A Noncooperative Game Approach to OSNR Optimization in Optical Networks

Lacra Pavel

Abstract—**We consider a game theory framework for power control in optical networks. Channel optical signal-to-noise ratio (OSNR) optimization is formulated as an m-player noncooperative game, based on a general network OSNR model. Conditions for existence and uniqueness of the game equilibrium solution are given. An iterative algorithm for power control is proposed, that uses only channel specific feedback measurements and is shown to converge to the equilibrium solution.**

Index Terms—**Game theory, iterative power control, Nash games, optical networks.**

I. INTRODUCTION

Optimization-based approach for control of communication networks has attracted considerable attention in recent years. Game-theoretic models have been employed for flow optimization (congestion control) and power control allocation in wireless networks. There exists a rich literature on these problems as reflected in works such as [2]–[4], [8], [10], [20].

Control of optical networks via an optimization-based approach is a new area for which there is not much known to date. This area is placed in the context of evolution of optical communications from statically designed, point-to-point links, toward wavelength-division multiplexed (WDM) networks. Essential problems are stability and optimal channel performance in optical networks of general topology, online reconfigurable [12], [19]. These are some of the interesting questions that can be formulated and addressed within the rigorous control theory approach, [14], [15].

In this note we formulate a game-theoretic model for optical networks, specifically related to channel optical signal-to-noise ratio (OSNR) optimization. We introduce the problem from the mathematical model and game formulation. We follow by establishing existence of an equilibrium and giving conditions for its uniqueness, and then developing iterative control algorithms. Existent OSNR approaches, either static [7] or heuristic online procedures, [6], are developed for single links, not appropriate for reconfigurable optical networks. In large-scale optical networks it is difficult to maintain a centralized information system. In this sense cooperation among channels is impractical, and this leads to our approach based on noncooperative game theory [5] as a suitable framework. This problem is similar to power control via noncooperative game approaches in wireless networks, a topic which has been explored extensively in works such as [2], [8], and [20]. The problem needs to be formulated and addressed separately for optical networks, as there are several differences when compared to wireless networks [2], [8]. In wireless networks, channels are characterized by uncoupled loss/gain with single Tx to Rx links. Optical networks have specific features: cascaded amplified spans, accumulation and self-generation of optical noise, cross-talk and possible coupling and saturation [1].

Considering these differences and specific physical features of optical networks, relevant questions that we address in this note are: what is a natural way of formulating a tractable OSNR optimization game in optical networks of general topology? How is this game compared to the SIR optimization game in wireless networks? Is there a natural decomposition that leads to iterative algorithms with provable convergence, decentralized with respect to channels?

The main contribution of this note is to formulate and efficiently solve a noncooperative game in optical networks, toward optimizing channel OSNR. This appears to be the first work on this problem, for which a short version appeared in [16]. We concentrate firstly on problem formulation, derivation of the mathematical model and justification of the assumptions. We show that although based on different physical mechanisms, the OSNR model in optical networks is mathematically similar to the SIR model in wireless networks [2], [8], but has a richer system matrix structure for which the results from wireless cannot be applied. Conditions for uniqueness of the Nash equilibrium as well as proof techniques and conditions for selecting pricing parameters are different, in this sense generalizing the results in [2]. Our work provides a starting point for future contributions in the area of optical network control.

The note is organized as follows. In Section II, we develop a network OSNR model. In Section III, we formulate a noncooperative game between channels toward maximizing OSNR. We define channel utility as a logarithmic function that reflects channel's preference for OSNR, and a pricing function linear in transmitter power. We explicitly characterize the Nash equilibrium and give conditions for its uniqueness for the general network model. In Section IV we introduce an iterative power control algorithm that converges to the Nash equilibrium, and we discuss some pricing strategies. In Sections V and VI, we give a numerical example and conclusions.

II. NETWORK OSNR MODEL

Consider a network with a set of optical links $\mathcal{L} = \{1, \ldots, L\}$ connecting the optical nodes, where channel add/drop is realized. A set $\mathcal{M} = \{1, \ldots, m\}$ of channels are transmitted, corresponding to a set of multiplexed wavelengths. A link l has N_l cascaded optically amplified spans. Let $\mathcal{M}_l$ be the set of channels transmitted over link l. For a channel $i \in \mathcal{M}$ we denote by $\mathcal{R}_i$ its optical path, or collection of links, from source (Tx) to destination (Rx). Let u_i be the ith channel input optical power (at Tx), and $\mathbf{u} = [u_1, \ldots, u_m]^T$ the vector of all channels' input powers. Let s_i be the ith channel output power (at Rx), and n_i the optical noise power (in the ith channel bandwidth) at Rx. The ith channel optical signal-to-noise ratio (OSNR) is defined as $OSNR_i = s_i/n_i$. We develop a network OSNR model, extending the single link model in [7].

