H_∞ Optimal Control Design for Time Dependent Tones within Erbium-doped Fiber Amplifiers

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***Abstract*— In this paper we propose the application of H_∞ control within optical networks. Specifically, we aim to suppress the cross-gain modulation effects within Erbium-doped fiber amplifiers owed to pilot tone disturbances used for optical monitoring. Based on the characteristics of these pilot tone disturbances, we design an H_∞ controller to improve transient performance of optical channels within the network. Within this paper the control system design is described and simulation results are presented.**

I. Introduction

Driven by the rapid growth of the Internet and an increased breadth of online services, the demand for network capacity continues to grow. In response to this growth the application of Erbium-doped fiber amplifiers (EDFAs) within wavelength division multiplexed (WDM) systems has enabled significant improvements in network capacity, reliability and transparency. Unlike optoelectronic regenerators, EDFAs do not require costly sophisticated electronics while also providing high gain with a relatively low noise figure. Figure 1 illustrates a typical optical network with a mesh topology accommodating over 100 channels where several EDFAs can be concatenated within a given optical link. Despite the advantages associated with EDFAs, there remain a number of challenges which must be overcome as WDM networks increase in complexity. In particular, it is well known that EDFAs suffer from the effects of cross-gain modulation, whereby output signals suffer from unwanted power transients due to the variation of input signal powers entering the EDFA [1]-[3]. Up to this point these cross-gain modulation effects have been primarily analyzed in the case where channels are added and dropped, which is commonly due to network reconfiguration and/or network faults. Extending on this concept, we examine the characteristics of system disturbances due to pilot tones and their effects on optical network performance.

Recently, the use of small sinusoidal signals (pilot tones) have been proposed to monitor wavelength channels directly on the optical layer [4]-[7]. The use of pilot tones allows for supervisory information to be carried throughout the core network and recovered all-optically, providing a cost-effective alternative to previous signal monitoring solutions. Although, pilot tones have many advantages, their effectiveness to monitor optical signals may be impaired by the effects of cross-gain modulation, especially as the number of EDFAs and the number of channels and pilot tones increases in the network [8],[9]. Thus, it would be desirable to synthesize a controller to minimize or reject the effects of cross-gain modulation owed to these disturbances, while also ensuring that output channels closely track their respective input signals. Although effective control methods for the EDFA exist [10],[11], these methods have focused on pump or link control for a channel add/drop.

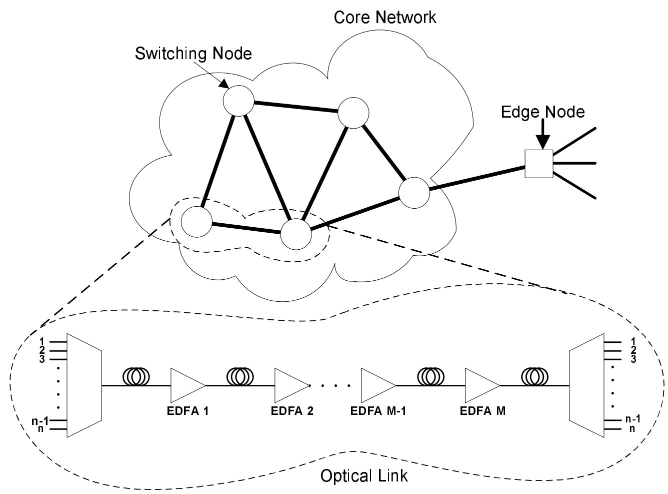

Fig. 1. Optical Network Topology

This paper studies the characteristics of input signal disturbances owed to pilot tones. Then by exploiting these characteristics an H_∞ controller is designed to reduce the unwanted effects of these disturbances. We provide simulation results for a single stage EDFA showing significant improvements in transient behaviour.

