Hierarchical Iterative Algorithm for a Coupled Constrained OSNR Nash Game

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Abstract— **This paper develops a hierarchical iterative OSNR algorithm based on a game theory framework. A Nash game is formulated between channels with channel utility related to maximizing channel optical signal-to-noise ratio (OSNR). The OSNR game has coupled utilities and coupled constraints, such that total power is kept below the nonlinearity threshold. Solving directly this game requires coordination among all channels and is impractical in networks. A duality approach is used instead, based on the recent theoretical results in [16]. This method offers a natural way to hierarchically decompose the coupled Nash game into a lower-level Nash game with no coupled constraints, and a higher-level link optimization problem for pricing parameters. The lower-level Nash game is analytically tractable, and its solution can be iteratively found via an algorithm decentralized with respect to channels. The price is adjusted at the network higher-level so that channels are induced to cooperate towards satisfying the coupled total power constraint.**

I. INTRODUCTION

Optical wavelength-division multiplexed (WDM) communication networks are evolving beyond statically designed links, so that relevant research questions are how to maintain network stability and optimal channel performance, [1]-[2]. Channel optical signal-to-noise ratio (OSNR) is an important performance factor, [3], as is its optimization, typically done statically [4]. Work on on-line decentralized OSNR optimization algorithms has been initiated very recently based on either optimization, [9]-[10], or game theory, [14], [15].

In recent years, game theory approaches have been used for optimization and control of networks [12]-[13], as an alternative to traditional network optimization [6]-[9]. For optical networks, such a game approach was first formulated in [14], where channel utility is related to maximizing channel OSNR. A closed-form Nash equilibrium solution was obtained and a decentralized iterative algorithm was developed. However no constraint on total channel power was not taken into account in [14]. Yet such constraints have to be considered in optical links, since all wavelength-multiplexed channels share the optical fiber, [3]. Total power launched into the fiber has to be below the nonlinearity threshold, which can be regarded as the link capacity constraint. In [15] the capacity constraint was indirectly considered by modifying each channel cost function. However the complexity of the cost function precluded finding an analytically tractable Nash solution and no iterative algorithms were developed to find it numerically.

The contribution of this paper is to explicitly consider the link power capacity constraint into the OSNR game. We formulate a coupled OSNR Nash game between channels with coupled utilities and coupled power constraint. Typically in wireless networks utilities are coupled, [12]-[13], while in congestion control constraints are coupled but utilities are not, [22]. The OSNR game is more general in that both utilities and constraints are coupled. Solving directly such a coupled Nash game requires coordination among possibly all channels and is impractical in networks. A natural decomposition that leads to decentralized algorithms is desirable.

A similar problem is flow optimization (congestion control) in networks, [20]- [22]. In [20], standard duality has been employed to decompose the optimization problem into a set of decoupled optimization problems for sources and a network problem for links. Instrumental was the property of utilities being decoupled and the system cost function being separable. Contrastingly, in optical networks the OSNR game with coupled power constraints is a genuine coupled Nash game: both utilities and constraints are coupled. This game is not separable in the standard sense and the approach in [20] cannot be applied.

Instead we propose to use the recent theoretical extension of duality results from optimization to coupled games, [16]. Our approach offers a natural way to hierarchically decompose the coupled Nash game into a lower-level Nash game with no coupled constraints between channels, and a higher-level network optimization problem for pricing. For simplicity we consider a single optical link. The multiple-link network case can be developed similarly.

The paper is organized as follows. In Section II we review the OSNR model and basic unconstrained OSNR Nash game, [14]. We also review recent results on general duality extension results for games, [16]. In Section III we formulate the OSNR Nash game with coupled constraints, based on total power capacity for all channels sharing a link. We prove that this OSNR Nash game with coupled constraints can be decomposed into a lower-level Nash game, and a higher-level system optimization problem for pricing. In Section IV we propose an iterative hierarchical algorithm to solve it. At the lower-level, for a given price received from the link the algorithm is decentralized with respect to channels. At the link level, the link pricing parameter is adjusted based on the measured total channel power, and acts as a coordination so that channels satisfy the total power constraint. An example is given in Section V and conclusions in Section VI.

II. BACKGROUND

A. OSNR Model and Basic Nash Game Formulation

We first review the analytical OSNR model and basic unconstrained Nash game as formulated in [14] for an optical link. Let an optical link be composed of N cascaded optical amplifiers (OAs), with a set $\mathcal{M} = \{1, ..., m\}$ of channels transmitted by wavelength-multiplexing. After every few tens of km of fiber (optical span), an OAs is used to amplify the optical power of all channels simultaneously, at the expense of introducing amplified spontaneous emission (ASE) noise. The amplifier's gain and ASE noise is wavelength-dependent so that each channel sees a different gain, G_i, and self-generated ASE noise power, ASE_i.

