

# OSNR Optimization in Optical Networks: Modeling and Distributed Algorithms via a Central Cost Approach

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**Abstract**—This paper addresses the problem of optical signal-to-noise ratio (OSNR) optimization in optical networks. An analytical OSNR network model is developed for a general multi-link configuration, that includes the contribution of amplified spontaneous emission and crosstalk accumulation. The network OSNR optimization problem is formulated such that all channels maintain a desired individual OSNR level, while input optical power is minimized. Conditions for existence and uniqueness of the optimal solution are given. An iterative, distributed algorithm for channel power control is proposed, which is shown to converge geometrically to the optimal solution. The algorithm is valid for general network configurations, and uses only local measurements or decentralized feedback. Convergence is proved for both synchronous and asynchronous operation, which is particularly important for adaptation in a dynamic environment.

**Index Terms**—Optical networks, optical signal-to-noise ratio, optimization, iterative control algorithms.

## I. INTRODUCTION

OPTICAL wavelength-division multiplexed (WDM) communication networks are evolving beyond statically designed point-to-point links. The goal is to realize reconfigurable networks with arbitrary topologies, while at the same time maintaining network stability, optimal channel transmission performance and quality of service (QoS), [1]-[4].

At the physical transmission level, one parameter that directly determines channel performance and QoS is the bit-error rate (BER). BER in turn, depends on optical signal-to-noise ratio (OSNR), dispersion and nonlinear effects, [5]. Typically in link optimization, OSNR is considered as the dominant performance parameter, with dispersion and nonlinearity effects being kept low by proper link design. Therefore, OSNR optimization can be directly translated to QoS optimization. The dominant impairment in OSNR optimization is given by noise accumulation in chains of optical amplifiers and its effect on OSNR, [7]. A typical approach for OSNR optimization uses a static budget of device and fiber impairments along a link, with significant tolerance margins added such that at least a desired OSNR value is achieved. In [8], a heuristic algorithm was proposed for on-line OSNR equalization in single point-to-point links, but its convergence was not considered. In [7],

OSNR equalization in a point-to-point link was formulated as a static optimization problem. The optimal channel optical power vector at the transmitter is found by solving an eigenvalue problem for the system transmission matrix, composed of link gains of all channels. The algorithm which is developed only for a single point-to-point link, is static and requires centralized, global link information.

The current OSNR optimization approaches, that are developed for single links, are not appropriate for reconfigurable optical networks, where the length of the links and the number of devices in a link are changing. Moreover, in these networks arbitrary topologies can be formed, different channels can travel via different optical paths, and also different channels can have different levels of QoS (OSNR) requirements. Therefore, it is desirable to adjust network parameters (optical power, gains) in an optimal way, based on on-line network feedback (from receiver to transmitter and various nodes). This dynamic optimization will result in increased network flexibility and capacity, so that the needed quality of service (QoS) is assured whenever a new channel is added/dropped. These observations justify the need for on-line OSNR optimization algorithms, that have provable convergence properties for general network configurations. Moreover, particularly useful are decentralized algorithms, such that channel power at the transmitter (Tx), can be adjusted based on feedback from the corresponding receiver (Rx) only, plus other channel specific measurements. This is the problem we address in this paper: OSNR optimization in optical networks and development of iterative power control algorithms. A short version appeared in [19].

This problem is similar to power control in wireless communication systems, a topic which has been explored extensively, via either centralized approaches, [9] - [14], or decentralized, noncooperative game approaches, [15] - [18]. There are several differences that make this dynamic network optimization problem more challenging in optical networks. In wireless networks, channels are characterized by static loss/gain only, with single Tx to Rx links, or multiple links with no gains (ATM networks). Optical networks bring a new combination of challenges: amplified spans, multiple links, accumulation and self-generation of optical (ASE) noise, as well as crosstalk generated at the routing elements.

We consider this problem of developing algorithms for optimally re-adjusting network parameters such that all channel/routes maintain a desired QoS (OSNR), while at the same

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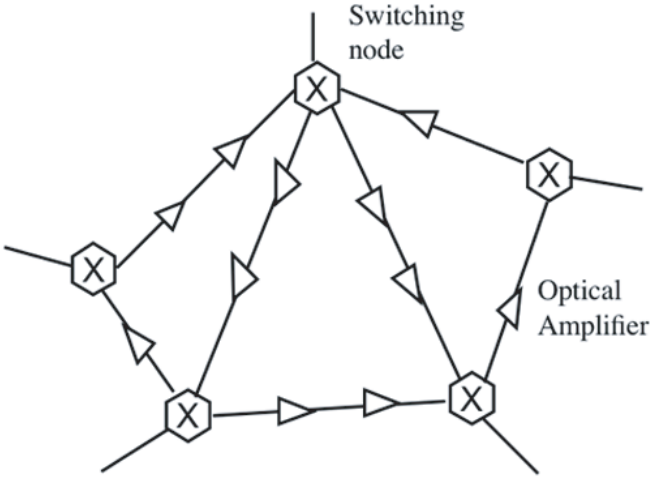


Fig. 1. Optical network configuration.

time taking into account specific constraints as imposed by dispersion and nonlinearity. The paper is organized as follows. In Section II we develop the analytical model for OSNR in a generic multi-link configuration, both as recursive relations and end-to-end relations. In Section III we formulate the static OSNR optimization problem for a large-scale network, and characterize the optimal solution. In Section IV we develop an iterative algorithm for adjusting power levels and we give conditions for its convergence to the optimal solution. The proposed algorithm is valid for general network configurations, and uses only local measurements or decentralized feedback. In Section V we prove that the algorithm is convergent also in the asynchronous case, which is particularly important for adaptation in a dynamic environment. A numerical example is given in Section V and conclusions are given in Section VI.

## II. NETWORK OSNR MODEL

Consider a generic optical network configuration (Figure 1), with a set of optical links  $\mathcal{L} = \{1, \dots, L\}$  connecting the optical nodes (for channel add/drop (OADM) or cross-connect (OXC)). A link  $l$  is composed of  $N_l$  equal length, cascaded optically amplified spans. A total set of channels,  $\mathcal{M} = \{1, \dots, m\}$ , corresponding to a set of wavelengths, are transmitted across the network. This is realized by intensity modulation of the wavelengths, and then by multiplexing the wavelengths to be transmitted across the same link. We denote by  $\mathcal{M}_l$  the set of channels transmitted over link  $l$ ,  $l \in \mathcal{L}$ . For a channel  $i \in \mathcal{M}$ , corresponding to wavelength  $\lambda_i$ , we denote by  $\mathcal{R}_i$  its optical path, or collection of links, from source (Tx) to destination (Rx). For the  $i^{\text{th}}$  channel, we denote by  $u_i$ ,  $s_i$ , and  $n_i$  the optical power of the input signal (at Tx), output signal (at Rx) and output noise (at Rx), respectively, in the  $i^{\text{th}}$  channel bandwidth. We also denote by  $\mathbf{u} = [u_1, \dots, u_m]^T$  the vector of input powers for all channels.

In the following we develop a network OSNR model for a generic multi-link configuration, following the approach in [7]. As in [7], [20], dispersion and nonlinearity effects are considered to be limited, and the dominant impairment is amplified spontaneous emission (ASE) noise in amplifiers. Optical amplifiers are used to amplify the optical power

of all channels in a link simultaneously. Because of their wavelength-dependent gain, ASE noise is also wavelength-dependent, and hence different channels will have different OSNR levels.

The following relations will be used. An optically amplified span is composed of optical fiber and an optical amplifier. For the  $k^{\text{th}}$  span, of the  $l^{\text{th}}$  link, the optical span transmission,  $h_{l,k,i}$ , for channel  $i$  is given by

$$h_{l,k,i} = G_{l,k,i} L_{l,k} \quad \forall k = 1, \dots, N_l \quad (1)$$

where  $G_{l,k,i}$  is the amplifier gain (typically wavelength/channel dependent), and  $L_{l,k}$  is optical fiber loss (typically channel independent). For channel  $i$ , we denote by  $p_{l,k,i}$  and  $v_{l,k,i}$ , the signal and the noise power, respectively, at the output of the  $k^{\text{th}}$  span on the  $l^{\text{th}}$  link.

We use the following assumption with respect to ASE noise.

(A.i.1): ASE noise power does not participate in amplifier gain saturation.

ASE noise power generated in the  $k^{\text{th}}$  amplifier on the  $l^{\text{th}}$  link, for the  $i^{\text{th}}$  channel, is given as [5]

$$ASE_{l,k,i} = 2n_{sp} [G_{l,k,i} - 1] h\nu_i B_o \quad (2)$$

where  $n_{sp} > 1$  is the amplifier excess noise factor,  $h$  is the Planck constant,  $B_o$  is the optical bandwidth, and  $\nu_i$  is the optical frequency corresponding to wavelength  $\lambda_i$ .

The optical signal-to-noise ratio (OSNR) for the  $i^{\text{th}}$  channel,  $i \in \mathcal{M}$ , is defined as

$$OSNR_i = \frac{s_i}{n_i} \quad (3)$$

where  $s_i$  and  $n_i$  are the channel's signal, and noise power at the output (at Rx), respectively. We consider only forward propagation of signal and noise in steady-state, [7], i.e., we do not consider amplifier gain dynamics, [27], [28]. This is justified for an OSNR optimization problem where power adjustments need to be made at steady-state, after updating network topology.

We will consider separately two cases depending on the amplifier operation mode: gain control mode (Lemma 1), and power control mode (Lemma 2). The case of gain control mode leads to a simpler mathematical model, such that OSNR on a particular channel does not depend on the power of the rest of the channels. The case of power control mode is more representative, as this is the amplifier mode typically used in optical links. In this mode, the same total power is launched into each span of a link, leading to uniform total power distribution across a link, which is limiting the nonlinear effects, [20], [5]. However this case leads to a more complex mathematical model; the inherent scaling on the total power translates into coupling between all channels' powers, and OSNR on a particular channel does depend on all other channels' powers.