II. Pilot Tones

Small amplitude sinusoidal signals or gain modulations, commonly referred to as pilot tones, are used to carry supervisory information in WDM optical networks. Performed at each switching node, a small portion of the signal is tapped off the incoming signals and the corresponding pilot tone can be extracted all-optically, without the use of costly multiplexers. In many cases, the pilot tone itself is frequency modulated to carry useful binary data specific to the optical channel such as source, destination and designated path. In general, with a pilot tone on a given i^{th} channel, the channel's optical power takes the following form:

$$P_i(t) = P_0 \cdot [1 + m_i \cdot sin(2\pi(f_{c,i} + \Delta f \cdot d_i(t))t)] \quad (1)$$

where P_0 is the nominal optical power, m_i is the modulation depth, $f_{c,i}$ is the pilot tone carrier frequency, Δf is the peak frequency deviation and $d_i(t)$ is the binary (NRZ) data carried on the pilot tone.

Typical pilot tone carrier frequencies should be selected such that they do not interfere with the natural slow gain dynamics of the EDFA, while also remaining below the occupied optical transmission bandwidth. This has led to the majority of pilot tone frequencies to be selected between 10 and 100kHz [5],[6]. However, as optical bit rates continue to increase to Gb/s and even Tb/s, tones may be selected into the MHz range, assuming suitable optical filters are available to capture the pilot tones and their modulated data. The amplitude, or modulation depth, of such pilot tones should also be considered. In order to avoid power penalty at the transmitter and receiver, the amplitude of the pilot tones should be limited to 10% of the data level [5],[6].

III. EDFA Model

The EDFA's average inversion level can be used to describe the transient gain behaviour of the EDFA. Both Sun [12] and Bononi [13] each introduced time dependent gain models for the EDFA's average inversion level, which in turn determines the dynamics of output power signals. Using the model provided by Sun, the dynamics of the average inversion level $\overline{N}_2(t)$ are related to the n input channel powers $p_{i,IN}(t)$ and pump power $p_p(t)$ by the nonlinear differential equation:

$$\begin{aligned} \frac{d\overline{N}_2(t)}{dt} &= \frac{-\overline{N}_2(t)}{\tau} - \frac{1}{\rho SL}\sum_{i=1}^{n}\left[e^{\bar{g}_i(t)L} - 1\right] p_{i,IN}(t)... \\ & \quad - \frac{1}{\rho SL}\left[e^{\bar{g}_p(t)L} - 1\right] p_p(t) \end{aligned} \quad (2)$$

where $i = 1...n, p$ and the average exponential gain coefficient is described by

$$\bar{g}_i(t) = (\gamma_{i,Er} + \alpha_{i,Er})\overline{N}_2(t) - \alpha_{i,Er} \quad (3)$$

where τ is the fluorescence time, ρ is the number density of Er atoms, S is the cross sectional area of the fiber core, L is the length of the Erbium fiber and $\gamma_{i,Er}$ and $\alpha_{i,Er}$ are the emission and absorption parameters, respectively, of the Erbium fiber. Using the expression for the gain coefficient from (3), the output channel powers $p_{i,OUT}(t)$ are obtained using

$$p_{i,OUT}(t) = e^{\bar{g}_i(t)L} p_{i,IN}(t) \quad (4)$$

Given the numerical relation between the average inversion level, input power and output power, a measure for the EDFA's average total gain is given by

$$y_G(t) = \frac{\sum_{i=1}^{n} p_{i,OUT}(t)}{\sum_{i=1}^{n} p_{i,IN}(t)} \quad (5)$$

Note that the average total gain provides a suitable measurement variable for controlling output power. That is, by ensuring that the average total gain remains constant, output power will remain relatively constant. Figure 2 illustrates a

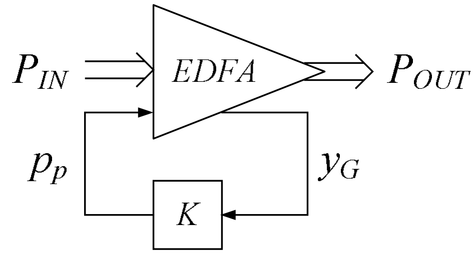

Fig. 2. EDFA Control Configuration (p_{IN}: input channel power, p_{OUT}: output channel power, p_p: pump power, y_G: average total gain)