Let u_i and $n_{0,i}$ denote the input signal and noise optical power (at Tx), respectively, for the i^{th} channel. Similarly, let $p_{N,i}$, $n_{N,i}^{out}$ be output signal and noise optical power (at Rx). As in [4], [5], [14], the following non-restrictive assumptions are used: all spans in the link have equal length, L, and all the amplifiers in a link have the same spectral shape, G_i. The results can be modified easily for the relaxed assumptions of unequal spans. Optical amplifiers are operated in automatic power control (APC) mode such that a target total power is maintained. This mode compensates variations in fiber-span loss across a link [4]. Total power target is selected to be below the threshold for nonlinear effects [5]. For spans of equal length, this threshold and hence the total power target P_0 is the same. Then at the output of each intermediary span the following condition is automatically satisfied,

$$\sum_{j=1}^{m} p_{k,j} = P_0 \qquad \forall k = 1, \dots, N \tag{1}$$

The i^{th} channel OSNR, defined as $OSNR_i = \frac{p_{N,i}}{n_{N,i}^{out}}$, is given as in the following result which generalizes [4].

Lemma 1: [14] The channel OSNR at the link output is

$$OSNR_i = \frac{u_i}{n_{0,i} + \sum_{j \in \mathcal{M}} \Gamma_{i,j}\, u_j}$$

where $\mathbf{\Gamma} = [\Gamma_{ij}]$ is $(m \times m)$ system matrix with

$$\Gamma_{i,j} = \sum_{v=1}^{N} \frac{G_j^v}{G_i^v}\, \frac{ASE_{v,i}}{P_0} \qquad \forall\, i, j \in \mathcal{M}$$

and $ASE_{v,i}$ is ASE noise self-generated at the v^{th} optical amplifier, associated with the i^{th} channel.

The OSNR model (Lemma 1) is similar to the SIR model in wireless networks, but has a richer system matrix structure. Based on this model, a noncooperative Nash game can be formulated between channels towards OSNR maximization. Let $\mathbf{u} = [u_1, \dots, u_i, \dots, u_m]^T$ be the vector of channel powers at the Tx (action), and $\mathbf{u_{-i}} = [u_1, \dots, u_{i-1}, u_{i+1}, \dots, u_m]^T$ the vector obtained by deleting the i^{th} element, with $u_i \in \Omega$, $\Omega = [u_{min}, u_{max}]$. We denote compactly $\mathbf{u} = [u_i]$, or alternatively, $\mathbf{u} = (\mathbf{u_{-i}}, u_i) \in \Omega^m$, where $\Omega^m = \Omega \times \dots \times \Omega$, $\mho = \Omega^m$ is a rectangular, closed and bounded action set.

Each channel is player that minimizes its individual cost function $\overline{J}_i$ over $u_i \in \Omega$, by adjusting its own channel power at Tx, in response to the other channels' actions. The cost function $\overline{J}_i$ is taken as the difference between a pricing function and a utility function (indicating preference for higher ONSR), and defined as, [14],

$$\overline{J}_i(\mathbf{u_{-i}}, u_i) = \alpha_i\, u_i - \beta_i U_i(\mathbf{u_{-i}}, u_i) \tag{2}$$

where $\alpha_i u_i$ is a linear pricing term and U_i is channel utility function taken as

$$U_i(\mathbf{u_{-i}}, u_i) = \ln(1 + \frac{a_i}{\frac{1}{OSNR_i} - \Gamma_{i,i}}) \tag{3}$$

with $\alpha_i, \beta_i > 0$ as weighting factors. Equivalently,

$$U_i(\mathbf{u_{-i}}, u_i) = \ln(1 + a_i\, \frac{u_i}{X_{-\mathbf{i}}}) \tag{4}$$

where $X_{-\mathbf{i}} = \sum_{j \neq i} \Gamma_{i,j} u_j + n_{0,i}$, and $a_i > 0$ is a channel specific parameter (for scalability). U_i is monotone in OSNR, so that higher utility (low cost) means higher OSNR.