Each kth span in lth link is composed of optical fiber with wavelength independent loss $L_{l,k}$, and an optical amplifier with wavelength dependent gain $G_{l,k,i}$. Channel i transmission is [7]

$$h_{l,k,i} = G_{l,k,i}\, L_{l,k} \qquad \forall k = 1, \ldots, N_l. \tag{1}$$

Optical amplifiers simultaneously amplify many channels, but introduce spontaneous emission noise (ASE), [21]. ASE is wavelength-dependent and accumulates across cascaded amplifiers. At kth amplifier on lth link, ASE power for the ith channel is given as [1]

$$ASE_{l,k,i} = 2n_{sp}\,[G_{l,k,i} - 1]\, h\nu_i\, B_o \tag{2}$$

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The author is with Department of Electrical and Computer Engineering, University of Toronto, Toronto, ON M5S 3G4, Canada (e-mail: pavel@control.toronto.edu).

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where $n_{sp} > 1$ is amplifier excess noise factor, h is Planck constant, B_o is optical bandwidth, and ν_i is optical frequency corresponding to wavelength i. As in [7] we assume that

A.i.1) ASE noise power does not participate in amplifier gain saturation.

If noise power at amplifier input is $v_{in,i}$, then noise power at amplifier output is [1], [7]

$$v_{out,i} = v_{in,i}\, G_{l,k,i} + ASE_{l,k,i}. \tag{3}$$

We consider only forward propagation of signal and noise in steady-state, [7], i.e., we do not consider amplifier gain dynamics, [21], [18]. This is justified for an OSNR optimization problem where power adjustments need to be made at steady-state, after updating network topology.

Lemma 1: Let $p_{l,k,i}$, $v_{l,k,i}$ denote signal and noise power, respectively, at the output of the kth span on lth link. Let u_i, $n_{0,i}$ denote the ith channel's signal and noise power, respectively, at Tx. Under (A.i.1), the ith channel OSNR, along a path $\mathcal{R}_i$, is given as

$$OSNR_i = \frac{u_i}{n_{0,i} + \sum\limits_{l\in\mathcal{R}_i}\sum\limits_{k=1}^{N_l} \frac{1}{\prod_{q=1}^{l-1}\mathbf{T}_{q,i}} \frac{ASE_{l,k,i}}{\mathbf{H}_{l,k,i}}} \tag{4}$$

while

$$\begin{aligned} p_{l,k,i} &= p_{l,0,i}\mathbf{H}_{l,k,i} \\ v_{l,k,i} &= v_{l,0,i}\mathbf{H}_{l,k,i} + \sum_{r=1}^{N_l} ASE_{l,r,i} \prod_{q=r+1}^{k} h_{l,q,i} \end{aligned} \tag{5}$$

where $\mathbf{H}_{l,k,i} = \prod_{q=1}^{k} h_{l,q,i}$, $\mathbf{T}_{l,i} = \mathbf{H}_{l,N_l,i}$, are the intermediary and full lth link transmission.

Proof: The proof follows by recursively using (1) and (3) (see [16] and [17]). ■

When the network is in steady-state, optical amplifiers are typically operated in automatic power control (APC) mode, [1]: The same total power is launched into each span of a link, this uniform distribution limiting the nonlinear effects [11]. In what follows, we also assume that

A.i.2) all the amplifiers in a link have the same spectral shape, with the same total power target.

A.i.2) is representative for uniformly designed optical links, [1]. All optical spans in a link use fiber with the same loss coefficient, hence the same span length is supported before optical amplification is needed. APC mode ensures equal span launching power $P_{0,l}$, selected below the threshold for nonlinear effects, [7], [11]. Then, at the output of each span

$$\sum_{j\in\mathcal{M}_l} p_{l,k,j} = P_{0,l} \qquad \forall\, l \in \mathcal{R}_i \quad \forall k = 1,\ldots,N_l. \tag{6}$$

This leads to a more complex mathematical model; the inherent scaling on the total power translates into coupling between all channels' powers; OSNR depends on all channels' powers.