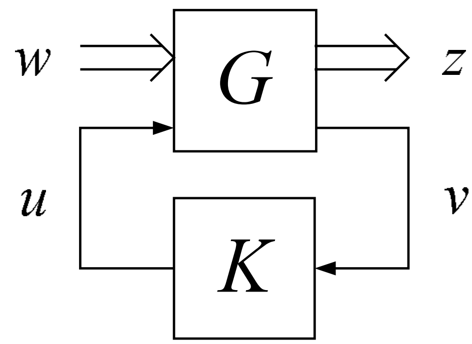

Fig. 3. General Control Configuration where u is control variable, v is the measurement variable, w are the disturbances, and z is the performance signal

typical EDFA, where average total gain is used to adjust the pump power driving the EDFA. In order to minimize the effects of cross gain modulation EDFA gain control has been shown to effectively recover gain and power on existing channels through electronic pump adjustments. By linearizing equations (2)-(5) around a steady state operating point ($\overline{N}_{2,0}, p_{0,OUT}, p_{0,IN}, p_{p0}, y_{G0}$) the single stage EDFA, supporting n channels, can be described as a multi-input-multi-output system [14] where we have $p_{IN}, p_{OUT} \in \mathbb{R}^n$ and $y_G, p_p \in \mathbb{R}$

$$\begin{bmatrix} p_{OUT} \\ y_G \end{bmatrix} = G(s)\begin{bmatrix} p_{IN} \\ p_p \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}\begin{bmatrix} p_{IN} \\ p_p \end{bmatrix} \quad (6)$$

with a state-space realization

$$G : \begin{cases} \dot{\overline{N}_2}(t) &= A\overline{N}_2(t) + B_1 p_{IN} + B_2 p_p \\ p_{OUT} &= C_1\overline{N}_2(t) + D_{11} p_{IN} + D_{12} p_p \\ y_G &= C_2\overline{N}_2(t) + D_{21} p_{IN} + D_{22} p_p \end{cases} \quad (7)$$

where $D_{12} = 0_{n\times 1}$ and $D_{22} = 0$. Note that the signals in (6) and (7) represent normalized deviations from the chosen steady state operating point.

IV. H$_\infty$ Optimal Control

The H$_\infty$ optimal control problem is well known to provide simple, reliable and robust control and the synthesis of H$_\infty$ optimal controllers are readily available in MATLAB based on the state-space solutions in [15]. It is quickly observed that the general control configuration shown in Figure 3 for the H$_\infty$ control problem is well suited for control of the EDFA. Considering the general control configuration the H$_\infty$ control objective is to find a stabilizing controller which minimizes the H$_\infty$-norm of the closed-loop transfer function from w to z that is given by the linear fractional transformation

$$z = T_{zw}w = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}w \quad (8)$$

Thus the H$_\infty$ control problem is formalized as synthesizing a controller K such that

$$\|T_{zw}\|_\infty < \gamma_{zw} \quad (9)$$

for some value $\gamma_{zw} > 0$.

V. Control Design

The plant G from the general control problem in Figure 3 is selected as the EDFA state space model in (7). It follows that the average inversion level is taken as the internal state, $x \Rightarrow \overline{N}_2(t)$, the disturbance signal is selected as the input channel powers $w \Rightarrow p_{IN}$, the measurement signal $v \Rightarrow y_G$ and the control signal $u \Rightarrow p_p$. However the original plant G is modified so that the performance variable is taken as the average inversion $z \Rightarrow \overline{N}_2(t)$, as opposed to the output channel powers.

Given a pilot tone disturbance with the following properties:

- Disturbance signals reside in a specified bandwidth $[f_{min}, f_{max}]$ where $f_{max} < TransmissionBitRate$ (typically on the order of Gb/s and Tb/s)
- Disturbance signals have a maximum modulation index m_{max}

we now provide a methodology for designing a suitable H$_\infty$ controller.

A. Plant Augmentation

In order to meet the H$_\infty$ assumptions in [15], the original EDFA model must augmented. Specifically the EDFA model in (7) must be augmented to ensure that D_{12} has full column rank. In order to meet this assumption, $\hat{z}$ is introduced:

$$\hat{z} = \begin{bmatrix} z \\ \tilde{z} \end{bmatrix} = \begin{bmatrix} C_1x + D_{11}w + D_{12}u \\ \alpha u \end{bmatrix} \quad (10)$$

where α is a design parameter to be selected. This gives the augmented plant $G \Rightarrow \hat{G}$ with modified state space matrices

$$\hat{D_{11}} = \begin{bmatrix} D_{11} \\ 0 \end{bmatrix}, \hat{D_{12}} = \begin{bmatrix} D_{12} \\ \alpha \end{bmatrix} \quad (11)$$

The selection of α represents a control penalty, limiting the control signal the designer is able to provide to the EDFA. Therefore, the selection of α should reflect the EDFA's pump characteristics, most notably the pump response time and maximum deliverable pump power.