Definition 1: Consider an m-player channel game, each minimizing the cost function $\overline{J}_i$, over $u_i \in \Omega$. A vector $\mathbf{u}^* \in \Omega^m$ is called a Nash equilibrium (NE) solution if

$$\overline{J}_i(\mathbf{u}_{-i}^*, u_i^*) \leq \overline{J}_i(\mathbf{u}_{-i}^*, u_i) \qquad \forall u_i \in \Omega \qquad \forall i$$

for any given $\mathbf{u}_{-i}^*$. Definition 1 specifies that $\mathbf{u}^*$ is an NE when u_i^* is the solution to the individual optimization problem $\overline{J}_i$ for channel i, given all channels on its path have equilibrium power levels, $\mathbf{u}_{-i}^*$, [17]. NE optimality means that a player cannot improve its performance function by acting unilaterally. An NE solution can be viewed as a fixed-point vector solution $\mathbf{u}^* = [u_i^*]$ to the set of minimization problems $\overline{J}_i$ (2).
The following assumption guarantees that the NE solution is inner (interior to $\mho$).

(A.1) $u_i = u_{min}$ is not a solution to the minimization of the cost function $\overline{J}_i$, i.e., $\overline{J}_i((\mathbf{u}_{-i}, u_{min}) > \overline{J}_i((\mathbf{u}_{-i}, u_i)$, $\forall u_i \neq u_{min}$, for any given $\mathbf{u}_{-i}$. Similarly, $u_i = u_{max}$ is not a solution to the minimization of the cost function $\overline{J}_i$, i.e., $\overline{J}_i((\mathbf{u}_{-i}, u_{max}) > \overline{J}_i((\mathbf{u}_{-i}, u_i)$, for any given $\mathbf{u}_{-i}$.

Utility U_i, (4), is a twice continuously differentiable, monotone increasing and strictly concave function in u_i. Thus the cost function $\overline{J}_i$, (2), is strictly convex on Ω and a minimizing u_i^* exists. Channel specific parameters $\alpha_i, \beta_i > 0$ indicate a channel's willingness to pay the price and desire to maximize its OSNR, and can be selected such that (A.1) holds, for e.g. if $\alpha_i << \beta_i$. Conditions for uniqueness of the NE solution to this unconstrained OSNR game are given next in terms of the scalability parameters a_i.

Theorem 1: [14] The m-player game problem with individual cost functions $\overline{J}_i$, (2), admits a unique NE solution $\mathbf{u}^*$ if a_i are selected such that the diagonal dominance condition

$$\sum_{j \neq i} \Gamma_{i,j} < a_i, \qquad \forall i \in \mathcal{M}$$

holds. The unique NE solution $\mathbf{u}^*$ is inner and is given as

$$\mathbf{u}^* = \widetilde{\mathbf{\Gamma}}^{-1}\, \tilde{\mathbf{b}}$$

where $\widetilde{\mathbf{\Gamma}} = [\widetilde{\Gamma}_{i,j}]$ and $\tilde{\mathbf{b}} = [\tilde{b}_i]$ are defined as $\widetilde{\Gamma}_{i,i} = a_i$, $\widetilde{\Gamma}_{i,j} = \Gamma_{i,j}, j \neq i$, $\tilde{b}_i = \frac{a_i \beta_i}{\alpha_i} - n_{0i}$. and $\mathbf{\Gamma} = [\Gamma_{i,j}]$ is system matrix defined in Lemma 1.

Note that the basic OSNR game does not take into account total power constraints on channel powers at Tx. This is needed in optical links to limit nonlinearity effects. We extend this OSNR game formulation to explicitly address this coupled constraint in Section III.

B. Duality for Nash Games with Coupled Constraints

We review next some recent theoretical results on general Nash games with coupled constraints, [16]. Consider an m-player Nash game with general individual cost functions $J_i : \mho \to R$, $\mho := \Omega^m = \Omega \times \ldots \times \Omega$, subject to an additional coupled constraint

$$g(\mathbf{u}) \leq 0, \qquad \mathbf{u} \in \mho$$

where $g : \mho \to R$ is constraint function. Such an $\mathbf{u} \in \mho$ is called a feasible vector (NG-feasible). An NE solution can be defined with respect to individual costs J_i or, by using the concept of "system-like" cost function, [17]. This is a two-argument function $\widetilde{J} : \mho \times \mho \to R$, defined as

$$\widetilde{J}(\mathbf{u}; \mathbf{x}) := \sum_{i=1}^{m} J_i(\mathbf{u}_{-i}, x_i), \qquad \forall \mathbf{x} \in \mho \tag{5}$$

that we call the NG-game cost function. The NG-game cost function (5) has the special property of being separable in the second argument $\mathbf{x}$ for every given $\mathbf{u}$. Similarly, the constraint g can be augmented into an equivalent two-argument form, $\widetilde{g}$,

$$\widetilde{g}(\mathbf{u}; \mathbf{x}) = \sum_{i=1}^{m} g(\mathbf{u}_{-i}, x_i) \tag{6}$$

As before we write $\mathbf{u}, \mathbf{x} \in \mho$, as $\mathbf{u} = [u_i]$, $\mathbf{x} = [x_i]$, $u_i, x_i \in \Omega$. The form in (6) has also the special separability property. NG-feasibility is then equivalent to $\widetilde{g}(\mathbf{u}; \mathbf{u}) \leq 0$.