The following lemma gives the OSNR network model for systems in the gain control mode, generalizing the single link result in [7] to an arbitrary multi-link network,

*Lemma 1:* Let  $u_i$ , and  $n_{0,i}$  be the  $i^{\text{th}}$  channel's signal, and noise power at Tx, respectively. Under (A.i.1), the  $i^{\text{th}}$  channel

OSNR, along a path  $\mathcal{R}_i$ , is given as

$$OSNR_i = \frac{u_i}{n_{0,i} + \sum_{l \in \mathcal{R}_i} \sum_{k=1}^{N_l} \frac{1}{\prod_{q=1}^{l-1} \mathbf{T}_{q,i}} \frac{ASE_{l,k,i}}{\mathbf{H}_{l,k,i}}} \quad (4)$$

where  $\mathbf{H}_{l,k,i} = \prod_{q=1}^k h_{l,q,i}$ , and  $\mathbf{T}_{l,i} = \mathbf{H}_{l,N_l,i}$ , are the intermediary and, respectively, the full transmission for  $l^{th}$  link.

*Proof:* Appendix I.

For the second case we consider that amplifiers are operated in automatic power control mode. This is the typical operation mode for optical amplifiers when the network is in steady-state, [5], such that a constant total power is maintained. Keeping a constant total power, or a constant total launching power after each span, compensates variations in fiber-span loss across a link, [7]. Moreover if the total power target  $P_{0,l}$  is selected to be below the threshold for nonlinear effects, an optimal gain distribution is achieved across the link, [20]. The following assumption is used for simplicity.

(A.i.2) All the amplifiers in a link have the same spectral shape, with the same total power target.

This is a reasonable assumption if all spans in a link have the same length, i.e., for uniformly designed optical links, [5]. Then at the output of each span

$$\sum_{j \in \mathcal{M}_l} p_{l,k,j} = P_{0,l} \quad \forall l \in \mathcal{R}_i \quad \forall k = 1, \dots, N_l \quad (5)$$

Mathematically, the automatic power control mode, can be seen as performing a scaling of the gain such that the same total power is at the output of each amplifier. This power scaling introduces coupling between channel powers, and channel OSNR is no longer independent of other channels' powers.

*Lemma 2:* Under (A.i.1)-(A.i.2) for amplifiers in power control mode, the OSNR for the  $i^{th}$  channel is written as

$$OSNR_i = \frac{u_i}{n_{0,i} + \sum_{j \in \mathcal{M}} \Gamma_{i,j} u_j} \quad \forall i \in \mathcal{M} \quad (6)$$

where the full ( $m \times m$ ) system matrix  $\Gamma$  is defined as  $\Gamma = [\Gamma_{i,j}]$

$$\Gamma_{i,j} = \sum_{l \in \mathcal{R}_i} \sum_{k=1}^{N_l} \frac{G_{l,j}^k}{G_{l,i}^k} \left( \prod_{q=1}^{l-1} \frac{\tilde{\mathbf{T}}_{q,j}}{\mathbf{T}_{q,i}} \right) \frac{ASE_{l,k,i}}{P_{0,l}}, \quad j \in \mathcal{M}_l$$

and  $\Gamma_{i,j} = 0$ , if  $j \notin \mathcal{M}_l$ .

*Proof:* Appendix I.

In the following the OSNR model in Lemma 2 is extended to include crosstalk terms due to WDM components at the optical nodes (OADM or OXC), such as optical filters, demultiplexers, add/drop modules, routers or switches, [5]. We consider only the heterowavelength (out-of-band) crosstalk due to incomplete filtering. A similar approach could be used to include the homowavelength crosstalk (in-band) in the space switches following a model as in [2].

Let the  $l^{th}$  optical node, associated with the  $l^{th}$  link, be characterized by an insertion loss  $L_{sw,l}$  (typically channel independent), and a crosstalk ratio,  $X_{he}$ . The crosstalk ratio  $X_{he}$  is a measure of the out-of-band crosstalk and represents the fraction of total power leaked into a specific channel from all the other channels, [5]. We assume also that an optical

amplifier is present at the  $l^{th}$  optical node, characterized by a gain  $G_{X,l,i}$  and an ASE noise power denoted by  $ASE_{X,l,i}$ . The following assumption will be used.

(A.i.3) Cascaded in-line amplifiers have a significantly higher contribution to the ASE accumulation than a single OXC stage.

Since an OXC stage follows after  $N_l$  (typically up to 10) in-line amplifiers with accumulated ASE noise, (A.i.3) is a reasonable assumption. It basically means that the OXCs effect on the OSNR degradation is mostly due to the cross-talk accumulation and not due to ASE noise accumulation. This assumption allows in turn for a simpler mathematical form of the complete OSNR model, with the ASE noise component (due to amplifiers) separated from the cross-talk component (due to OXC) as seen in the following result.

*Lemma 3:* Under (A.i.1)-(A.i.3), the OSNR for the  $i^{th}$  channel, including both ASE accumulation and crosstalk accumulation effects, is given as

$$OSNR_i = \frac{u_i}{n_{0,i} + \sum_{j \in \mathcal{M}} \tilde{\Gamma}_{i,j} u_j} \quad \forall i \in \mathcal{M} \quad (7)$$

where the system matrix  $\tilde{\Gamma}$  is given as  $\tilde{\Gamma} = [\tilde{\Gamma}_{i,j}]$  where

$$\tilde{\Gamma}_{i,j} = \sum_{l \in \mathcal{R}_i} \sum_{k=1}^{N_l} \frac{G_{l,j}^k}{G_{l,i}^k} \left( \prod_{q=1}^{l-1} \frac{\tilde{\mathbf{T}}_{q,j}}{\mathbf{T}_{q,i}} \right) \frac{ASE_{l,k,i}}{P_{0,l}} + \sum_{l \in \mathcal{R}_i} \left( \prod_{q=1}^l \frac{\tilde{\mathbf{T}}_{q,j}}{\mathbf{T}_{q,i}} \right) \mathbf{X}_{he}, \quad \text{if } j \neq i, j \in \mathcal{M}_l$$

$\tilde{\Gamma}_{i,j} = \Gamma_{i,j}$ , if  $j = i$ ,  $\tilde{\Gamma}_{i,j} = 0$ , if  $j \neq i, j \notin \mathcal{M}_l$ , and where  $\tilde{\mathbf{T}}_{l,i} = \mathbf{T}_{l,i} G_{X,l,i} L_{sw,l}$ .

*Proof:* Appendix I.

*Remark 1:* The new matrix  $\tilde{\Gamma} = [\tilde{\Gamma}_{i,j}]$  (Lemma 3), is similar to  $\Gamma = [\Gamma_{i,j}]$ , (Lemma 2, with the last extra term being the specific contribution of crosstalk accumulation. Therefore, in the following we will use the general notation  $\Gamma = [\Gamma_{i,j}]$  for the system matrix, with  $\Gamma$  being as in either Lemma 2, or Lemma 3.

Note that mathematically the OSNR model bears a striking similarity to the wireless SIR model, [9], except that the system matrix has non-zero diagonal elements, while the off-diagonal elements are dependent on specific optical network parameters such as link and span gains.

Also another specific feature is the fact that cross-coupling terms in OSNR due to other channels exist not only due to cross-talk as in the wireless case. Even with no OXCs, i.e., no cross-talk, ASE accumulation in optical links leads to cross-coupling terms due to the specific mode of amplifier operation in optical networks, that requires total power scaling.

### III. NETWORK OSNR OPTIMIZATION: CENTRAL COST FORMULATION

In this section we use the general network OSNR model in Lemma 2 (or Lemma 3) to formulate the static OSNR optimization problem for a large-scale network.

A special OSNR optimization problem, of maximizing the minimum OSNR, was considered in [7] for a single link case. This is similar to the problem of maximizing the minimum signal-to-interference ratio (SIR) in wireless networks, [9],

[13], [14]. The problem can be translated into an OSNR (SIR) equalization problem and formulated as an eigenvalue problem for the system matrix. The solution can be based on the Perron-Frobenius theorem for the maximum eigenvalue of a matrix with nonnegative elements, [29]. Solving this eigenvalue problem can give the optimal transmitter power vector as the eigenvector corresponding to the maximum modulus eigenvalue. However, thermal or external noise is neglected, and more importantly, the method is not appropriate for developing on-line algorithms.

In the following, we consider a different OSNR optimization problem, similar to the SIR optimization problem considered in [12], [11] for wireless networks. We will exploit the parallel between the mathematical form of the OSNR equations and the signal-to-interference equations in the wireless case. Specifically we formulate the performance objective to be that of ensuring that the OSNR set of all channels is above a predefined set of targets, as determined by QoS constraints. This approach allows full flexibility in setting individual per channel targets and also recognizes the presence of external (input) noise. Moreover, if a feasible solution exists, then there exists a unique solution which minimizes transmitter power, obtained by solving a system of linear equations. A specific feature of this approach is the fact that it leads to iterative (on-line) algorithms with geometric convergence as we will show in the following sections.

We would like to emphasize that the resulting algorithms for optical networks are mathematically the same as in the wireless context, as well as their convergence properties.