B. Disturbance Weighting

The selection of the disturbance weighting is strictly based on characteristics of the input disturbances of the system. Specifically, using the known transfer function for a second order bandpass filter the disturbance weighting W_w is used to describe the frequency content of pilot tone disturbances:

$$W_w = \frac{k_w \cdot w_H \cdot s}{(s + w_L)(s + w_H)} + \alpha_w \quad (12)$$

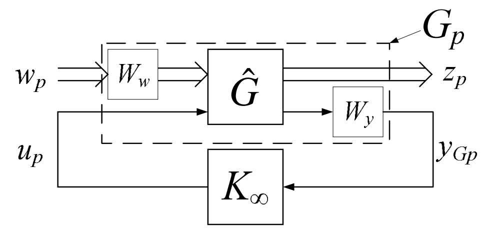

Fig. 4. Weighted augmented EDFA for H$_\infty$ controller synthesis

Based on this format we propose that the following parameters be selected accordingly:

- Choose w_L and w_H to describe the frequency bandwidth (rad/s) where the input signal resides. ie. $w_L = 2\pi \cdot f_{min}$ and $w_H = 2\pi \cdot f_{max}$
- Choose α_w to describe the maximum amplitude or modulation depth of the input disturbances. ie. $\alpha_w = 1 + m_{max}$
- Choose $k_w < 1$ to describe the amount of cross gain modulation within the disturbances.

C. Integral Control

It is a desired performance objective to minimize the steady state error of output signals. This can be accomplished by ensuring that the average total gain y_G is maintained. In order to achieve this, integral control should be imposed on the resulting H$_\infty$ controller. But, in order to meet the H$_\infty$ assumptions, the selected weighting function W_y must be proper, in the form:

$$W_y = \frac{\beta_y s + k_y}{s + \alpha_y} \quad (13)$$

Based on the above form, we select the following parameters:

- Choose $k_y = 1$ as the controller gain
- We select the bandwidth in which integral control should be imposed. That is, we choose $\beta_y << \frac{1}{f_{max}}$ and $\alpha_y << f_{min}$ such that integral control is imposed over the same bandwidth of the input disturbances.

D. Weighted Plant

From the augmented plant $\hat{G}$, a weighted augmented plant is created using the weighting functions $W_w(s)$, to describe the input signal disturbances, and $W_y(s)$, to impose integral control for minimum steady state error. This gives the new augmented plant G_p as depicted in Figure 4. Given this new plant G_p, the design objective is to find a stabilizing controller K_∞ such that the H$_\infty$ norm is minimized. That is

$$\|T_{z_p w_p}\|_\infty < \gamma_\infty \quad (14)$$

for some $\gamma_\infty > 0$. With the weighted augmented plant G_p, a K_∞ controller is formulated according to the H$_\infty$ algorithm in [15]. Then, combining the resulting controller with the imposed measurement weighting W_y, we arrive at our desired controller K which should then be applied to the original plant G, as shown in Figure 5.

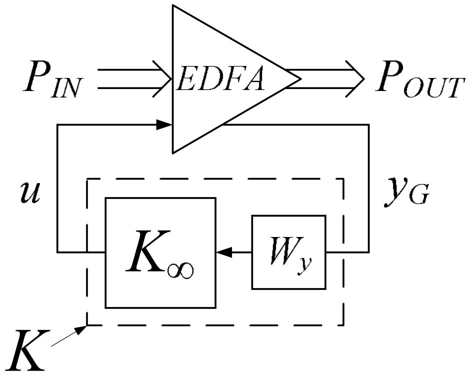

Fig. 5. Application of designed H_∞ controller upon original EDFA model

E. H_∞ Synthesis Parameters

In synthesizing an optimal H_∞ controller, it is required to select γ_{min} and γ_{max}. Thus based on our specified performance objectives and our plant model we choose $\gamma_{min,max}$ accordingly:

- From the H_∞ control algorithm we choose $\gamma_{min} = \bar{\sigma}(D_{11})$, where $\bar{\sigma}()$ is the maximum singular value.
- We now select $\gamma_{max} = \bar{\sigma}(D_{11}) + \epsilon$, where ϵ is the maximum allowable optical signal power deviation at the receiver.