Lemma 2: Under A.i.1) and A.i.2), the OSNR for the ith channel is given as

$$OSNR_i = \frac{u_i}{n_{0,i} + \sum\limits_{j\in\mathcal{M}} \Gamma_{i,j}\, u_j} \qquad \forall\, i \in \mathcal{M} \tag{7}$$

where the full $(m \times m)$ system matrix $\mathbf{\Gamma} = [\Gamma_{i,j}]$ is defined as

$$\Gamma_{i,j} = \sum_{l\in\mathcal{R}_i}\sum_{k=1}^{N_l} \frac{G_{l,j}^k}{G_{l,i}^k}\left(\prod_{q=1}^{l-1}\frac{\mathbf{T}_{q,j}}{\mathbf{T}_{q,i}}\right)\frac{ASE_{l,k,i}}{P_{0,l}}, \qquad j \in \mathcal{M}_l$$

and $\Gamma_{i,j} = 0$, if $j \notin \mathcal{M}_l$.

Proof: Using (5) and the notations in Lemma 1 recursively into (6) yields

$$\sum_{j\in\mathcal{M}_l}\left(\prod_{q=1}^{l-1}\mathbf{T}_{q,j}\right)\mathbf{H}_{l,k,j}\, u_j = P_{0,l} \qquad \forall\, l \in \mathcal{R}_i. \tag{8}$$

Since all amplifiers in a link have the same shape by A.i.2), $G_{l,k,i}$ can be decomposed as $G_{l,k,i} = G_{l,i}\cdot\alpha_{l,k}$, where $\alpha_{l,k}$ is the loss of a variable optical filter, adjusted to achieve constant total output power $P_{0,l}$, [7]. Then, using this with (1) and notations in Lemma 1 yields

$$\mathbf{H}_{l,k,i} = G_{l,i}^k \prod_{q=1}^{k}\tilde{\alpha}_{l,q} \quad \tilde{\alpha}_{l,q} = \alpha_{l,q}\, L_{l,q}. \tag{9}$$

Substituting (9) for $\mathbf{H}_{l,k,i}$ into (8) we can find the wavelength independent part as

$$\prod_{q=1}^{k}\tilde{\alpha}_{l,q} = \frac{P_{0,l}}{\sum\limits_{j\in\mathcal{M}_l} G_{l,j}^k\left(\prod_{q=1}^{l-1}\mathbf{T}_{q,j}\right)u_j} \qquad \forall l\in\mathcal{R}_i, k = 1,\ldots,N_l. \tag{10}$$

Using (9), (10) and Lemma 1, it follows that *OSNR*, (4), can be rewritten as (7). ■

Note that in [17], this approach is used to include the cross-talk accumulation generated at the optical nodes, in addition to ASE noise accumulation.

III. Network OSNR Optimization: Noncooperative Game Formulation

Next, we use Lemma 2 to formulate the network OSNR optimization problem as a noncooperative game, [13], [5]. Each channel is a player that minimizes its own cost function (or maximizes its utility function), by adjusting its transmission power, in response to the other channels' actions. Recall that $\mathbf{u}$ denotes the vector of channel input powers and denote by $\mathbf{u}_{-i}$ the vector obtained by deleting the ith element from $\mathbf{u}$, $\mathbf{u}_{-i} = [u_1 \ldots u_{i-1}, u_{i+1} \ldots u_m]^T$. The relevant concept is the noncooperative Nash equilibrium (NE) [13], [5].

Definition 1: Consider an m-player game, with each player minimizing the cost function J_i, over $u_i^* \in [0, u_{\max}]$. Then a vector $\mathbf{u}^*$ is called a Nash equilibrium (NE) solution of this game if

$$J_i(\mathbf{u}^*) \le \inf_{u_i\in[0,u_{\max}]} J_i(u_i, \mathbf{u}_{-i}^*) \qquad \forall i.$$

Thus $\mathbf{u}^*$ is an NE when u_i^* is solves individual optimization problem J_i, given all channels on its path have equilibrium power levels, $\mathbf{u}_{-i}^*$. Existence of an NE solution depends on existence of a common intersection point for all players' reaction curves, [5]; uniqueness depends on the specific problem. In the following, we show that while similar in some respects to wireless case [2], [8], the NE solution for optical networks needs a more general uniqueness condition.

