ECE 1649 — Adaptive Control
Project 2
Monday February 10, 1997

Problem 1. (50 points) Consider the following unstable plant:

$$\hat{P}(s) = \frac{\hat{y}_p(s)}{\hat{u}(s)} = \frac{s+2}{(s-1)(s^2+s+10)}. \tag{1}$$

Design a model reference adaptive controller for this problem, using the output-error method developed in class. Assume that the plant order is known (it is 3), the relative degree is known (it is 2), and that the *sign* of the high frequency gain, k_p, is known (it is positive). The plant's six other parameters are assumed to be unknown to the controller. ($\hat{P}(s)$ is provided so you may conduct simulation experiments.) Let the reference model be

$$\hat{M}(s) = \frac{\hat{y}_m(s)}{\hat{r}(s)} = \frac{4}{(s+2)^2}. \tag{2}$$

Simulate the system for to the following two reference inputs:

- $r(t) = \sin(2t)$, and
- $r(t) =$ square wave of period π seconds.

For all simulations, use a *normalized* gradient for the parameter estimation tuning function. Show simulation plots of (only) the output errors, $r(t)$, the plant outputs, the parameters, and the Euclidean norm of the parameter error.

Problem 2. (10 points) Simulate exactly the same controller that you derived in Problem 1, for the plant

$$\hat{P}(s) = \frac{\hat{y}_p(s)}{\hat{u}(s)} = \frac{s+2}{(s+1)(s^2+s+10)}. \tag{3}$$

This should involve only slight modification of the simulation code describing the plant dynamics; the controller dynamics should be unchanged. Comment on the differences in the two sets of simulations.

Problem 3. (40 points) The output error MRAC considered in Problems 1 and 2 tunes six parameter estimates $\theta_1(t), \ldots, \theta_6(t)$, plus one estimate of the high-frequency gain, $k_1(t)$. For the each plant (1) and (3), compute the set of "true" controller parameters $\theta_1^*, \ldots, \theta_6^*, k_1^*$ that give perfect model matching. Prove that these are unique. Is your answer a function of the state-variable filter $1/\lambda(s)$? Compute the transfer function $\hat{y}_p(s)/\hat{r}(s)$ when $\theta_i(t) = \theta_i^*$, $i = 1, \ldots, 6$ and $k_1(t) = k_1^*$. How many pole-zero cancelations are there?