It was recently shown that a Nash game J_i with coupled constraints g is equivalent to a *constrained* minimization of the NG-game cost function, $\widetilde{J}$, (5), (6) with respect to the second argument, that admits a fixed-point solution (Lemma 1, [16]). This procedure is written compactly as

$$\mathbf{u}^* = \left[\ \arg\min_{\mathbf{x} \in \mho, \widetilde{g}(\mathbf{u};\mathbf{x}) \leq 0}, \widetilde{J}(\mathbf{u}; \mathbf{x}) \ \right]\Big|_{\mathbf{x}=\mathbf{u}} \tag{7}$$

and

$$\widetilde{J}(\mathbf{u}^*; \mathbf{u}^*) = \left[\ \min_{\mathbf{x} \in \mho, \widetilde{g}(\mathbf{u};\mathbf{x}) \leq 0} \widetilde{J}(\mathbf{u}; \mathbf{x}) \ \right] |_{\mathbf{x}=\mathbf{u}} \tag{8}$$

The compact notation in (7,8) indicates that the first step is a constrained minimization of $\widetilde{J}$, with respect to the second argument $\mathbf{x}$ for any given $\mathbf{u}$. From the parameterized solution $\mathbf{x} = \phi(\mathbf{u})$, the next step is to find a fixed-point solution (indicated by setting $\mathbf{x} = \mathbf{u}$), which yields $\mathbf{u}^*$. This $\mathbf{u}^*$ satisfies

$$\widetilde{J}(\mathbf{u}^*; \mathbf{u}^*) \leq \widetilde{J}(\mathbf{u}^*; \mathbf{x}), \quad \forall \mathbf{x} \in \mho, \quad \widetilde{g}(\mathbf{u}^*; \mathbf{x}) \leq 0 \tag{9}$$

with $\widetilde{g}(\mathbf{u}^*; \mathbf{u}^*) \leq 0$. The individual components of $\mathbf{u}^* = [u_i^*]$ constitute an NE solution. As in standard constrained optimization [19], a two-argument Lagrangian function that incorporates the constraints can be defined for $\widetilde{J}$,

$$\widetilde{L}(\mathbf{u}; \mathbf{x}; \mu) = \widetilde{J}(\mathbf{u}; \mathbf{x}) + \mu^T \, \widetilde{g}(\mathbf{u}; \mathbf{x}) \tag{10}$$

with $\mu \geq 0$, $\widetilde{g}(\mathbf{u}; \mathbf{u}) \leq 0$. Also a dual cost function $D(\mu)$ is defined as

$$D(\mu) := \left[\ \min_{\mathbf{x} \in \mho} \widetilde{L}(\mathbf{u}; \mathbf{x}; \mu) \ \right]\Big|_{\mathbf{x}=\mathbf{u}} \tag{11}$$

where minimization on the right-hand side is done in a fixed-point sense, as in (8). The dual NG problem is defined as that of *maximizing* $D(\mu)$ subject to $\mu \geq 0$, with

$$D^* = \max_{\mu \geq 0} D(\mu) \tag{12}$$

Theorem 2: [16] $(\mathbf{u}^*; \mu^*)$ is an optimal NE solution - multiplier pair in the sense of (9), (12), if and only if

$$NG-feasibility \quad \mathbf{u}^* \in \mho \qquad \widetilde{g}(\mathbf{u}^*; \mathbf{u}^*) \leq 0$$

$$Dual\,feasibility \quad \mu^* \geq 0$$

$$\begin{array}{ll} Lagrangian & \mathbf{u}^* = \left[\ \arg\min_{\mathbf{x} \in \mho} \widetilde{L}(\mathbf{u}; \mathbf{x}; \mu^*) \ \right]\Big|_{\mathbf{x}=\mathbf{u}} \\ optimality & \end{array}$$

$$\begin{array}{ll} Complementary & \mu^* \widetilde{g}(\mathbf{u}^*; \mathbf{u}^*) = 0 \\ slackness & \end{array}$$

For convex cost functions and constraints, based on the special separability of both NG-game cost function (5) and constraints (6), the dual cost function $D(\mu)$ (11, (10) can be decomposed as

$$\begin{aligned} D(\mu) &= \sum_{i=1}^{m} \left[\ \min_{x_i \in \Omega} L_i(\mathbf{u}_{-i}, x_i, \mu) \ \right] |_{x_i = u_i} \\ &= \sum_{i=1}^{m} L_i(\mathbf{u}_{-i}^*(\mu), u_i^*(\mu), \mu) \end{aligned} \tag{13}$$

where $L_i(\mathbf{u}_{-i}, x_i, \mu) = J_i(\mathbf{u}_{-i}, x_i) + \mu \, g(\mathbf{u}_{-i}, x_i)$. $\mathbf{u}^*(\mu) = [u_i^*(\mu)] \in \Omega^m$ is a fixed-point solution to the set of minimizations on the right-hand side of (13).

