications are satisfied:

   - The controller $C(s)$ must be first-order, Lag-type, with

     $$|C(j\omega)| \leq 1$$

     for all finite, nonzero frequencies ω;
   - The phase margin θ_m must satisfy

     $$50° < \theta_m < 80°;$$

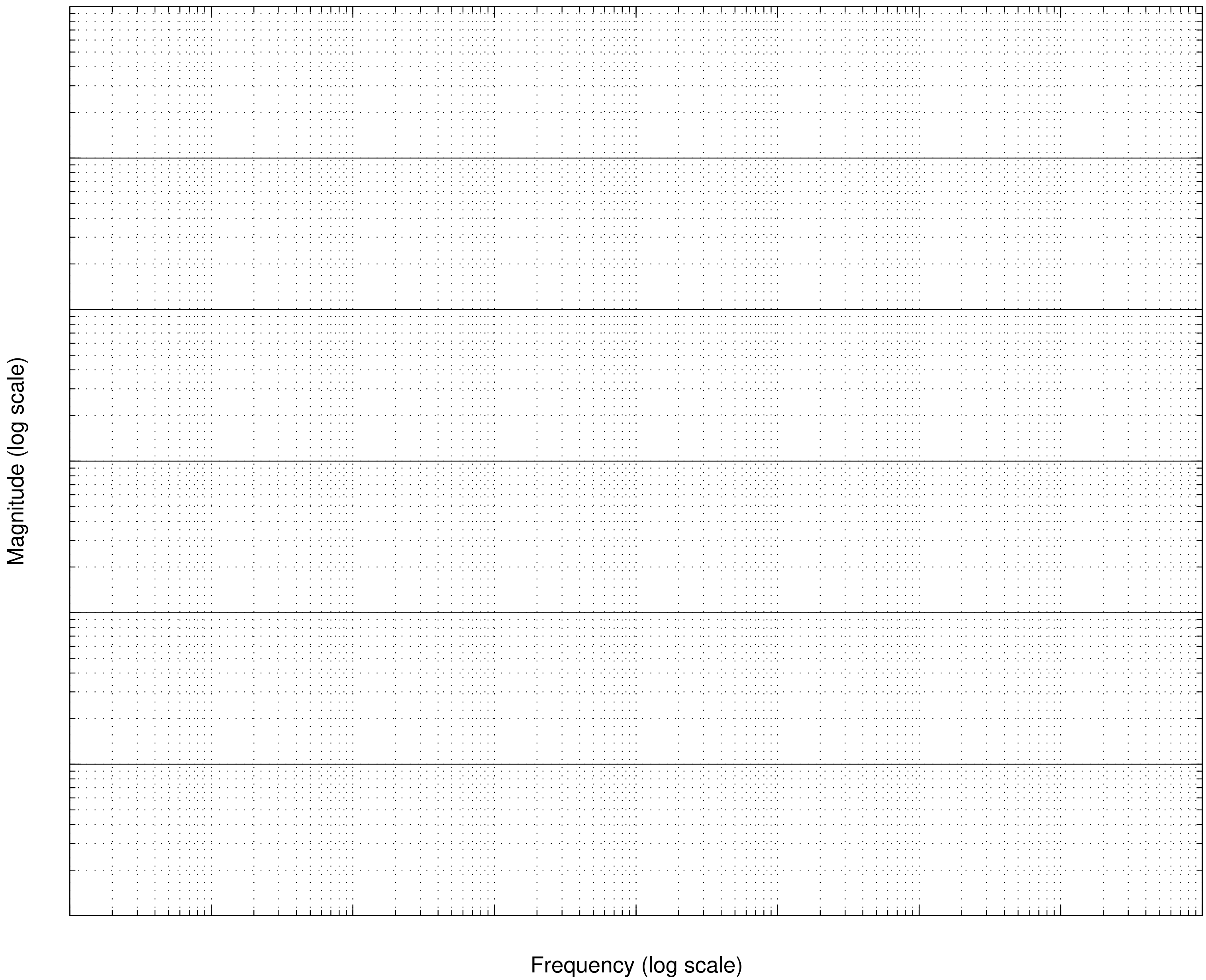
   - The maximum tracking error, $e_{max} = \max |e(t)|$, must satisfy

     $$e_{max} < \frac{1}{316}$$

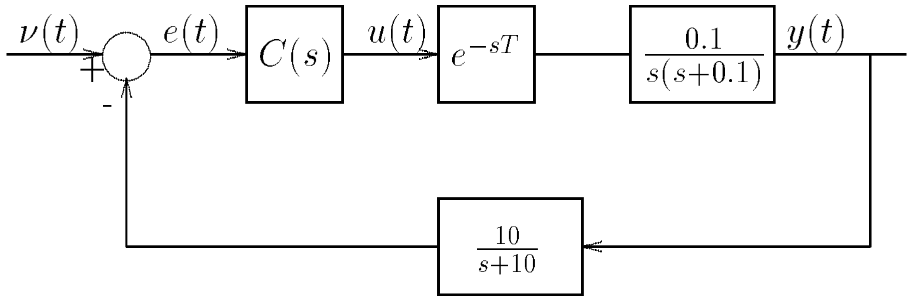
     when $\nu(t) = \sin(\omega t)$, for all $\omega < 0.001$;
   - The bandwidth of the closed-loop system, ω_b, must satisfy $0.1 < \omega_b$;
   - The controller must meet all performance specifications robustly, using the multiplicative uncertainty bound $l_m(\omega)$ from item 2.

   To receive full credit, your design must include one Bode magnitude plot showing $l_m^{-1}(w)$, the tracking performance specification, and the designed loop transfer function $C(s) \cdot P(s)$.