We consider the problem of finding an input (transmitter) power vector so that the OSNR of all channels satisfy

$$OSNR_i \geq \hat{\gamma}_i \quad \forall i \in \mathcal{M} \quad (8)$$

where the set of targets,  $\hat{\gamma}_i$ , can be predefined as needed by QoS requirements for each channel. Moreover, the minimum power satisfying this constraint is sought. Therefore, using (6) or (7), the optimization problem can be formulated as the minimization of the central cost function

$$\min \sum_{i \in \mathcal{M}} u_i \quad (9)$$

such that

$$\frac{u_i}{\sum_{j \in \mathcal{M}} \Gamma_{i,j} u_j + n_{0,i}} \geq \hat{\gamma}_i \quad \forall i \in \mathcal{M}$$

This requirement is translated into

$$u_i \geq \sum_{j \in \mathcal{M}} \hat{\gamma}_i \Gamma_{i,j} u_j + \hat{\gamma}_i n_{0,i}, \quad \forall i \in \mathcal{M}$$

Recall that  $\mathbf{n}_0$  stands as the input noise power at the Tx, and also can be considered to include also some other external noise such as thermal noise. Equivalently, in matrix-vector form, from the above we need to have

$$\mathbf{u} \geq \hat{\Gamma} \mathbf{u} + \hat{\mathbf{n}}_0 \quad \text{and} \quad \mathbf{u} \geq 0 \quad \forall i \in \mathcal{M} \quad (10)$$

where

$$\hat{\Gamma} = \begin{bmatrix} \hat{\gamma}_1 & \cdots & 0 \\ & \ddots & \\ 0 & \cdots & \hat{\gamma}_m \end{bmatrix} \cdot \Gamma, \quad \mathbf{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}$$

$$\hat{\mathbf{n}}_0 = \begin{bmatrix} \hat{\gamma}_1 & \cdots & 0 \\ & \ddots & \\ 0 & \cdots & \hat{\gamma}_m \end{bmatrix} \cdot \mathbf{n}_0$$

We call the set of OSNR targets  $\hat{\gamma}_i, i = 1, \dots, m$  a feasible set, if there is a nonnegative finite vector  $\mathbf{u}$  that satisfies (10). In the foregoing  $\hat{\Gamma}$  is the network transmission matrix, normalized (weighted) by  $\hat{\gamma}_i$ , while  $\mathbf{u}$ , and  $\hat{\mathbf{n}}_0$  are the vectors of input signal and weighted noise power, respectively.

The existence of a nonnegative solution for (10) is specified by the following standard result based on Perron's Theorem, [29], [32], which can be proved by using the Jordan canonical form of  $\hat{\Gamma}$ .

*Theorem 1:* [29] Let  $\rho(\hat{\Gamma})$  be the spectral radius of the nonnegative matrix  $\hat{\Gamma}$ , defined as the magnitude of the largest eigenvalue of  $\hat{\Gamma}$ . Then, the following statements are equivalent:

- (1) There exists  $\mathbf{u} > 0$  such that  $(I - \hat{\Gamma}) \mathbf{u} \geq \hat{\mathbf{n}}_0$
- (2)  $\rho(\hat{\Gamma}) < 1$
- (3)  $(I - \hat{\Gamma})^{-1} = \sum_{k=0}^{\infty} \hat{\Gamma}^k$  exists and is positive component-wise. ■

*Remark 2:* As a intuitive justification of the feasibility theorem, note that if  $\lambda$  is any eigenvalue of  $\hat{\Gamma}$  such that  $|\lambda| < 1$ , that corresponds to a positive eigenvector  $\mathbf{v}$ ,

$$\hat{\Gamma} \mathbf{v} = \lambda \mathbf{v}$$

then for any positive scalar  $\alpha > 0$  we have

$$(I - \hat{\Gamma}) \alpha \mathbf{v} = \alpha(1 - \lambda) \mathbf{v}$$

Therefore  $\alpha$  can be chosen arbitrarily large to ensure that the right-hand side of the foregoing is larger than or equal to the noise vector  $\hat{\mathbf{n}}_0$ , and hence the feasibility condition (10) holds.

Moreover, if the targets are feasible, i.e., if  $\rho(\hat{\Gamma}) < 1$  (by Theorem 1), it can be shown that the vector that satisfies (10) with equality minimizes the sum of the power transmitted, [10]. Therefore, if  $\rho(\hat{\Gamma}) < 1$ , the optimal power vector  $\mathbf{u}^*$  is the solution of

$$\mathbf{u} = \hat{\Gamma} \mathbf{u} + \hat{\mathbf{n}}_0 \quad (11)$$

or,

$$\mathbf{u}^* = (I - \hat{\Gamma})^{-1} \hat{\mathbf{n}}_0$$

*Remark 3:* An alternative sufficient condition for existence of the optimal solution is that

$$\sum_j |\hat{\Gamma}_{i,j}| < 1, \quad \forall i \in \mathcal{M}$$

Using this inequality and Gershgorin's Theorem, [30], it can be shown that all eigenvalues of  $\hat{\Gamma}$  are inside the unit disk, so that  $\rho(\hat{\Gamma}) < 1$ , and hence the optimal solution  $\mathbf{u}^*$  exists.

Note that setting  $\mathbf{u}^*$  as in (11) is a static and centralized approach. In the next section we show how this formulation can be used to develop iterative and distributed algorithms.

#### IV. NETWORK OSNR OPTIMIZATION: ITERATIVE ALGORITHM - SYNCHRONOUS CASE

In the following we use the static optimization problem formulated in the previous section, to develop an iterative network algorithm for adjusting power levels. The proposed algorithm is distributed and autonomous, i.e., uses decentralized feedback and only local measurements. We show that, for the synchronous updating of all channels' power levels, this distributed algorithm converges with a geometric rate to the optimal solution  $\mathbf{u}^*$ .

Recall (11) and consider the following update equation

$$\mathbf{u}(n+1) = \widehat{\Gamma} \mathbf{u}(n) + \widehat{\mathbf{n}}_0 \quad n = 1, 2, 3, \dots \quad (12)$$

where

$$\mathbf{u}(n) = \begin{bmatrix} u_1(n) \\ \vdots \\ u_m(n) \end{bmatrix}$$

with  $u_i(n)$  being the input optical power for the  $i^{\text{th}}$  channel at time index  $n$ .

This is a centralized algorithm that requires, at each iteration  $n$ , knowledge of the full gain matrix of the optical network  $\widehat{\Gamma}$ , together with measurement of the current power allocation  $\mathbf{u}(n)$  for all channels.

In the following, a distributed algorithm is defined, that uses only individual, local, measurements for each channel  $i^{\text{th}}$ .

Note that a standard assumption in adaptive control is that system parameters are stationary between any updates, [31]. Similarly here, system parameters change at network reconfiguration, and updates have to be performed after all network topology changes have been propagated. Therefore, we assume that

(A.ii.1) Channel gains and the "interference" terms due to other channel's power variation are stationary between power updates.

Now note that from (12), the updates for the  $i^{\text{th}}$  channel can be written component-wise as

$$u_i(n+1) = \hat{\gamma}_i \left( \sum_{j \in \mathcal{M}} \Gamma_{i,j} u_j(n) + n_{0,i} \right)$$

However from (6), (A.ii.1) we have that

$$\sum_{j \in \mathcal{M}} \Gamma_{i,j} u_j(n) + n_{0,i} = \frac{u_i(n)}{OSNR_i(n)} \quad (13)$$

Based on the foregoing consider then the following distributed algorithm.

*Algorithm:*

$$u_i(n+1) = (1 - \mu) u_i(n) + \mu \hat{\gamma}_i \frac{u_i(n)}{OSNR_i(n)} \quad (14)$$

where  $\mu > 0$  is an update (adjustment) parameter.

*Remark 4:* Note that this algorithm is distributed and autonomous as it relies only on locally available information. By this updating rule, the current transmitter power should be adjusted to be proportional to the previous power, plus by a  $\hat{\gamma}_i$  equal to the ratio between the target OSNR,  $\hat{\gamma}_i$ , and the measured OSNR at the receiver.

The adjustment factor  $\mu$  acts as a weighting factor between the previous update and the current measurement, and can

influence the transient response during convergence of the algorithm. A larger  $\mu$  leads to faster convergence, while a smaller  $\mu$  even though leads to slower convergence, can impose smoother variation of the channel powers.

The following theorem establishes convergence conditions for algorithm (14).

*Theorem 2:* Assume that an optimal solution exists, i.e.,  $\rho(\widehat{\Gamma}) < 1$ . Then algorithm (14) is globally stable and converges to the optimal solution,  $\mathbf{u}^*$ , if

$$0 < \mu < \frac{2}{1 + \rho(\widehat{\Gamma})} \quad (15)$$

Moreover, the rate of convergence of the algorithm is geometrical.

*Proof:* The proof is presented in Appendix II, after some preliminary matrix results are reviewed.

*Remark 5:* Note that for  $\mu = 1$ , (14) reduces to

$$u_i(n+1) = \hat{\gamma}_i \frac{u_i(n)}{OSNR_i(n)} \quad (16)$$

which is similar to the wireless case, [12], [14], and, for the case of equal OSNR targets, to the algorithm proposed heuristically in [8].

#### V. NETWORK OSNR OPTIMIZATION: ITERATIVE ALGORITHM - ASYNCHRONOUS CASE

In this section we prove that the algorithm (14) converges to the optimal solution in the asynchronous case also. This is particularly important for adaptation in dynamic, large-scale networks, whereby channels update their powers asynchronously.

We follow an approach as in [11], based on the worst case delay. We assume that asynchronous operation can occur in two cases. Firstly, all channels can have the same update rate, but not all channels are synchronously updated at the same instance. Secondly, different channels can have different update rates. Both cases fit within the formulation described below.