III. OSNR GAME WITH COUPLED CONSTRAINTS & DECOMPOSITION

In this section we consider the OSNR game and explicitly take into account the link power capacity constraint. The basic unconstrained OSNR game, [14], did not consider total power constraints on channel powers at Tx. Yet such power capacity constraints have to be taken into account as all wavelength-multiplexed channels in a link share the optical fiber, [3]. In order to limit the nonlinear effects, the total power launched into the fiber has to be below the nonlinearity threshold. This can be regarded as the link power capacity constraint. By (1) this is satisfied at the output of each intermediary span, but not at the Tx.

Consider now that each channel minimizes a cost function $\overline{J}_i$, (2), over $u_i \in \Omega$, where U_i is as in (4), or (3), with the additional coupled inequality constraint,

$$\sum_{j=1}^{m} u_j \leq P_0 \tag{14}$$

based on the total power capacity for all channels in a link. An NE solution $\mathbf{u}^* \in \Omega^m$ to the OSNR game (2) has to satisfy the additional coupled constraint (14).

Recall that the basic OSNR game in Section II.A had a rectangular action set $\mho = \Omega^m$. Each player had a separable action set Ω, so that players could choose independently their actions within Ω. In the present game formulation with the coupled constraint (14), the action sets are no longer separable. Instead a coupled action set $\overline{\Omega}$ exists

$$\overline{\Omega} = \{\mathbf{u} \in \Omega^m \quad | \quad \sum_{j=1}^{m} u_j \leq P_0\}$$

A player i cannot choose independently his action, but has to choose it from the projection set $\overline{\Omega}_i(\mathbf{u}^*_{-i})$, [17],

$$\overline{\Omega}_i(\mathbf{u}^*_{-i}) = \{x_i \in \Omega \quad | \quad \sum_{j \neq i}^{m} u^*_j + x_i \leq P_0\} \tag{15}$$

which is coupled. Since cost functions $\overline{J}_i$, (2), are strictly convex and $\overline{\Omega}$ is compact and convex action set (although coupled), by Theorem 4.4, [17]), an NE solution exists for this OSNR game with coupled constraints. An NE solution for this game with coupled constraints satisfies

$$\overline{J}_i(\mathbf{u}^*_{-i}, u^*_i) \leq \overline{J}_i(\mathbf{u}^*_{-i}, x_i) \qquad \forall x_i \in \overline{\Omega}_i(\mathbf{u}^*_{-i}) \qquad \forall i \tag{16}$$

for any given $\mathbf{u}^*_{-i}$. However, due to the coupling in $\overline{\Omega}_i(\mathbf{u}^*_{-i})$ (15) solving directly for an NE solution requires coordination among possibly all channels, which is impractical.

Note that this OSNR game with constraint (14) is in the class of games with coupled constraints. We will use duality results as reviewed in Section II.B towards solving it. Specifically, we show next that for the coupled constrained OSNR game $\overline{J}_i(\mathbf{u}_{-i}, u_i)$, (2), (14), a natural hierarchical decomposition can be obtained.

Theorem 3: Consider the coupled OSNR Nash game with cost functions $\overline{J}_i(\mathbf{u}_{-i}, u_i)$, (2), subject to the linear constraint (14). Then the associated dual cost function $D(\mu)$ defined as

$$D(\mu) := \left[\; \min_{\mathbf{x} \in \mho} \widetilde{L}(\mathbf{u}; \mathbf{x}; \mu) \; \right]\Big|_{\mathbf{x}=\mathbf{u}}$$

is decomposed as

$$D(\mu) = \sum_{i=1}^{m} \overline{L}_i(\mathbf{u}^*_{-i}(\mu), u^*_i(\mu), \mu) + \sum_{i=1}^{m} \mu\, (\mathbf{e}^T \mathbf{u}^*_{-i} - P_0) \tag{17}$$

where $\mathbf{u}^*(\mu) = [u^*_i(\mu)] \in \Omega^m$ is an NE solution to the *modified* Nash game with cost functions $\overline{L}_i$

$$\overline{L}_i(\mathbf{u}_{-i}, x_i, \mu) = \overline{J}_i(\mathbf{u}_{-i}, x_i) + \mu\, x_i, \qquad i \in \mathcal{M} \tag{18}$$

or,

$$\overline{L}_i(\mathbf{u}_{-i}, u_i, \mu) = \overline{J}_i(\mathbf{u}_{-i}, u_i) + \mu\, u_i, \qquad i \in \mathcal{M}$$

which has no coupled constraints.