We define a channel utility function U_i that reflects the channel's preference for maximizing OSNR_i. We define also a linear pricing term, so that each channel i minimizes the cost function

$$J_i(u_i, \mathbf{u}_{-i}) = \alpha_i\, u_i - \beta_i U_i(u_i, \mathbf{u}_{-i}), \qquad u_i \in [0, u_{\max}] \quad (11)$$

where channel power is limited to be below some $u_{\max}$ threshold on transmitter power. In the above, α_i, β_i are channel specific parameters, that quantify the willingness to pay the price and the desire to maximize its OSNR, respectively. Linear pricing is justified by recognizing that increasing the power may in fact degrade the OSNR of other channels, as seen from Lemma 2.

We consider an utility function appropriately defined for optical networks that is a generalization of the one used in [2]. The following assumptions are used.

A.ii.1) The utility function U_i is a continuously differentiable function in u_i, monotone increasing and strictly concave in u_i.

A.ii.2) $u_i = 0$, $u_i = u_{\max}$ are not solutions to the minimization of the cost function J_i.

We construct a utility function that satisfies A.ii.1) and A.ii.2). Rewrite OSNR_i (7) as

$$OSNR_i = \frac{u_i}{X_{-i} + \Gamma_{i,i}\, u_i} \quad \text{with} \quad X_{-i} = \sum_{j\neq i} \Gamma_{i,j}\, u_j + n_{0,i} \quad (12)$$

where X_{-i} denotes the total interference on channel ith due to other channels' power.

Note that OSNR_i, (12), is a strictly increasing function with respect to u_i, and tends to $1/\Gamma_{i,i}$, for infinite channel power. Even though a different physical mechanism is present (ASE noise accummulation), relation (12) bears a striking similarity with the wireless SIR model, [2], [8]. The system matrix in the SIR model [2], [8] has a special structure with equal rows, which was instrumental in the uniqueness results. In contrast, for optical networks $\mathbf{\Gamma}$ (Lemma 2) is a full general structure matrix, with coupling due to all channels and all spans. Moreover, OSNR_i is no longer a linear function of u_i and unlike the wireless case, a direct logarithmic utility of SNR cannot be used. For the general full matrix $\mathbf{\Gamma}$, we define a more general utility function

$$U_i = \ln\left(1 + a_i\, \frac{\mathrm{OSNR}_i}{1 - \Gamma_{i,i}\, \mathrm{OSNR}_i}\right), \qquad a_i > 0 \qquad (13)$$

where a_i is a channel specific parameter. Note that (13) is monotone in OSNR, so that maximizing utility is related to maximizing channel OSNR. Equivalently, using (12)

$$U_i = \ln\left(1 + a_i\, \frac{u_i}{X_{-i}}\right) \qquad (14)$$

where X_{-i} is given as in (12), as a function of the full system-matrix $\mathbf{\Gamma}$. This dependence will be instrumental in the following results. Therefore, the cost function to be minimized is

$$J_i(u_i, \mathbf{u}_{-i}) = \alpha_i u_i - \beta_i \ln\left(1 + a_i\, \frac{u_i}{X_{-i}}\right). \qquad (15)$$

From (14), it follows immediately that U_i satisfies A.ii.1). Using (15), it can be shown that there exists a nonempty interval from which to select β_i/α_i, such that A.ii.2) holds.

The following result characterizes the Nash equilibrium (NE) solution.

Theorem 3: The m-player game problem with individual cost functions J_i, (15), admits a unique NE solution $\mathbf{u}^*$ if a_i are selected such that

$$\sum_{j\neq i} \Gamma_{i,j} < a_i \qquad \forall i. \qquad (16)$$

Proof: From (11), (15) and A.ii.1) it follows directly that $\partial^2 J_i/\partial {u_i}^2 > 0$. Since the cost function J_i is strictly convex in u_i, there exists a minimizing u_i^*, for any given $\mathbf{u}_{-i}$, such that

$$J(u_i^*, \mathbf{u}_{-i}) < J(u_i, \mathbf{u}_{-i}) \qquad \forall u_i \neq u_i^*$$

on the closed and bounded (compact) set $[0, u_{\max}]$. Furthermore, by A.ii.2), u_i^* is inner.