VI. Simulation Results

For our numerical simulation we consider a single stage 16 channel C-band EDFA. Under nominal operating conditions all input channels are ON, operating with an average optical power of 0.1995mW with the EDFA providing an average gain of 14dB per channel. The EDFA is pumped at 980nm with a nominal pump power of 150mW. Through the use of MATLAB simulations, this section outlines the numerical values used for the EDFA model and presents the selection of model parameters W_w, W_y and α used for controller design. Finally, the transient response of the closed loop EDFA is observed under channel add/drop and pilot tones disturbances for three different control cases:

1) Uncontrolled case ($u = 0$)
2) PID Controller as defined in [14] (K_{PID})
3) H_∞ Controller (K)

A. Control Design Results

We now summarize the process for selecting the parameter values for controller design. In particular we consider the selection of W_w, W_y and α. Initially the disturbance weighting in (12) is selected with the parameters $k_w = 1 \times 10^6$, $B_d = 2\pi(1 \times 10^8)$, $w_d = 2\pi(1 \times 10^5)$ and $\alpha_w = 1$ which gives the weighting function two poles placed at -628.32 and -6.2832×10^8. Also, following the transfer function form in (13), $W_y(s)$ is selected with the parameters $k_y = 1$, $\beta_y = 1 \times 10^{-5}$, $\alpha_y = 1 \times 10^{-5}$.

With the selected weightings and choice of $\alpha = 1 \times 10^{-3}$, a 5^{th} order controller K_∞ is synthesized via MATLAB.

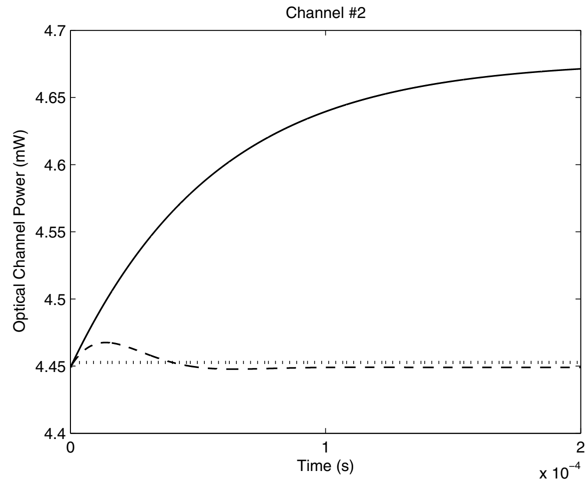

Fig. 6. EDFA Transient response for a channel drop on channel #1. Power excursion on surviving channel #2 for uncontrolled EDFA (solid line), PID controlled EDFA (dashed line) and H_∞ controlled EDFA (dotted line)

B. Step Response

To simulate the step response we apply a channel drop on channel 1. We then view the transient response of the output power on channel 2 (cross response) for the different control cases, as shown in Figure 6. In terms of performance we are specifically interested in the settling time and maximum power excursion. Immediately, the simulation results show significant improvements by both controllers in response time and signal power. However, the H_∞ controller exhibits a response time of $4.2\mu s$, compared to a response time of 0.1ms for the PID controlled EDFA. Also the H_∞ controller improves upon the maximum power excursion of the PID controller by approximately 80%.

However this improved performance does come at a cost. In Figure 7 we show how quickly the pump control must react to achieve such improved performance. The PID Controller requires a change in pump power of approximately 7.5mW within 0.1ms, while the H_∞ controller requires a 6.5mW drop in pump power within $4.2\mu s$.