Proof: We apply duality results to the game $\overline{J}_i$, (2), with coupled constraint (14). Since each cost function $\overline{J}_i$, (2), is convex in u_i and the constraints are linear, (13) holds for the associated $D(\mu)$. Then we obtain

$$D(\mu) = \sum_{i=1}^{m} \left[\; \min_{x_i \in \Omega} L_i(\mathbf{u}_{-i}, x_i, \mu) \; \right] |_{x_i = u_i} \tag{19}$$

for $L_i(\mathbf{u}_{-i}, x_i, \mu) = \overline{J}_i(\mathbf{u}_{-i}, x_i) + \mu\, g(\mathbf{u}_{-i}, x_i)$, where $\overline{J}_i$ is given here as in (2). We also rewrite the linear constraint (14), $g(\mathbf{u}) \leq 0$, $g(\mathbf{u}) = \sum_{j=1}^{m} u_j - P_0 \leq 0$ in a two-argument form. Therefore, for $\mathbf{u} = (\mathbf{u}_{-i}, x_i)$, we can write

$$g(\mathbf{u}_{-i}, x_i) = \mathbf{e}^T \mathbf{u}_{-i} + x_i - P_0 \tag{20}$$

where $\mathbf{e} = [1, \ldots, 1]^T$ is the $(m-1) \times 1$ all ones vector. Using (20) in the foregoing we see that here L_i is given as

$$L_i(\mathbf{u}_{-i}, x_i, \mu) = \overline{J}_i(\mathbf{u}_{-i}, x_i) + \mu\, x_i + \mu(\mathbf{e}^T \mathbf{u}_{-i} - P_0) \tag{21}$$

with $\overline{J}_i$ as in (2). Therefore, $D(\mu)$ is decomposed as in (19) with L_i as in (21).

We can further decompose this as follows. On the right-hand side of (19) we have to perform first minimization of L_i with respect to x_i, and then find a fixed point solution (as procedurally done in (8)). Note that in L_i, (21), only the first two terms depend on x_i. Thus using (21) into (19) and isolating the terms that are independent of x_i, we can write

$$D(\mu) = \sum_{i=1}^{m} \min_{x_i \in \Omega} [\overline{L}_i(\mathbf{u}_{-i}, x_i, \mu)]|_{x_i = u_i} + \sum_{i=1}^{m} \mu\, (\mathbf{e}^T \mathbf{u}_{-i} - \, P_0) \tag{22}$$

with

$$\overline{L}_i(\mathbf{u}_{-i}, x_i, \mu) = \overline{J}_i(\mathbf{u}_{-i}, x_i) + \mu\, x_i \tag{23}$$

as in (18). After minimization in each subproblem i on the right-hand side of (22) we obtain x^*_i as function of $\mathbf{u}_{-i}$, that we denote $x^*_i = x_i(\mathbf{u}_{-i})$, so that

$$x_i(\mathbf{u}_{-i}) = \arg \min_{x_i \in \Omega} [\overline{L}_i(\mathbf{u}_{-i}, x_i, \mu)], \qquad \forall i \in \mathcal{M} \tag{24}$$

with $\overline{L}_i$ defined as in (23). Since we search for a fixed point solution we set simultaneously

$$x_i(\mathbf{u}_{-i}) = u_i, \qquad \forall i \in \mathcal{M}$$

and solve the m equations for a vector solution $\mathbf{u} = [u_i]$. We denote such a solution as $\mathbf{u}^*(\mu) = [u^*_i]$, or $\mathbf{u}^*(\mu) = (\mathbf{u}^*_{-i}, u^*_i)$, which depends on μ. Hence as a fixed-point minimization, substituting for this $\mathbf{u}^*(\mu)$ on the right-hand side of (22) yields $D(\mu)$ as in (17). Such a fixed-point vector solution $\mathbf{u}^* = [u^*_i]$ to the set (24) is an NE solution to the Nash game with cost functions, $\overline{L}_i$, (23) or (18) and the last part of the claim follows.