Formally, assume for example, that channel  $j^{\text{th}}$  makes an update followed by an update on channel  $i^{\text{th}}$  at a later time. Suppose that this later time is earlier than the time at which the effect of  $j^{\text{th}}$  channel update is propagated to the  $i^{\text{th}}$  channel. Therefore, effectively channel  $i^{\text{th}}$  is performing an update that depends on an outdated value of the  $j^{\text{th}}$  channel power. Thus, for the asynchronous case we have that the OSNR at instant  $(n)$ ,  $OSNR_i(n)$ , is a function of the delayed powers of the other channels, i.e.,

$$\frac{u_i(n)}{OSNR_i(n)} = \sum_{j \in \mathcal{M}} \Gamma_{i,j} u_j(n - \tau_{i,j}(n)) + n_{0,i}$$

We will use this to study convergence in the asynchronous case. Let  $\mathcal{A}(n)$  denote the collection of indices of channels making concurrent updates, i.e., a subset of  $1, 2, \dots, m$ . Thus, using the foregoing relation, for the asynchronous operation, instead of (14), we have if  $i \in \mathcal{A}(n)$

$$u_i(n+1) = (1 - \mu) u_i(n) + \quad (17)$$

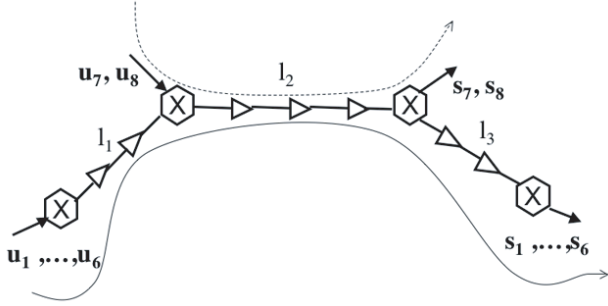


Fig. 2. Example network configuration.

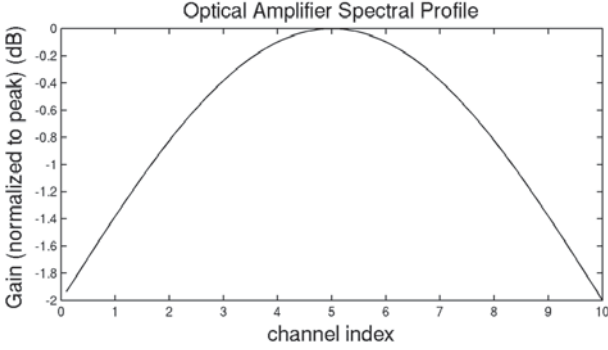


Fig. 3. Amplifier gain profile.

$$+\mu \hat{\gamma}_i \left( \sum_{j \in \mathcal{M}} \Gamma_{i,j} u_j(n - \tau_{i,j}(n)) + n_{0,i} \right)$$

and  $u_i(n+1) = u_i(n)$ , if  $i \notin \mathcal{A}(n)$ .

The delay terms  $\tau_{i,j}(n)$  are nonnegative, bounded integers, which can be appropriately defined to cover all the asynchronously cases discussed above. The following assumptions will be used.

(A.iii.1) The delay terms are uniformly bounded, i.e., there exists  $\tau_0$  such that

$$\tau_{i,j}(n) \leq \tau_0 < \infty \quad \forall n \geq 0, \quad i, j \in \mathcal{M}$$

(A.iii.2) There exists finite  $\pi$  such that every channel has an update at least once every  $\pi$ .

The following result gives convergence conditions for the asynchronous algorithm, (17).

*Theorem 3:* Assume that  $\rho(\hat{\Gamma}) < 1$  holds in addition to (Aiii.1) and (Aiii.2). Then if

$$0 < \mu < \frac{2}{1 + \rho(\hat{\Gamma})} \quad (18)$$

then the asynchronous algorithm (17) converges to  $\mathbf{u}^*$ , with a geometrical rate, i.e.,

$$\|e_\tau(n)\|_{\tau,v,\infty} \leq \alpha^q \|e_\tau(0)\|_{\tau,v,\infty}$$

where

$$\alpha = |1 - \mu| + \mu \rho(\hat{\Gamma}), \quad \text{and} \quad q = \left\lfloor \frac{n}{\tau_0 + \pi} \right\rfloor$$

*Proof:* Appendix III.

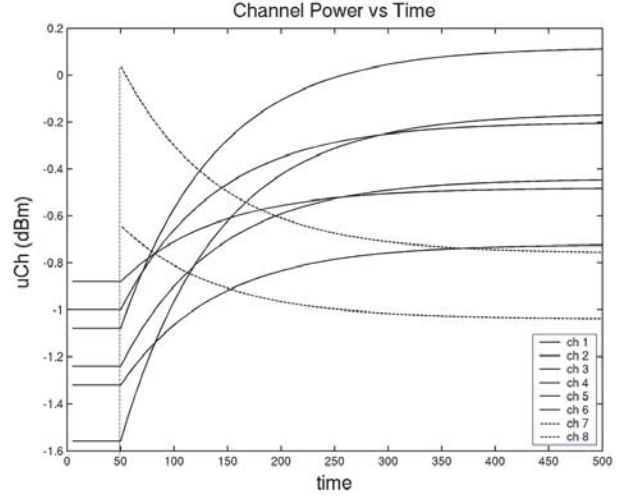


Fig. 4. Channel power evolution in time.

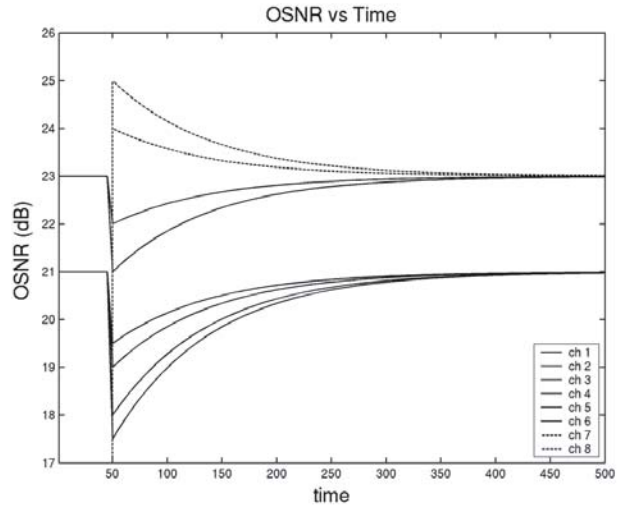


Fig. 5. OSNR evolution in time.

## VI. NUMERICAL EXAMPLE

In this section we describe a MATLAB simulation used to exemplify the iterative algorithm. We considered a basic network configuration (Fig. 2), with three links and ten amplified spans per link, with a span loss of 15 dB. For simplicity we ignore the crosstalk. We assume that  $m = 8$  channels can be transmitted, distributed from 1554 nm to 1561 nm, with channel separation of 1 nm. Each optical amplifier has a parabolic spectral gain profile as in Fig. 3, a noise figure of 5.2 dB and a total output power of 8 dBm. In Fig.3 channel index from 1 to 8 indicate channel location within the wavelength range from 1554 nm to 1561 nm. Within the set of 8 channels there are two levels of desired OSNR, a 21 dB level desired on the first 4 channels, and a 23 dB OSNR level on the other 4 channels. We consider initially that only the first 6 channels were present and we set the optimal power vector (static) so that the desired levels are satisfied, with channel power distributed around the -1dBm per channel (see Fig. 4). At step  $t = 50$  the network is reconfigured such that two new channels, 7 and 8, are added that pass only through link

$l_2$ . With transmitter powers maintained at the same level as before the add event, the OSNR for the existing channels has a sudden drop at  $t = 50$  (see Fig. 5), due to the extra channels that share the link. Using the iterative algorithm and synchronous updating to adjust all channel powers, the channel OSNR levels converge back to the desired values of 21dB and 23dB, respectively. Fig. 4 and 5 show the evolution in time of channel power and OSNR, respectively.

## VII. CONCLUSION

In this paper we considered OSNR optimization in optical networks and development of iterative algorithms for optimally re-adjusting network parameters, such that all channel/routes maintain a desired QoS (OSNR). We developed an analytical model for OSNR in a generic multi-link configuration, both as recursive relations and end-to-end relations. We formulated a static OSNR optimization problem for a generic large-scale network, and characterized the optimal solution. We developed an iterative distributed algorithm for adjusting channel power levels, that was shown to converge geometrically to the optimal solution. The algorithm is valid for general network configurations, and uses only local measurements or decentralized feedback. We proved that the algorithm is convergent also in the asynchronous case, which is particularly important for adaptation in a dynamic environment. An interesting future direction is extension of this approach to include the total power limit as a parameter to be optimized, as well as investigation of non-cooperative game theory approaches.

## APPENDIX I

### PROOF OF LEMMAS 1, 2 AND 3

#### *Proof of Lemma 1:*

The proof follows by developing end-to-end (Tx to Rx) propagation relations for signal and noise, i.e.,  $s_i$  and  $n_i$ . Recall that for the  $i^{th}$  channel,  $p_{l,k,i}$  and  $v_{l,k,i}$ , denote the signal and the noise power, respectively, at the output of the  $k^{th}$  span on the  $l^{th}$  link. Using (1) it follows

$$\begin{aligned} p_{l,k,i} &= h_{l,k,i} \cdot p_{l,k-1,i} \\ v_{l,k,i} &= h_{l,k,i} \cdot v_{l,k-1,i} + ASE_{l,k,i}, \quad k = 1, \dots, N_l \end{aligned}$$

with  $ASE_{l,k,i}$ , as in (2). Using these relations recursively after  $k$  we can write

$$p_{l,k,i} = p_{l,0,i} \prod_{q=1}^k h_{l,q,i} \quad (19)$$

$$v_{l,k,i} = v_{l,0,i} \prod_{q=1}^k h_{l,q,i} + \sum_{r=1}^{N_l} ASE_{l,r,i} \prod_{q=r+1}^k h_{l,q,i}$$

We will use the following notations and conventions. The signal power at the output of the  $l^{th}$  link, denoted by  $s_{l,i}$ , is taken by convention to be the same as the signal power at the output of the  $N_l^{th}$  span, i.e.,  $p_{l,N_l,i}$ . A similar convention holds for noise, so that

$$s_{l,i} = p_{l,N_l,i} \quad \text{and} \quad n_{l,i} = v_{l,N_l,i} \quad (20)$$

Also, the signal power at the output of the  $(l-1)^{th}$  link,  $s_{l-1,i}$ , is identical to the signal power at the input of the  $l^{th}$

link, which, by convention, is taken as the output of the  $0^{th}$  span on the  $l^{th}$  link, i.e.,  $p_{l,0,i}$ . Therefore,

$$s_{l-1,i} = p_{l,0,i} \quad \text{and} \quad n_{l-1,i} = v_{l,0,i} \quad (21)$$

where the  $(l-1)^{th}$  link is the preceding link on the  $\mathcal{R}_i$  route of the  $i^{th}$  channel.