To find u_i^* we solve the necessary conditions $\partial J_i/\partial u_i = 0$. From (15), we obtain

$$a_i\, u_i^* + X_{-i}^* = \frac{a_i \beta_i}{\alpha_i} \qquad \forall i \qquad (17)$$

which defines the ith player's reaction curve. From [5, Th. 4.3], the vector solution of (17) is an NE solution to the m-player game. Use the definition of X_{-i}^*, (12), to rewrite (17) as

$$a_i\, u_i^* + \sum_{j\neq i} \Gamma_{i,j} u_j^* = \frac{a_i \beta_i}{\alpha_i} - n_{0,i} \qquad \forall i.$$

Equivalently, in matrix form, this is written as

$$\widetilde{\mathbf{\Gamma}}\, \mathbf{u}^* = \tilde{\mathbf{b}} \qquad (18)$$

where matrix $\widetilde{\mathbf{\Gamma}}$ and vector $\tilde{\mathbf{b}}$ are defined as $\widetilde{\mathbf{\Gamma}} = [\widetilde{\Gamma}_{i,j}]$ and $\tilde{\mathbf{b}} = [\tilde{b}_i]$ with

$$\widetilde{\Gamma}_{i,j} = \begin{cases} a_i, & j = i \\ \Gamma_{i,j}, & j \neq i \end{cases} \qquad \tilde{b}_i = \frac{a_i \beta_i}{\alpha_i} - n_{0,i}.$$

Therefore, a unique NE solution $\mathbf{u}^*$ exists if the matrix $\widetilde{\mathbf{\Gamma}}$ is invertible. Recall that $\mathbf{\Gamma}$ is a positive-entry matrix. If (16) holds, then $\widetilde{\mathbf{\Gamma}}$ (18) is strictly diagonally dominant; from Gershgorin's Theorem [9] it follows that $\widetilde{\mathbf{\Gamma}}$ is invertible, and the unique NE solution is $\mathbf{u}^* = \widetilde{\mathbf{\Gamma}}^{-1}\, \tilde{\mathbf{b}}$. ■

Remark 1: For a given $\mathbf{\Gamma}$, the a_i factors can be selected such that the diagonal dominance condition (16) holds on $\widetilde{\mathbf{\Gamma}}$. If $\mathbf{\Gamma}$ is itself diagonal dominant, then a possible choice is $a_i = \Gamma_{i,i}$. Once $\mathbf{\Gamma}$ changes (e.g., upon reconfiguration) these factors can be adjusted to satisfy (16).

Remark 2: For the special case when $\widetilde{\Gamma}_{i,j} = 1$, $a_i = L$, (16) recovers the simple condition in [2]. However for a full system matrix $\mathbf{\Gamma}$, the general condition in Theorem 3 is needed.

IV. Network OSNR Optimization: Iterative Algorithm

Next, we propose a decentralized iterative algorithm, and we show that this algorithm converges to the NE solution if the diagonal dominance condition in Theorem 3 holds. Consider

$$u_i(n+1) = \frac{\beta_i}{\alpha_i} - \frac{X_{-i}(n)}{a_i} \qquad \forall i \qquad (19)$$

as recursive relation for updating transmitter power level, based on (17). Thus, (19) corresponds to a parallel adjustment scheme, [5], whereby each player responds optimally to the previously selected action of the other players. Relation (19) requires the total interference factor X_{-i}, which from (12) depends on all channel powers, u_j, and all channel gains, i.e., centralized information. However using (12) we can express (19) in terms of OSNR_i , i.e.,

$$u_i(n+1) = \frac{\beta_i}{\alpha_i} - \frac{1}{a_i}\left(\frac{1}{\mathrm{OSNR}_i(n)} - \Gamma_{i,i}\right)u_i(n). \tag{20}$$

This corresponds to a decentralized algorithm, since the only information fedback is the individual channel $OSNR_i$, which can be measured in real-time, and the channel "gain," $\Gamma_{i,i}$.

The following result gives convergence conditions for this algorithm.

Lemma 4: If (16) holds, then algorithm (20) converges to the NE solution.

Proof: Let $e_i(n) = u_i(n) - u_i^*$ where $\mathbf{u}^* = [u_i^*]$ is the NE solution. Using (17), (19), and (20) yields

$$e_i(n+1) = -\frac{1}{a_i}\sum_{j\neq i}\Gamma_{i,j}\, e_j(n) \tag{21}$$

so that

$$\| \mathbf{e}(n+1) \|_\infty = \max_i |e_i(n+1)| \leq \max_i \left(\frac{1}{a_i}\sum_{j\neq i}\Gamma_{i,j}|e_j(n)|\right)$$

where we used the fact that $\mathbf{\Gamma}$ is component-wise positive. Using $|e_j(n)| \leq \| \mathbf{e}(n) \|_\infty, \forall j$ yields

$$\| \mathbf{e}(n+1) \|_\infty \leq \max_i \left(\frac{1}{a_i}\sum_{j\neq i}\Gamma_{i,j}\right) \| \mathbf{e}(n) \|_\infty .$$

Using (16) on the right-hand side of the foregoing yields

$$\| \mathbf{e}(n+1) \|_\infty < \| \mathbf{e}(n) \|_\infty \qquad \forall n = 0, 1, 2, \ldots.$$

By a contraction mapping argument this shows that $\mathbf{e}(n) \to 0$ and $\mathbf{u}(n) \to \mathbf{u}^*$.