Magnitude Plot for Problem 4



Problem 5. (30 points) For the following control system, the delay T is unknown, but satisfies $0 \leq T \leq 1$.



It is also known that

$$|e^{-j\omega T} - 1| < \frac{3\omega}{\omega + 1}$$

for all $\omega > 0$ and for all $0 \leq T \leq 1$. Design a controller $C(s)$ to satisfy the following specifications.

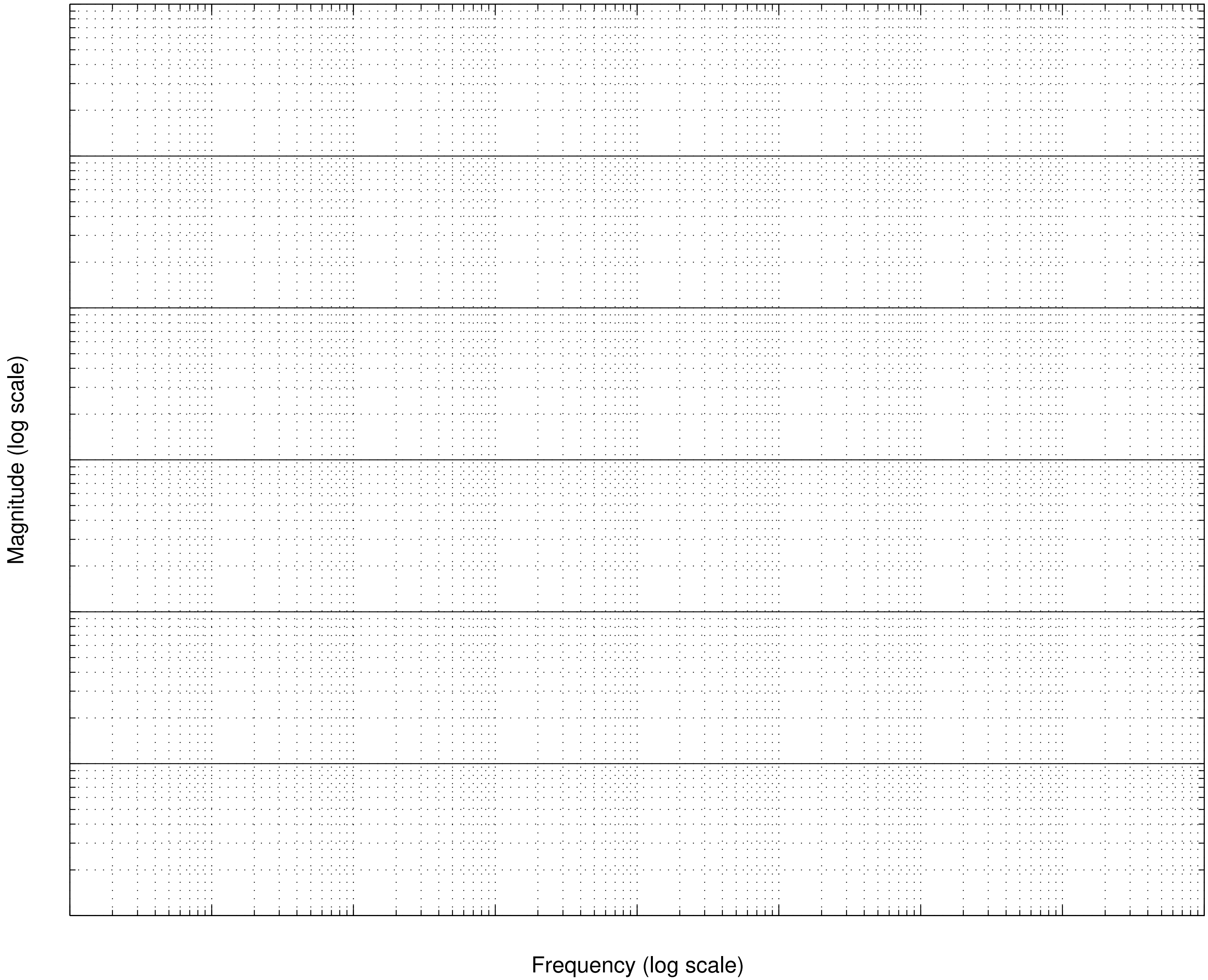
1. $C(s)$ must be stable, proper, and no greater than fourth-order.
2. The phase margin θ_m must satisfy

$$50^\circ \leq \theta_m \leq 80^\circ.$$

3. The system must track a step input at $\nu(t)$ with zero steady-state error, for all $0 \leq T \leq 1$.
4. The closed-loop system must track a sinusoidal input $\nu(t) = \sin(\omega t)$ with $\max|e(t)| \leq 0.01$ for all $\omega < 0.00316$ and all $0 \leq T \leq 1$.
5. The closed-loop system must have a bandwidth of at least 0.01 radians per second.

To receive full credit, your complete design must appear on a single Bode magnitude plot.

Magnitude Plot for Problem 5



The following formulae are for the second order system

$$H(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

$$\begin{aligned}
\text{percent overshoot} &= 100e^{-\frac{\delta\pi}{\sqrt{1-\delta^2}}} && (1)\\
\text{damped frequency} &= \omega_d = \omega_n \cdot \sqrt{1-\delta^2} && (2)\\
\text{settling time} &= t_s = \frac{3}{\delta\omega_n} && (3)\\
\text{peak magnitude} &= \mathrm{M}_p = \frac{1}{2\delta\sqrt{1-\delta^2}} && (4)\\
\text{peak frequency} &= \omega_p = \omega_n\sqrt{1-2\delta^2} && (5)\\
\text{rule of thumb... } \mathrm{M}_p &\simeq \frac{1}{\sin(\theta_m)} && (6)
\end{aligned}$$