Therefore, for the signal and noise power at the output of the  $l^{th}$  link output,  $s_{l,i}$  and  $n_{l,i}$ , we can write from (20), (19) and (21),

$$s_{l,i} = \mathbf{T}_{l,i} s_{l-1,i} \quad (22)$$

$$n_{l,i} = \mathbf{T}_{l,i} n_{l-1,i} + \sum_{r=1}^{N_l} \frac{\mathbf{T}_{l,i}}{\mathbf{H}_{l,r,i}} ASE_{l,r,i}$$

where the notations in the lemma were used. Recall that  $u_i = s_{0,i}$  is the input signal power and  $n_{0,i}$  is the input noise power. Then, by using (22) recursively after  $l$ , we obtain for  $s_i$  and  $n_i$  at the end of the path  $\mathcal{R}_i$ ,

$$\begin{aligned} s_i &= \prod_{l \in \mathcal{R}_i} \mathbf{T}_{l,i} u_i \\ n_i &= \prod_{l \in \mathcal{R}_i} \mathbf{T}_{l,i} n_{0,i} \\ &+ \sum_{l \in \mathcal{R}_i} \left( \prod_{q=l:q \in \mathcal{R}_i} \mathbf{T}_{q,i} \right) \sum_{k=1}^{N_l} \frac{1}{\mathbf{H}_{l,k,i}} ASE_{l,k,i} \end{aligned}$$

Substituting these two relations into the definition of  $OSNR_i$ , (3), yields (4). ■

#### *Proof of Lemma 2:*

From (19), (21) and (22) used recursively, we can write for  $p_{l,k,i}$ ,

$$p_{l,k,i} = \left( \prod_{l-1 \in \mathcal{R}_i} \mathbf{T}_{l-1,i} \right) \prod_{r=1}^k h_{l,r,i} u_i$$

Then, using the foregoing and the notations in the lemma, we see that (5) becomes

$$\sum_{j \in \mathcal{M}_l} \left( \prod_{q=1}^{l-1} \mathbf{T}_{q,j} \right) \mathbf{H}_{l,k,j} u_j = P_{0,l} \quad \forall l \in \mathcal{R}_i \quad (23)$$

Since all amplifiers in a link have the same shape by (A.i.2),  $G_{l,k,i}$  can be decomposed as

$$G_{l,k,i} = G_{l,i} \cdot \alpha_{l,k} \quad (24)$$

where  $\alpha_{l,k}$  is the loss of a variable optical attenuator, adjusted to achieve constant total output power  $P_{0,l}$ , [7]. Then, using (1), (24) and the notations in the lemma we can write

$$\mathbf{H}_{l,k,i} = G_{l,i}^k \prod_{q=1}^k \tilde{\alpha}_{l,q}, \quad \tilde{\alpha}_{l,q} = \alpha_{l,q} L_{l,q} \quad (25)$$

Substituting (25) for  $\mathbf{H}_{l,k,i}$  into (23) we can find the wavelength independent part as

$$\prod_{q=1}^k \tilde{\alpha}_{l,q} = \frac{P_{0,l}}{\sum_{j \in \mathcal{M}_l} G_{l,j}^k \left( \prod_{q=1}^{l-1} \mathbf{T}_{q,j} \right) u_j} \quad (26)$$

for all  $l \in \mathcal{R}_i, k = 1, \dots, N_l$ . Then, using (25, 26) into (4) yields

$$OSNR_i = \frac{u_i}{n_{0,i} + \sum_{l \in \mathcal{R}_i} \sum_{k=1}^{N_l} \frac{1}{\prod_{q=1}^{l-1} \mathbf{T}_{q,i}} \frac{ASE_{l,k,i}}{P_{0,i}} \xi_k}$$

for

$$\xi_k = \sum_{j \in \mathcal{M}_l} \frac{G_{l,j}^k}{G_{l,i}^k} \left( \prod_{q=1}^{l-1} \mathbf{T}_{q,j} \right) u_j$$

After some manipulation the foregoing can be rewritten as

$$OSNR_i = \frac{u_i}{n_{0,i} + \sum_{j \in \mathcal{M}} \mathbf{\Gamma}_{i,j} u_j} \quad \forall i \in \mathcal{M}$$

with  $\mathbf{\Gamma}$  defined as in (6).  $\blacksquare$

*Proof of Lemma 3:* Let  $\mathbf{x}_i$  be the crosstalk power, that falls within the bandwidth of the  $i^{\text{th}}$  channel, at the output (at Rx). With the crosstalk included the OSNR, (3), is written as

$$OSNR_i = \frac{s_i}{\mathbf{n}_i + \mathbf{x}_i} \quad (27)$$

Recall that in Lemma 2 any link-to-link connection was taken to be ideal. In order to include the crosstalk generated at OXC nodes, this has to be changed so that the recursive link relations for signal and noise power, (22), are modified as

$$s_{l,i} = \tilde{\mathbf{T}}_{l,i} s_{l-1,i} \quad (28)$$

$$n_{l,i} = \tilde{\mathbf{T}}_{l,i} n_{l-1,i} + \sum_{r=1}^{N_l} \tilde{\mathbf{T}}_{l,i} \frac{ASE_{l,r,i}}{\mathbf{H}_{l,r,i}} + ASE_{X,l,i}$$

where  $\tilde{\mathbf{T}}_{l,i} = \mathbf{T}_{l,i} G_{X,l,i} L_{sw,l}$  and where  $ASE_{X,l,i}$  is the ASE generated at the  $l^{\text{th}}$  OXC.

Similarly, the crosstalk power term  $\mathbf{x}_{l,i}$  added at the  $l^{\text{th}}$  OADM/OXC on the  $i^{\text{th}}$  channel is given as

$$\mathbf{x}_{l,i} = \tilde{\mathbf{T}}_{l,i} \mathbf{x}_{l-1,i} + \sum_{j \neq i, j \in \mathcal{M}_l} \mathbf{X}_{he} \tilde{\mathbf{T}}_{l,j} s_{l-1,j} \quad (29)$$

In the foregoing relation the first term is due to the crosstalk component propagated from the previous OADM/OXC node, while the second one represents the contribution due to the out-of-band crosstalk (incomplete filtering). With the foregoing relations, notice that the total optical power in the  $i^{\text{th}}$  channel's bandwidth, at the output of the  $l^{\text{th}}$  link is given as

$$y_{l,i} = s_{l,i} + n_{l,i} + \mathbf{x}_{l,i}$$

where all contributions are included, i.e., signal, ASE noise and crosstalk.

Recall that  $s_{0,i} = u_i$  and without loss of generality take  $\mathbf{x}_{0,i} = 0$ . Using (28, 29) recursively after  $l$ , after some manipulations, we can write for signal, noise and crosstalk terms, i.e.,  $s_i$ ,  $\mathbf{n}_i$  and  $\mathbf{x}_i$ , at the end of the path  $\mathcal{R}_i$ ,

$$\begin{aligned} s_i &= \prod_{l \in \mathcal{R}_i} \tilde{\mathbf{T}}_{l,i} u_i \\ \mathbf{n}_i &= \prod_{l \in \mathcal{R}_i} \tilde{\mathbf{T}}_{l,i} n_{0,i} + \sum_{l \in \mathcal{R}_i} \prod_{q=l+1:q \in \mathcal{R}_i} \tilde{\mathbf{T}}_{q,i} ASE_{X,l,i} \\ &\quad + \sum_{l \in \mathcal{R}_i} \left( \prod_{q=l:q \in \mathcal{R}_i} \tilde{\mathbf{T}}_{q,i} \right) \sum_{k=1}^{N_l} \frac{1}{\mathbf{H}_{l,k,i}} ASE_{l,k,i} \\ \mathbf{x}_i &= \sum_{j \neq i, j \in \mathcal{M}_l} \sum_{l \in \mathcal{R}_i} \prod_{q=l+1:q \in \mathcal{R}_i} \tilde{\mathbf{T}}_{q,i} \mathbf{X}_{he} \prod_{q=1:l} \tilde{\mathbf{T}}_{q,j} u_j \end{aligned}$$

By (A.i.3), we can neglect the third term in  $\mathbf{n}_i$ , i.e., the ASE component due to the OXC. Now using the foregoing relations into (27) it can be immediately shown that we can write

$$OSNR_i = \frac{u_i}{n_{0,i} + \sum_{l \in \mathcal{R}_i} \sum_{k=1}^{N_l} \frac{1}{\prod_{q=1}^{l-1} \tilde{\mathbf{T}}_{q,i}} \frac{ASE_{l,k,i}}{\mathbf{H}_{l,k,i}} + \eta_i}$$

with

$$\eta_i = \sum_{j \neq i, j \in \mathcal{M}_l} \sum_{l \in \mathcal{R}_i} \left( \prod_{q=1}^l \frac{\tilde{\mathbf{T}}_{q,j}}{\tilde{\mathbf{T}}_{q,i}} \right) \mathbf{X}_{he} u_j$$

The foregoing relation is similar to Lemma 1, with the extra third term at the denominator. Then, following the same approach as in the proof of Lemma 2, to replace for  $\mathbf{H}_{l,k,i}$  in the above, we can rewrite  $OSNR_i$  as

$$OSNR_i = \frac{u_i}{n_{0,i} + \sum_{j \in \mathcal{M}} \tilde{\mathbf{\Gamma}}_{i,j} u_j} \quad \forall i \in \mathcal{M}$$

where the new matrix  $\tilde{\mathbf{\Gamma}}$  is defined as in (7) and the claim is proved.  $\blacksquare$

## APPENDIX II PROOF OF THEOREM 2

In this appendix we first review some useful matrix results, and then we give a proof for Theorem 2.