■

A. Discussion on Pricing Strategies

Next we discuss parameter selection via pricing strategies, for the logarithmic utility (15).

1) Proportional Pricing: This is the case when pricing parameters α_i are selected to be proportional to the system matrix entry for channel i, $\Gamma_{i,i}$, i.e., $\alpha_i = \Gamma_{i,i}k_i$, and all β_i are equal to 1. We will show that the scaling factors k_i can be selected such that all channel achieve some desired OSNR level, γ_i^*. Using this with (12) and (17) for the NE solution, we can write

$$\frac{1}{\gamma_i^*}u_i^* = (\Gamma_{i,i} - a_i)u_i^* + \frac{a_i}{\Gamma_{i,i}}\frac{1}{k_i} \qquad \forall i.$$

In matrix vector form, we have

$$\mathbf{\Sigma}\,\mathbf{u}^* = \mathbf{v} \quad \text{with} \quad \mathbf{\Sigma} = \mathrm{diag}(\epsilon_1, \ldots, \epsilon_i, \ldots \epsilon_m) \quad \mathbf{v} = [v_i] \tag{22}$$

where $\epsilon_i = 1/\gamma_i^* + a_i - \Gamma_{ii}$, and $v_i = (a_i/\Gamma_{i,i})(1/k_i)$. Using the NE solution, (18), $\mathbf{u}^* = \tilde{\mathbf{\Gamma}}^{-1}\tilde{\mathbf{b}}$, and $\tilde{\mathbf{b}} = \mathbf{v} - \mathbf{n}_0$, we get after some manipulation

$$(I - \tilde{\mathbf{\Gamma}}\,\mathbf{\Sigma}^{-1})\mathbf{v} = \mathbf{n}_0 \tag{23}$$

assuming that $\epsilon_i \neq 0$. Then if $\rho(\tilde{\mathbf{\Gamma}}\,\mathbf{\Sigma}^{-1}) < 1$, where ρ is the matrix spectral radius, (23) has a unique solution $\mathbf{v}$. Moreover, for $\epsilon_i = \epsilon \neq 0$, using (22), we can rewrite this condition as an upper bound condition on the desired OSNR level γ_i^*, i.e.,

$$\gamma_i^* < \frac{1}{\rho(\tilde{\mathbf{\Gamma}}) + (\Gamma_{ii} - a_i)} \qquad \forall i. \tag{24}$$

Therefore, if (24) holds $\mathbf{v}$ can be uniquely found as in (23), and hence the k_i factors can be found such that the desired OSNR level γ_i^* is achieved. This corresponds to a centralized pricing strategy, and as in [2], shows the trade-off between "gain" of the system matrix, in terms of the spectral radius, and the level of OSNR achieved. Unlike [2], for the general model $\mathbf{\Gamma}$ these parameters and the upper bound for the desired OSNR level are different for each channel.

2) Decentralized Pricing: For the case when β_i can be adjusted individually, we can show that if β_i satisfies the lower and upper bounds

$$\begin{aligned}\beta_i &> \frac{1 + (a_i - \Gamma_{i,i})\gamma_i^*}{1 - \Gamma_{i,i}\gamma_i^*}\,\frac{\alpha_i}{a_i}\,X_{-i} \\ \beta_i &< \frac{\alpha_i}{a_i}\left[u_{\max}\left(a_i + \sum_{j\neq i}\Gamma_{i,j}\right) + n_0\right] \qquad \forall i\end{aligned}$$

then each channel will achieve at least γ_i^*, i.e., $\mathrm{OSNR}_i > \gamma_i^*$, with $u_i \leq u_{\max}$ at each iteration.

