ECE 1648 Problem Set 3
February 1, 1999

Problem 11. Consider the linear system

$$\frac{Y(s)}{U(s)} = \frac{K \cdot (s^{n-r} + b_{n-r-1}s^{n-r-1} + \cdot + b_1 s + b_0)}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}.$$

Suppose the numerator and denominator polynomials are relatively prime (no pole-zero cancellations).

1. Express the system in controllable canonical form. Write - out A, b, c, when the system is realized as

$$\begin{aligned} \dot{x} &= Ax + bu \\ y &= cx. \end{aligned}$$

   (Hint: the vectors b and c should be $b = [0\ 0\ \cdots\ 0\ K]^T$, $c = [b_0\ b_1\ \cdots\ b_{n-r-1}\ 0\ \cdots\ 0]$.)

2. Compute the normal form. What is the coordinate transformation $z = Tx$ from controllable canonical form to normal form, in terms of the numerator and denominator coefficients.

3. The normal form can be written

$$\begin{aligned} \dot{\boldsymbol{\xi}} &= R\boldsymbol{\xi} + S\boldsymbol{\eta} + \bar{K}u \\ \dot{\boldsymbol{\eta}} &= P\boldsymbol{\xi} + Q\boldsymbol{\eta} \end{aligned}$$

   where $\bar{K}$, R, S, P and Q are vectors and matrices of appropriate dimension. What are the zero dynamics? Prove that the eigenvalues of P are exactly the r zeros of the original system. If we define $u = -(R\boldsymbol{\xi} + S\boldsymbol{\eta})/K$, then where are the closed-loop poles and zeros?

Problem 12. For the rotational inverted pendulum defined in Problem 10, consider the pendulum link angle q_2 as the output. That is, define the scalar output function as $y = h(\boldsymbol{x}) = x_2$, where $\boldsymbol{x} = [q_1\ q_2\ \dot{q}_1\ \dot{q}_2]^T$. Now, linearize the input-output dynamics. What is the relative degree? Are the zero dynamics stable? Using your favorite simulation package, simulate the system for the initial condition $\boldsymbol{x}_0 = [0\ 0.1\ 0\ 0]^T$. (Note that the origin is when the pendulum is inverted, at its unstable equilibrium.)