We firstly review the standard  $l_\infty$  vector norm and matrix norm, [30]. For any  $m$ -dimensional vector  $\mathbf{x}$  with components  $x_i$ , we denote by  $\|\mathbf{x}\|_\infty$  the standard  $l_\infty$  vector norm, defined as

$$\|\mathbf{x}\|_\infty = \max_{1 \leq i \leq m} |x_i| \quad (30)$$

For a matrix  $\mathbf{A}$ , we denote by  $\|\mathbf{A}\|_\infty$  the corresponding induced matrix norm,

$$\|\mathbf{A}\|_\infty = \max_{\mathbf{x} \neq 0} \frac{\|\mathbf{A}\mathbf{x}\|_\infty}{\|\mathbf{x}\|_\infty} \quad (31)$$

which can be shown [30] to be equivalent to,

$$\|\mathbf{A}\|_\infty = \max_i \left( \sum_j |A_{i,j}| \right) \quad (32)$$

For component-wise nonnegative matrices we review the corresponding weighted  $l_\infty$  norm, defined in relation with one of its eigenvectors. Let  $\mathbf{A}$  be an  $(m \times m)$  matrix that is component-wise nonnegative and non-identically zero. Let  $\lambda_i, i = 1, \dots, m$  denote its eigenvalues and let  $\rho(\mathbf{A})$  be its spectral radius, [30], defined as

$$\rho(\mathbf{A}) = \max_i |\lambda_i(\mathbf{A})| \quad (33)$$

Since  $\mathbf{A}$  is nonnegative, by Perron-Frobenius Theorem, [29], it has a strictly positive eigenvalue  $\lambda_+$ , with the corresponding eigenvector  $\mathbf{v}$ ,  $\mathbf{v}^T = [v_1 \dots v_m]$ , having positive entries,  $v_i > 0$ , for all  $i$ . Moreover  $\lambda_+$  is a *simple* eigenvalue, i.e., non-repeated, and it is the largest eigenvalue in absolute value, i.e.,  $\rho(\mathbf{A}) = \lambda_+$ . Therefore, we can write that

$$\mathbf{A}\mathbf{v} = \rho(\mathbf{A})\mathbf{v}$$

Next we review the weighted  $l_\infty$  norm for vectors and matrices, defined in relation with the eigenvector  $\mathbf{v}$ . We define  $\mathbf{D}_v$  as the following diagonal scaling matrix associated with  $\mathbf{v}$

$$\mathbf{D}_v = \begin{bmatrix} \frac{1}{v_1} & \cdots & 0 \\ & \ddots & \\ 0 & \cdots & \frac{1}{v_m} \end{bmatrix}$$

Then, for any  $m$ -dimensional vector  $\mathbf{x}$ , we denote by  $\|\mathbf{x}\|_{v,\infty}$  the weighted  $l_\infty$  norm, defined as the standard  $l_\infty$  vector norm of the vector weighted by  $\mathbf{D}_v$ , i.e.,

$$\|\mathbf{x}\|_{v,\infty} = \|\mathbf{D}_v \mathbf{x}\|_\infty \quad (34)$$

or, equivalently using (30) on the right-hand side

$$\|\mathbf{x}\|_{v,\infty} = \max_{1 \leq i \leq m} \frac{|x_i|}{v_i}$$

For matrix  $\mathbf{A}$ , we denote by  $\|\mathbf{A}\|_{v,\infty}$  the corresponding induced weighted matrix norm

$$\|\mathbf{A}\|_{v,\infty} = \max_{\mathbf{x} \neq \mathbf{0}} \frac{\|\mathbf{A}\mathbf{x}\|_{v,\infty}}{\|\mathbf{x}\|_{v,\infty}} \quad (35)$$

The following result can be proved for the weighted matrix norm.

*Lemma 4:* Let  $\mathbf{A}$  be a component-wise non-negative matrix and  $\mathbf{D}_v$  its corresponding scaling matrix. Then the following relations hold

$$\begin{aligned} (i) \quad & \|\mathbf{A}\|_{v,\infty} = \|\mathbf{D}_v \mathbf{A} \mathbf{D}_v^{-1}\|_\infty \\ (ii) \quad & \rho(\mathbf{A}) = \|\mathbf{D}_v \mathbf{A} \mathbf{D}_v^{-1}\|_\infty \\ (iii) \quad & \|\mathbf{A}\mathbf{x}\|_{v,\infty} \leq \rho(\mathbf{A}) \|\mathbf{x}\|_{v,\infty} \quad \forall \mathbf{x} \end{aligned}$$

*Proof:* (i) From (35, 34) we can write

$$\|\mathbf{A}\|_{v,\infty} = \max_{\mathbf{x} \neq \mathbf{0}} \frac{\|\mathbf{D}_v \mathbf{A} \mathbf{x}\|_\infty}{\|\mathbf{D}_v \mathbf{x}\|_\infty} = \max_{\tilde{\mathbf{x}} \neq \mathbf{0}} \frac{\|\mathbf{D}_v \mathbf{A} \mathbf{D}_v^{-1} \tilde{\mathbf{x}}\|_\infty}{\|\tilde{\mathbf{x}}\|_\infty}$$

where  $\tilde{\mathbf{x}}$  is the weighted vector defined as  $\tilde{\mathbf{x}} = \mathbf{D}_v \mathbf{x}$ . Therefore, indeed using (31) we see that

$$\|\mathbf{A}\|_{v,\infty} = \|\mathbf{D}_v \mathbf{A} \mathbf{D}_v^{-1}\|_\infty$$

where  $\|\cdot\|_\infty$  is the standard induced  $\infty$  matrix norm.

(ii) Let  $\mathbf{M} = \mathbf{D}_v \mathbf{A} \mathbf{D}_v^{-1}$  and compute  $\|\mathbf{M}\|_\infty$  using (32), i.e.,

$$\|\mathbf{M}\|_\infty = \max_i \left( \sum_j |M_{i,j}| \right)$$

Using the definition of  $\mathbf{D}_v$  it follows that

$$\sum_j |M_{i,j}| = \frac{1}{v_i} \sum_j A_{i,j} v_j \quad \forall i$$

since all terms on the right-hand side are non-negative. Now, recall that  $\rho(\mathbf{A})$  is the spectral radius of  $\mathbf{A}$ , and  $\mathbf{v}$  is the corresponding eigenvector, so that component-wise we have

$$\sum_j A_{i,j} v_j = \rho(\mathbf{A}) v_i \quad \forall i$$

From the two foregoing relations it follows that

$$\sum_j |M_{i,j}| = \rho(\mathbf{A}) \quad \forall i$$

and hence,

$$\rho(\mathbf{A}) = \|\mathbf{M}\|_\infty = \|\mathbf{D}_v \mathbf{A} \mathbf{D}_v^{-1}\|_\infty$$

(iii) From (35) it follows that

$$\|\mathbf{A}\mathbf{x}\|_{v,\infty} \leq \|\mathbf{A}\|_{v,\infty} \|\mathbf{x}\|_{v,\infty} \quad \forall \mathbf{x}$$

which together with (i) and (ii) gives (iii) as required.  $\blacksquare$

Next we use these preliminary results to prove Theorem 2.

*Proof of Theorem 2:* Using (13) we see that (14) is equivalent to

$$\begin{aligned} u_i(n+1) &= (1-\mu)u_i(n) \\ &\quad + \mu \hat{\gamma}_i \left( \sum_{j \in \mathcal{M}} \Gamma_{i,j} u_j(n) + n_{0,i} \right) \end{aligned}$$

or, in matrix-vector form,

$$\mathbf{u}(n+1) = (1-\mu)\mathbf{u}(n) + \mu \left( \hat{\Gamma} \mathbf{u}(n) + \hat{\mathbf{n}}_0 \right) \quad (36)$$

with  $\hat{\Gamma}$  defined as in (10). Recall that  $\mathbf{u}^*$  satisfies (11), and define

$$\mathbf{e}(n) = \mathbf{u}(n) - \mathbf{u}^*$$

We will prove that the vector  $\mathbf{e}(n)$  converges to 0, for all initial conditions.

Using the foregoing relation and (11) into (36) yields, after some manipulation,

$$\mathbf{e}(n+1) = \mathbf{B} \mathbf{e}(n) \quad \text{with} \quad \mathbf{B} = I - \mu \left( I - \hat{\Gamma} \right) \quad (37)$$

Now (37) is a linear difference equation, with the solution

$$\mathbf{e}(n) = \mathbf{B}^n \mathbf{e}(0)$$

A necessary and sufficient condition such that  $\mathbf{e}(n) \rightarrow 0$ , for all initial conditions  $\mathbf{e}(0)$ , is that all eigenvalues of  $\mathbf{B}$  are inside the unit circle, [30]. To show this, we will relate the eigenvalues of  $\mathbf{B}$ , (37), to those of  $\hat{\Gamma}$ , (10). Specifically we show there is a one-to-one correspondence between the eigenvalue/eigenvector pairs of  $\mathbf{B}$  and  $\hat{\Gamma}$ .