3) Minimum OSNR *Level:* Finally, consider the case when all $u_i = u_{\max}$ and $\mathrm{OSNR}_i \geq \gamma_{\min}$. Using (7), it can be shown that a sufficient condition for this is

$$\gamma_{\min} \leq \frac{1}{\max_i\left(\sum_j \Gamma_{i,j}\right)}$$

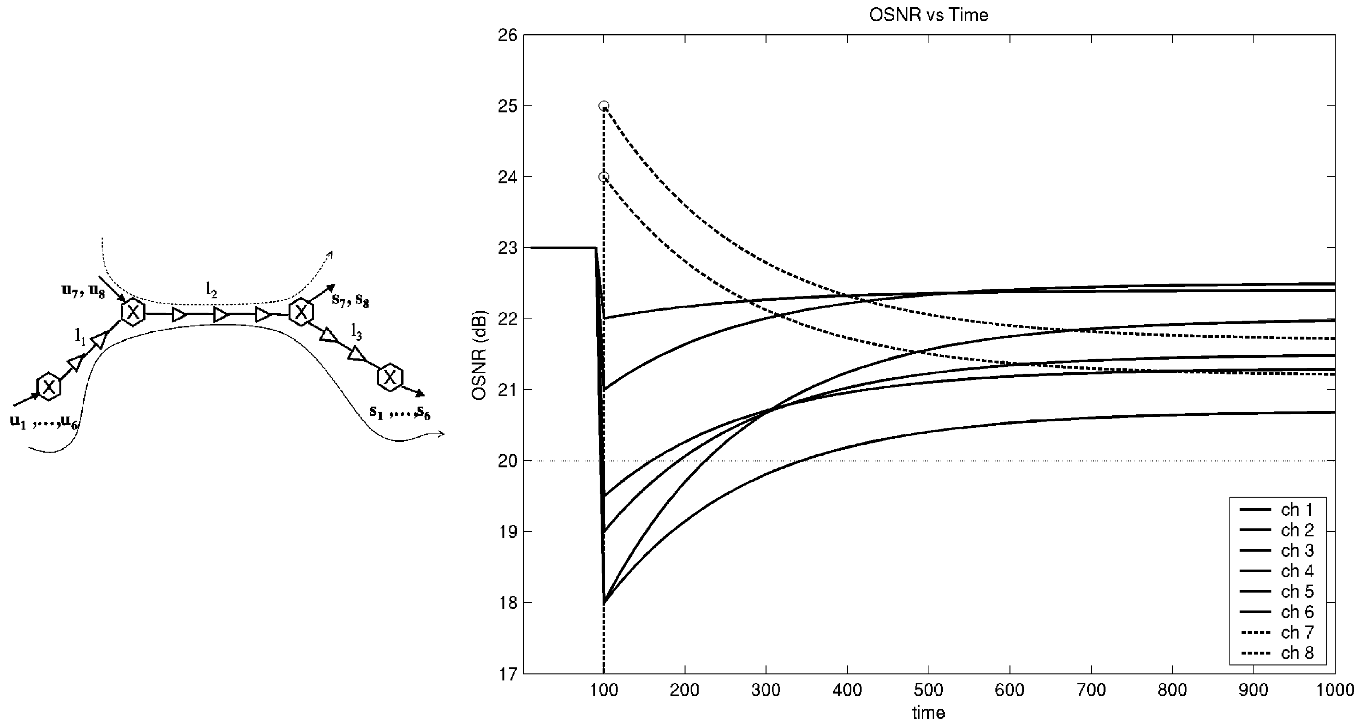
which shows the tradeoff between minimum achievable OSNR, $\gamma_{\min}$, and the norm (gain) of system matrix Γ. Unlike the wireless case, [2], the system matrix Γ plays an essential role, i.e., we cannot obtain a condition independent of the system matrix.

V. Numerical Example

A MATLAB simulation was used for a basic network configuration [Fig. 1(a)] with three links, ten amplified spans per link, each optical amplifier with a parabolic gain profile as in [7]. Assume that initially only six channels were present and the optimal power vector was obtained as in [7], for equal 23 dB OSNR. At step $n = 100$ the network is reconfigured and two new channels 7 and 8 are added to pass only through link l_2. With transmitter powers maintained as before, the OSNR for the existing channels has a sudden drop at $n = 100$ [see Fig. 1(b)], due to the extra channels sharing the link. If the iterative algorithm (20), with $\beta_i = 1$, $a_i = \Gamma_{i,i}$ and α_i proportional to $\Gamma_{i,i}$, is used to adjust all channel powers, channel OSNR levels converge to new steady-state values [Fig. 1(b)], and all of them satisfy the goal of at least 20 dB.