■

Remark 1: Theorem 3 gives a hierarchical decomposition into a lower-level modified Nash game with cost functions $\overline{L}_i$ (18) with no coupled constraints, and a higher-level optimization problem used for coordination. For $D(\mu)$ as in (17), D^* can be obtained by the one-dimensional optimization

$D^* = \max_{\mu \geq 0} D(\mu)$. In general, $\mathbf{u}^*(\mu)$, may not be NE optimal (for the given μ) unless the slackness conditions hold. However by Theorem 2 there exists a $\mu^* \geq 0$ such that $\mathbf{u}^*(\mu^*) = [u_i^*(\mu^*)]$ is NE optimal. Hence we can solve the dual NG problem and find μ^*. A sufficient condition is that the dual cost $D(\mu)$ is strictly concave in μ, for $\mathbf{u}^*(\mu)$ as obtained from the lower-level game.

Remark 2: The decomposition in Theorem 3 has also the interpretation of a two-level hierarchical game, [17]. The upper-level is a Stackelberg game with the link being the leader that sets the link pricing μ and the channels being the m followers. In this game, the system (link) sets the link price and the channels follow with appropriate actions such that the link (dual) cost is maximized. At the lower-level the channels play an m-player noncooperative game between themselves. For a given Lagrangian price as set by the network, player i chooses the action u_i to minimize its own cost function $\overline{L}_i$, (18), and the link price acts as a coordination signal.

IV. HIERARCHICAL ITERATIVE ALGORITHM

In this section, based on the decomposition results in Theorem 3, we propose an iterative hierarchical algorithm to solve the ONSR Nash game $\overline{J}_i$, (2), with coupled constraints, (14) or (15).

By Theorem 2 applied to the coupled OSNR game $\overline{J}_i$, (2), (14), $(\mathbf{u}^*, \mu^*)$ is an optimal NE solution - Lagrange multiplier pair if and only if $\mathbf{u}^*$ is NG-feasible, $\mu^* \geq 0$, $\mu^*(\sum_{i=1}^m \mathbf{u}_i^* - P_0) = 0$ (slackness condition) and the Lagrangian optimality condition

$$\mathbf{u}^* = \left[\ \arg\min_{\mathbf{x}\in\mho} \widetilde{L}(\mathbf{u};\mathbf{x};\mu^*) \ \right]\Big|_{\mathbf{x}=\mathbf{u}} \qquad (25)$$

holds. The solution can be found based on the dual problem, or alternatively we can exploit the linear constraint and adjust the price to satisfy the slackness condition. This is the approach that we take, specifically based on price adjustment at the higher-level, and on explicit solution of the lower-level channel game.

We solve the Lagrangian optimality condition (25) for $\mathbf{u}^*(\mu)$ and we impose the NG-feasibility condition

$$\sum_{i=1}^{m} u_i^*(\mu) \leq P_0, \qquad u_i^* \in \Omega, \qquad i \in \mathcal{M} \qquad (26)$$

By Theorem 3 and (17), we see that $\mathbf{u}^*(\mu)$ solving (25) is in fact an NE solution to the modified Nash game (18) with no coupled constraints.

From (18, 2) it can be immediately seen that $\overline{L}_i$, (18), has the same form as $\overline{J}_i$, (2) with a modified pricing parameter $\overline{\alpha}_i = \alpha_i + \mu$. Thus in the presence of coupled power constraints (14), the overall pricing parameter $\overline{\alpha}_i$ has two components: a channel set pricing α_i, and a link price μ, common to all channels due to coupled constraint.

Therefore for each given μ, with $\overline{\alpha}_i = \alpha_i + \mu$, the NE solution $\mathbf{u}^*(\mu)$ to the lower-level game $\overline{L}_i$ (18) is unique, by Theorem 1, and can be obtained as

$$\mathbf{u}^*(\mu) = \widetilde{\mathbf{\Gamma}}^{-1}\,(\frac{1}{\overline{\alpha}_i}\mathbf{b}_0 - \mathbf{n}_0) \qquad (27)$$

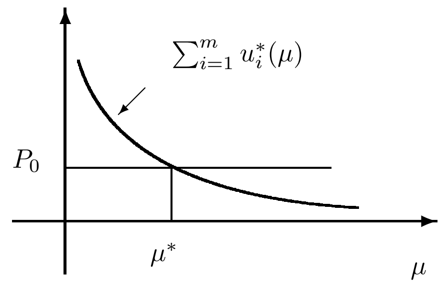

Fig. 1. Graphical plot of total power versus price

where $\mathbf{b}_0 = [a_i\beta_i]$, $\mathbf{n}_0 = [n_{0i}]$.