Let  $(\lambda_i(\hat{\Gamma}), \mathbf{w})$  be an eigenvalue/eigenvector pair of  $\hat{\Gamma}$ . Then using the definition of  $\mathbf{B}$  in (37) we see that

$$\mathbf{B} \mathbf{w} = \mathbf{w} - \mu \left( \mathbf{w} - \lambda_i(\hat{\Gamma}) \mathbf{w} \right)$$

where  $\hat{\Gamma} \mathbf{w} = \lambda_i(\hat{\Gamma}) \mathbf{w}$  was used. Therefore,

$$\mathbf{B} \mathbf{w} = \lambda_{i,B} \mathbf{w}$$

for

$$\lambda_{i,B} = 1 - \mu(1 - \lambda_i(\hat{\Gamma})) \quad (38)$$

Thus for any  $(\lambda_i(\hat{\Gamma}), \mathbf{w})$  eigenvalue/eigenvector pair of  $\hat{\Gamma}$ , it follows that  $(\lambda_{i,B}, \mathbf{w})$  with  $\lambda_{i,B}$  as in (38), is an eigenvalue/eigenvector pair of  $\mathbf{B}$ .

Similarly, let  $(\bar{\lambda}_i(\mathbf{B}), \mathbf{v})$  be an eigenvalue/eigenvector pair of  $\mathbf{B}$ . Then  $\mathbf{B} \mathbf{v} = \bar{\lambda}_i(\mathbf{B}) \mathbf{v}$ , which using the definition of  $\mathbf{B}$  in (37) on the left-hand side yields

$$\mathbf{v} - \mu \mathbf{v} + \mu \hat{\Gamma} \mathbf{v} = \bar{\lambda}_i(\mathbf{B}) \mathbf{v}$$

or, equivalently,

$$\hat{\Gamma} \mathbf{v} = \bar{\lambda}_{i,\Gamma} \mathbf{v}$$

for  $\bar{\lambda}_{i,\Gamma}$  defined as

$$\bar{\lambda}_{i,\Gamma} = \frac{1}{\mu}(\bar{\lambda}_i(\mathbf{B}) - 1 + \mu)$$

so that  $(\bar{\lambda}_{i,\Gamma}, \mathbf{v})$  eigenvalue/eigenvector pair of  $\hat{\Gamma}$ .

The foregoing show that  $\mathbf{B}$  and  $\hat{\Gamma}$  have the same eigenvectors, while their eigenvalues are related by a one-to-one correspondence as

$$\lambda_i(\mathbf{B}) = 1 - \mu(1 - \lambda_i(\hat{\Gamma})) \quad (39)$$

Now, from (39), we can write

$$|\lambda_i(\mathbf{B})| \leq |1 - \mu| + \mu\rho(\hat{\Gamma}) \quad (40)$$

where the triangle inequality and (33) were used. Therefore  $|\lambda_i(\mathbf{B})| < 1$ , if the right hand side of (40) is less than 1, or equivalently, if

$$\mu\rho(\hat{\Gamma}) < \mu \quad \text{and} \quad \mu + \mu\rho(\hat{\Gamma}) < 2$$

Since  $\rho(\hat{\Gamma}) < 1$  and  $\mu + \mu\rho(\hat{\Gamma}) < 2$ , by the conditions in the theorem, the foregoing are satisfied. Therefore  $|\lambda_i(\mathbf{B})| < 1$ , and hence  $\mathbf{e}(n) \rightarrow 0$  as  $n \rightarrow \infty$  for all initial conditions, proving convergence of the algorithm.

In the following we show that the rate of convergence is geometrical. Note that using Lemma 4, (iii), (Appendix II), applied to  $\hat{\Gamma}$  we can write

$$\|\hat{\Gamma} \mathbf{e}(n)\|_{v,\infty} \leq \rho(\hat{\Gamma}) \|\mathbf{e}(n)\|_{v,\infty} \quad (41)$$

Recall now (37), i.e.,

$$\mathbf{e}(n+1) = (1 - \mu)\mathbf{e}(n) + \mu\hat{\Gamma}\mathbf{e}(n)$$

and take  $\|\cdot\|_{v,\infty}$ , as in (34), on both sides. Then using the triangle inequality it follows that

$$\|\mathbf{e}(n+1)\|_{v,\infty} \leq |1 - \mu| \|\mathbf{e}(n)\|_{v,\infty} + \mu \|\hat{\Gamma} \mathbf{e}(n)\|_{v,\infty}$$

Using (41) in the foregoing inequality yields

$$\|\mathbf{e}(n+1)\|_{v,\infty} \leq \alpha \|\mathbf{e}(n)\|_{v,\infty}$$

for  $\alpha = |1 - \mu| + \mu\rho(\hat{\Gamma})$ , and hence

$$\|\mathbf{e}(n)\|_{v,\infty} \leq \alpha^n \|\mathbf{e}(0)\|_{v,\infty}, \quad \forall n \geq 0$$

Since from (40) we have  $\alpha < 1$ , this shows that the rate of convergence is geometrical. ■

### APPENDIX III PROOF OF THEOREM 3

In this appendix we first extend some of the linear algebra concepts in Appendix II to delayed vectors, and then we give a proof for Theorem 3.

Consider the weighted norm, (34), and let us extend it to a delayed vector. Let  $\mathbf{x}(n)$  and  $\mathbf{x}_\tau(n)$  denote an  $m$ -dimensional vector and its delayed version, respectively,

$$\mathbf{x}(n) = \begin{bmatrix} x_1(n) \\ \vdots \\ x_m(n) \end{bmatrix} \quad \mathbf{x}_\tau(n) = \begin{bmatrix} x_1(n - \tau_1) \\ \vdots \\ x_m(n - \tau_m) \end{bmatrix}$$

where  $(n)$  is the time index and  $\tau_i$ , are channel delays, assumed bounded, i.e.,  $\tau_i \leq \tau_0$ , for all  $i$ . We denote by  $\|\cdot\|_{\tau,v,\infty}$ , the following norm,

$$\|\mathbf{x}_\tau(n)\|_{\tau,v,\infty} = \max_{\tau} \|\mathbf{x}_\tau(n)\|_{v,\infty} \quad (42)$$

or, equivalently,

$$\|\mathbf{x}_\tau(n)\|_{\tau,v,\infty} = \max_{0 \leq \tau \leq \tau_0} \max_{1 \leq i \leq m} \frac{|x_i(n - \tau_i)|}{v_i}$$

Note that component-wise from the foregoing we have that

$$|x_i(n - \tau_i)| \leq v_i \|\mathbf{x}_\tau(n)\|_{\tau,v,\infty} \quad (43)$$

for any  $\tau_i \leq \tau_0$  and all  $i$ .

The following result extends Lemma 4, (iii) in Appendix II.

*Lemma 5:* The following relation holds

$$\|\mathbf{A} \mathbf{x}_\tau\|_{\tau,v,\infty} \leq \rho(\mathbf{A}) \|\mathbf{x}_\tau\|_{\tau,v,\infty}$$

*Proof:* Using the definition (42) it follows that

$$\|\mathbf{A} \mathbf{x}_\tau\|_{\tau,v,\infty} = \max_{\tau} \|\mathbf{A} \mathbf{x}_\tau\|_{v,\infty}$$

For any fixed delays  $\tau_i$ ,  $i = 1, \dots, m$ , from Lemma 4, (iii), we have

$$\|\mathbf{A} \mathbf{x}_\tau\|_{v,\infty} \leq \rho(\mathbf{A}) \|\mathbf{x}_\tau\|_{v,\infty}$$

From the foregoing two relations it can be seen immediately that

$$\begin{aligned} \|\mathbf{A} \mathbf{x}_\tau\|_{\tau,v,\infty} &\leq \max_{\tau} \{\rho(\mathbf{A}) \|\mathbf{x}_\tau\|_{v,\infty}\} \\ &= \rho(\mathbf{A}) \max_{\tau} \|\mathbf{x}_\tau\|_{v,\infty} \end{aligned}$$

and the result follows by using again (42) on the right hand side. ■

Next using these preliminary results we give a proof for Theorem 3, based on the worst case delay  $\tau_0$ .

*Proof of Theorem 3:* Firstly, notice that if the update is made synchronously for all channels but with *old* or *outdated* information (delayed), than algorithm (17) is given similarly with  $\mathcal{A}(n) = 1, \dots, m$ . Therefore, using the definition of  $\hat{\Gamma}$ , from (17) we can write in matrix form

$$\mathbf{u}(n+1) = (1 - \mu)\mathbf{u}(n) + \mu \left( \hat{\Gamma} \mathbf{u}_\tau(n) + \hat{\mathbf{n}}_0 \right)$$

where  $\mathbf{u}_\tau(n)$  is the delayed power vector

$$\mathbf{u}_\tau(n) = \begin{bmatrix} u_1(n - \tau_1(n)) \\ \vdots \\ u_m(n - \tau_m(n)) \end{bmatrix}$$

with  $\tau_j = \max_i \tau_{i,j}$ , and by (A.iii.1),  $\tau_j \leq \tau_0$ , for all  $j$ .

Define as before  $\mathbf{e}(n) = \mathbf{u}(n) - \mathbf{u}^*$ , with  $\mathbf{u}^*$  as in (11). From the foregoing relations it can be shown that

$$\mathbf{e}(n+1) = (1 - \mu)\mathbf{e}(n) + \mu\hat{\Gamma}\mathbf{e}_\tau(n)$$

Using the triangle inequality and Lemma 5 it follows that

$$\begin{aligned} \|\mathbf{e}(n+1)\|_{\tau,v,\infty} &\leq |1 - \mu| \|\mathbf{e}(n)\|_{\tau,v,\infty} + \mu \|\hat{\Gamma} \mathbf{e}_\tau(n)\|_{\tau,v,\infty} \\ &\leq |1 - \mu| \|\mathbf{e}(n)\|_{\tau,v,\infty} + \mu\rho(\hat{\Gamma}) \|\mathbf{e}_\tau(n)\|_{v,\infty} \end{aligned}$$

and the proof can follow along the same lines as for the synchronous case (see proof of Theorem 2).