VI. Conclusion

In this note, we developed a framework for OSNR optimization in optical networks via a game theory approach. The optical OSNR model leads to a system matrix with a more general structure than in wireless networks. Under some reasonable assumptions on the utility function, we obtained uniqueness conditions and explicit expression for Nash



(a) (b)

Fig. 1. (a) Example network configuration. (b) OSNR evolution in time.

equilibrium. We developed an iterative network power control algorithm, based on decentralized feedback of channel parameters, and we shown it converges to the Nash equilibrium. It would be interesting to explore whether such an approach can be extended to other elements in addition to optical amplifiers. Another possible direction is adding extra constraints on the total power, as a capacity constraint so that nonlinearity is limited. Optical networks present new challenging questions for control and game theory. We hope that this study would initiate further research in the area.

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REFERENCES

[1] G. P. Agrawal, *Fiber-Optic Communication Systems*. New York: Wiley, 2002.

[2] T. Alpcan, T. Basar, R. Srikant, and E. Altman, "CDMA uplink power control as a noncooperative game," in *Proc. 40th IEEE Conf. Decision and Control*, 2001, pp. 197–202.

[3] T. Alpcan and T. Basar, "A hybrid systems model for power control in multicell wireless data networks," *Perform. Eval.*, vol. 57, no. 4, pp. 477–495, 2004.

[4] E. Altman, T. Basar, and R. Srikant, "Nash equilibria for combined flow control and routing in networks: Asymptotic behavior for a large number of users," *IEEE Trans. Autom. Control*, vol. 47, no. 6, pp. 917–930, Jun. 2002.

[5] T. Basar and G. J. Olsder, *Dynamic Noncooperative Game Theory*, 2nd ed. Philadelphia, PA: SIAM, 1999.

[6] A. R. Chraplyvy, J. A. Nagel, and R. W. Tkach, "Equalization in amplified WDM lightwave transmission systems," *IEEE Photon. Technol. Lett.*, vol. 4, no. 8, pp. 920–922, Aug. 1992.

[7] F. Forghieri, R. W. Tkach, and D. L. Favin, "Simple model of optical amplifier chains to evaluate penalties in WDM systems," *IEEE J. Lightw. Technol.*, vol. 16, no. 9, pp. 1570–1576, Sep. 1998.

[8] S. Gunturi and F. Paganini, "Game theoretic approach to power control in cellular CDMA," in *Proc. IEEE Vehicular Technology Conf.*, 2003, pp. 2362–2366.

[9] R. G. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1999.

[10] S. H. Low and D. E. Lapsley, "Optimization flow control – I : Basic algorithm and convergence," *IEEE/ACM Trans. Netw.*, vol. 7, no. 6, pp. 861–874, Dec. 1999.

[11] A. Mecozzi, "On the optimization of the gain distribution of transmission lines with unequal amplifier spacing," *IEEE Photon. Technol. Lett.*, vol. 10, no. 7, pp. 1033–1035, Jul. 1998.

[12] B. Mukherjee, "WDM optical communication networks: Progress and challenges," *IEEE J. Select. Areas Commun.*, vol. 18, no. 10, pp. 1810–1824, Oct. 2000.

[13] J. Nash, "Equilibrium points in $\mathbf{n}$-person games," in *Proc. Nat. Acad. Sci.*, vol. 36, 1950, pp. 48–49.

[14] L. Pavel, "A $\boldsymbol{\mu}$-analysis application to stability of optical networks," in *Proc. Amer. Control Conf.*, 2004, pp. 3956–3961.

[15] ——, "Dynamics and stability in optical communication networks: A system theoretic framework," *Automatica*, vol. 40, no. 8, pp. 1361–1370, 2004.

[16] ——, "Power control for OSNR optimization in optical networks: A noncooperative game approach," in *Proc. 43rd IEEE Conf. Decision and Control*, 2004, pp. 3033–3038.

[17] ——, "OSNR optimization via end-to-end power control: A central cost approach," in *Proc. IEEE INFOCOM*, 2005, pp. 1095–1105.

[18] ——, "Control design for transient power and spectral control in optical communication networks," in *Proc. IEEE Conf. Control Appl.*, 2003, pp. 415–422.

[19] B. Ramamurthy, D. Datta, H. Feng, J. P. Heritage, and B. Mukherjee, "Impact of transmission impairments on the teletraffic performance of wavelength-routed optical networks," *IEEE J. Lightw. Technol.*, vol. 17, no. 10, pp. 1713–1723, Oct. 1999.

[20] C. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 291–303, Feb. 2002.

[21] Y. Sun, J. L. Zyskind, and A. K. Srivastava, "Average inversion level, modeling, and physics of Erbium-doped fiber amplifiers," *IEEE J. Select. Topics Quantum Electron.*, vol. 3, no. 4, pp. 991–1007, Aug. 1997.