C. Pilot Tones

For the EDFA simulation, a pilot tone is modulated onto both channel 1 and channel 2. The pilot tones are selected with the following parameters: $f_{c,1} = 20kHz$, $f_{c,2} = 40kHz$, $m_1 = 0.05$, $m_2 = 20kHz$ and $\Delta f = 1kHz$. Also, random binary data $d_1(t)$ and $d_2(t)$ was generated to be modulated onto the pilot tones. To gauge the performance of the different control cases we are interested in minimizing the cross response on unmodulated channels 3 to 16. Figure 8 shows the transient response of the surviving channel #3

From the results in Figure 8 we observe significant improvements in performance when the H_∞ controller is used. The maximum power excursions on channel #3 are reduced by approximately 97% when the H_∞ controller is used, compared against the uncontrolled EDFA. In comparison, the PID controller reduces the maximum power excursions by 80%.

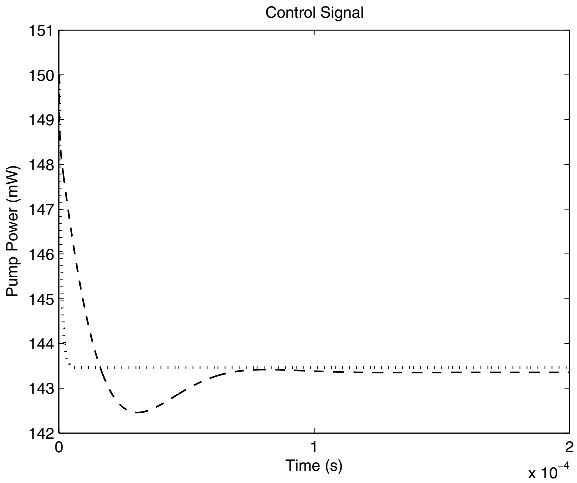

Fig. 7. Pump Power signal in response to a channel drop on channel #1: PID Controller (Dashed line), H_∞ Controller (Dotted Line).

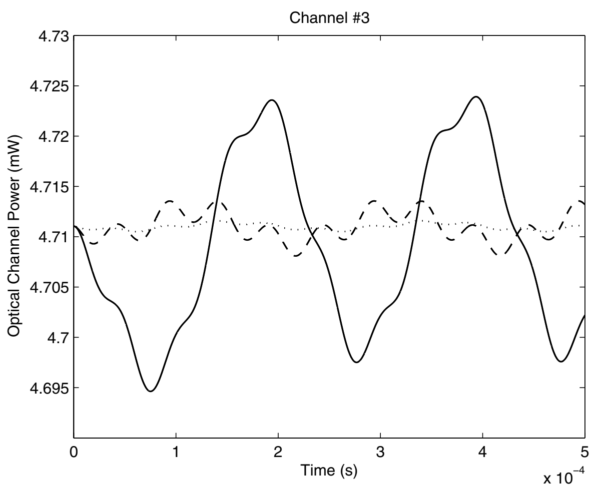

Fig. 8. EDFA pilot tone response for surviving channel #3 ($f_{c,1} = 20kHz$, $m_1 = 0.05$) and channel 2 ($f_{c,2} = 5.0kHz$, $m_2 = 0.10$) three different control cases: Uncontrolled Case (solid line), PID Controller (dashed line), H_∞ Controller (dotted Line).

VII. Conclusion

In this report we present a description and analytical model for pilot tone input disturbances. We also propose the use of an H_∞ controller to provide gain compensation for a single stage EDFA. For this control strategy a detailed design methodology is presented and implemented. Via MATLAB simulation, H_∞ control is shown to significantly improve network performance, when compared against an uncontrolled EDFA and a PID controlled EDFA. Specifically by improving the fast response times, the control schemes were shown to reduce power excursions on surviving channels due to these input disturbances.

The present paper examines the transient behavior of a single EDFA within a given optical link. It should be noted that in practice power excursions are shown to grow and become faster as they propagate through a series of EDFAs [12], stressing even more the need for effective control schemes. It is also well known that these power excursions are symptomatic of optical packet switching networks, where optical input powers are constantly changing [13]. Future work will address these packetized signal disturbances and the analysis of control performance will also be extended to chains of EDFAs. However, the control methodology reported in this work remains valid for a variety of system disturbances and network configurations.

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