In the following we give the proof for the totally asynchronous case, i.e., component-wise. Let

$$e_i(n) = u_i(n) - u_i^*$$

so that using (17, 11) we can write if  $i \in \mathcal{A}(n)$

$$e_i(n+1) = (1-\mu)e_i(n) + \mu \sum_{j \in \mathcal{M}} \hat{\Gamma}_{i,j} e_j(n - \tau_{i,j}(n))$$

and  $e_i(n+1) = e_i(n)$ , if  $i \notin \mathcal{A}(n)$ .

Then if  $i \in \mathcal{A}(n)$ , using the triangle inequality, we have

$$|e_i(n+1)| \leq |1-\mu| |e_i(n)| + \mu \sum_{j \in \mathcal{M}} \hat{\Gamma}_{i,j} |e_j(n - \tau_{i,j}(n))|$$

so that

$$|e_i(n+1)| \leq |1-\mu| \|e_\tau(n)\|_{\tau,v,\infty} v_i + \mu \|e_\tau(n)\|_{\tau,v,\infty} \sum_{j \in \mathcal{M}} \hat{\Gamma}_{i,j} v_j$$

where (43) (by (A.iii.1)) was used for both terms on the right-hand side.

Using the fact that  $(\rho(\hat{\Gamma}), \mathbf{v})$  is an eigenvalue-eigenvector pair, from the foregoing it follows that

$$|e_i(n+1)| \leq |1-\mu| \|e_\tau(n)\|_{\tau,v,\infty} v_i + \mu \rho(\hat{\Gamma}) \|e_\tau(n)\|_{\tau,v,\infty} \cdot v_i$$

Equivalently, if  $i \in \mathcal{A}(n)$ ,

$$|e_i(n+1)|/v_i \leq \alpha \|e_\tau(n)\|_{\tau,v,\infty} \quad (44)$$

for  $\alpha = |1-\mu| + \mu \rho(\hat{\Gamma})$  where  $\alpha < 1$  by the conditions in the theorem.

Note also that if  $i \notin \mathcal{A}(n)$

$$|e_i(n+1)|/v_i = |e_i(n)|/v_i \leq \|e_\tau(n)\|_{\tau,v,\infty} \quad (45)$$

or, using (43),

$$\|e_\tau(n+1)\|_{\tau,v,\infty} \leq \|e_\tau(n)\|_{\tau,v,\infty}$$

From (44,45) we see that, for any  $i$ ,  $\|e_\tau(n)\|_{\tau,v,\infty}$  is a non-increasing function of  $n$ , that is also bounded from below by 0. Therefore  $\|e_\tau(n)\|_{\tau,v,\infty}$  converges to 0.

In the following we show that the rate of convergence is geometrical. Let  $k$  be the index for the last update of channel  $i$  between  $n$  and  $(n + \pi)$ , (at least one exists by (Aiii.2)).

Then, for any  $n' \geq n + (\tau_0 + \pi)$ , we have to use the last updated value which is at the  $(k + 1)$  index (with  $\tau_0$  the maximum delay). Therefore we have

$$|e_i(n')|/v_i = |e_i(k+1)|/v_i \leq \alpha \|e_\tau(k)\|_{\tau,v,\infty}$$

for  $\forall n' \geq n + (\tau_0 + \pi)$ , where the last inequality followed from (44), taken at the two consecutive updates,  $k$  and  $(k+1)$ . Since  $k$  is the first update after index  $n$ , we have as in (45)

$$\|e_\tau(k)\|_{\tau,v,\infty} \leq \|e_\tau(n)\|_{\tau,v,\infty}$$

From the foregoing two relations we see that

$$|e_i(n')|/v_i \leq \alpha \|e_\tau(n)\|_{\tau,v,\infty}, \quad \forall n' \geq n + (\tau_0 + \pi)$$

so that

$$\|e_\tau(n')\|_{\tau,v,\infty} \leq \alpha \|e_\tau(n)\|_{\tau,v,\infty}, \quad \forall n' \geq n + (\tau_0 + \pi)$$

Using recursively the above relation it follows

$$\|e(n')\|_{\tau,v,\infty} \leq \alpha^q \|e(0)\|_{\tau,v,\infty}$$

with  $q = \lfloor \frac{n'}{\tau_0 + \pi} \rfloor$ . Since  $\alpha < 1$  this shows that the algorithm converges at a geometrical rate.  $\blacksquare$

## REFERENCES

- [1] B. Mukherjee, "WDM optical communication networks: progress and challenges," *IEEE J. Select. Areas Commun.*, vol. 18, pp. 1810-1824, 2000.
- [2] B. Ramamurthy, D. Datta, H. Feng, J. P. Heritage, and B. Mukherjee, "Impact of transmission impairments on the teletraffic performance of wavelength-routed optical networks," *IEEE J. Lightwave Technol.*, vol. 17, pp. 1713-1723, 1999.
- [3] L. Pavel, "Dynamics and stability in optical communication networks: a system theoretic framework," *Automatica*, vol. 40, no. 8, pp. 1361-1370, Aug. 2004.
- [4] L. Pavel, "A  $\mu$ -analysis application to stability of optical networks," in *Proc. American Control Conference 2004*, pp. 3956-3961.
- [5] G. P. Agrawal, *Fiber-Optic Communication Systems*. John Wiley & Sons, 2002.
- [6] M. Menif, L. A. Rusch, and M. Karasek, "Application of preemphasis to achieve flat output OSNR in time-varying channels in cascaded EDFAs without equalization," *IEEE J. Lightwave Technol.*, vol. 19, pp. 1440-1452, 2001.
- [7] F. Forghieri, R. W. Tkach, and D. L. Favin, "Simple model of optical amplifier chains to evaluate penalties in WDM systems," *IEEE J. Lightwave Technol.*, vol. 16, pp. 1570-1576, 1998.
- [8] A. R. Chraplyvy, J. A. Nagel, and R. W. Tkach, "Equalization in amplified WDM lightwave transmission systems," *IEEE Phot. T. Lett.*, vol. 4, pp. 920-922, 1992.
- [9] J. Zander, "Performance of optimum transmitter power control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 305-311, 1992.
- [10] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Select. Areas Commun.*, vol. 13, pp. 1341-1347, 1995.
- [11] D. Mitra, "An asynchronous distributed algorithm for power control in cellular radio systems," in *Proc. 4th WINLAB Workshop on 3rd Generation Wireless Info. Networks*, pp. 177-186, 1993.
- [12] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. Veh. Technol.*, vol. 42, pp. 641-646, 1993.
- [13] S. A. Grandhi, R. Vijayan, D. J. Goodman, and J. Zander, "Centralized power control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 42, pp. 466-468, 1993.
- [14] N. Bambos, S. C. Chen, and G. J. Pottie, "Radio link admission algorithm for wireless networks with power control and active link quality protection," in *Proc. IEEE INFOCOM 1995*, pp. 97-104.
- [15] D. Falomari, N. Mandayam, and D. Goodman, "A new framework for power control in wireless data networks: games, utility and pricing," in *Proc. Allerton Conf. on Communication, Control and Computing 1998*, pp. 546-555.
- [16] T. Alpcan, T. Basar, R. Srikant, and E. Altman, "CDMA uplink power control as a noncooperative game," in *Proc. 40th IEEE Conf. Decision and Control 2001*, pp. 197-202.
- [17] Chakrabarti and A. Mishra, "QoS issues in ad hoc wireless networks," *IEEE Commun. Mag.*, vol. 39, no. 2, pp. 142-148, Feb. 2001.
- [18] H. Ji and C.-Y. Huang, "Non-cooperative uplink power control in cellular radio systems," *Wireless Networks*, vol. 4, pp. 233-240, 1998.
- [19] L. Pavel, "Power control for OSNR optimization in optical networks: a distributed algorithm via a central cost approach," in *Proc. IEEE INFOCOM 2005*, pp. 1095-1105.
- [20] A. Mecozzi, "On the optimization of the gain distribution of transmission lines with unequal amplifier spacing," *IEEE Phot. T. Lett.*, vol. 10, pp. 1033-1035, 1998.
- [21] S. Turistyn, M. Fedoruk, V. Mezentssev, and E. Turitsyna, "Optimal power budget in quasi-linear dispersion managed fiber links," in *Proc. IEEE LEOS 2002*, pp. 701-702.

- [22] J. E. Ford and J. A. Walker, "Dynamic spectral power equalization using micro-optomechanics," *IEEE Phot. T. Lett.*, vol. 10, pp. 1440-1442, 1998.
- [23] C. R. Doerr, L. W. Stultz, *et al.*, "An automatic 40-wavelength channelized equalizer," *IEEE Phot. T. Lett.*, vol. 12, pp. 1195-1197, 2000.
- [24] L. Pavel, "Effect of equalization strategy on dynamic response of optical networks," in *Proc. IEEE LEOS 2002*, pp. 185-186.
- [25] M. Manna and C. Rocca, "Modeling of penalties on chains of optical amplifiers with equalizing filters," *IEEE J. Lightwave Technol.*, vol. 18, pp. 295-300, 2000.
- [26] L. Pavel, "A nested noncooperative OSNR game in optical links with dynamic gain filters," in *Proc. 44th IEEE Conf. Decision and Control 2005*, to appear.
- [27] Y. Sun, J. L. Zyskind, and A. K. Srivastava, "Average inversion level, modeling, and physics of Erbium-doped fiber amplifiers," *IEEE J. Select. Topics Quantum Electron.*, vol. 3, pp. 991-1007, 1997.
- [28] L. Pavel, "Control design for transient power and spectral control in optical communication networks," in *Proc. IEEE Conf. Control Applications 2003*, pp. 415-422.
- [29] H. Mync, *Nonnegative Matrices*. New York: Wiley, 1988.
- [30] R. G. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1999.
- [31] S. Haykin, *Adaptive Filter Theory*, 3rd Ed. Prentice-Hall, 1996.
- [32] C. R. MacCluer, "The many proofs and applications of Perron's theorem," *SIAM Review*, vol. 42, no. 3, pp. 487-498, 2000.



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