From (27), since $\overline{\alpha}_i = \alpha_i + \mu$, it is seen that all components of $\mathbf{u}^*(\mu)$ decrease with μ. The optimal link price μ^* can be obtained by maximizing $D(\mu)$, or easier, as the point of interception between the curve representing total power, $u_T^*(\mu) = \sum_{i=1}^m u_i^*(\mu)$ versus μ, with the level P_0 (see (26) and Fig. 1). This will ensure that the slackness condition holds and has the interpretation of a coordination mechanism. The optical link (coordinator) sets the link price at the optimal value μ^*. The channels respond by acting in their own interest and adjust their power levels to $u_i^*(\mu^*)$ that minimizes their own cost. This channel cost is the net cost of satisfying the total pricing (individual and total power pricing) minus its own utility function. Based on this interpretation we consider the following iterative hierarchical algorithm for both coordinating link price (higher-level) and channel powers (lower-level).

Link Algorithm

Every K iterations of the channel algorithm, the new link price μ is computed based on the received total power for all channels in the link $u_T(K) = \sum_{j=1} u_j(K)$ as

$$\mu(k+1) = [\mu(k) + \eta \quad (u_T(K) - P_0) \quad]^+ \qquad (28)$$

where η is the step-size and $[z]^+ = max\{z, 0\}$.

Note that this simple update relation for the price based on Fig. 1 is according to the supply and demand. It is decentralized and requires only a measurement of the total power into the link. Moreover under stationary channel powers, it corresponds to a simple gradient descent technique. This holds if the link price is adjusted slower than channel powers. At the higher level, $\mu(k)$ acts as a coordination price that aligns individual optimality with the system constraint, (14) or (26). This is the new link price communicated to the channels, who repeat K iterations of the following algorithm.

Channel Algorithm

Based on a received link price $\mu(k)$ from the link, the overall price is set as $\overline{\alpha}_i(k) = \alpha_i + \mu(k)$. Channel power vector is computed iteratively as

$$u_i(n+1) = \frac{\beta_i}{\overline{\alpha}_i(k)} - \frac{1}{a_i}\left(\frac{1}{OSNR_i(n)} - \Gamma_{i,i}\right)u_i(n) \qquad (29)$$

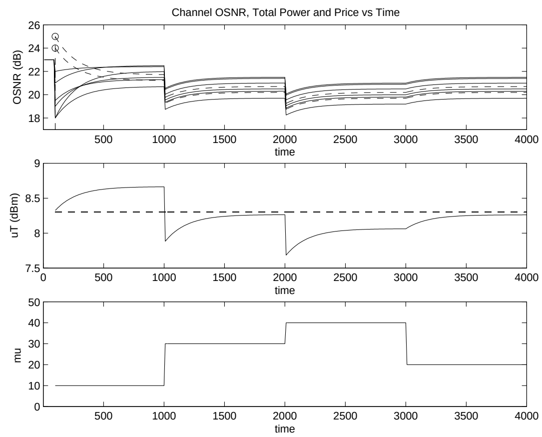

Fig. 2. OSNR, total power and price evolution in time

Note that this corresponds to implementing a decentralized algorithm, since the only information fedback is the individual channel $OSNR_i$, that can be measured in real-time, and the channel "gain", $\Gamma_{i,i}$. For a fixed μ ($\overline{\alpha}_i$) this algorithm converges to the optimal NE solution $\mathbf{u}^*(\mu)$ (27), [14]. This will also hold if the link price μ is updated much slower than channel powers. Even if the optimal solution is coupled as in (27), it can be iteratively computed by the decentralized algorithm (29). Thus at the lower-level game, individual channels do not have to coordinate with other channels.

V. NUMERICAL EXAMPLE

A MATLAB simulation was used for a link with ten amplified spans and eight channels. Each optical amplifier has a parabolic spectral gain profile and the link has a total power constraint of 8.3 dBm. Assume initially that only the first 6 channels were present and the optimal powers set for equalized OSNR. At step $t = 100$ two new channels 7 and 8 are added. With transmitter powers maintained at the same level as before the add event, the OSNR for the existing channels has a sudden drop at $t = 100$ (see Fig. 2), due to the extra channels sharing the link. If the channel iterative algorithm with $\beta_i = 1$, $a_i = \Gamma_{i,i}$, $\alpha_i = 0.1\Gamma_{i,i}$ and $\mu = 10$ is used to adjust all channel powers, then channel OSNR levels converge to high steady-state values (Fig. 2) but the total power limit is exceeded in the first interval. The link adjusts the link price μ via the link algorithm every $K = 100$ iterations (every 1000 time steps in Fig. 2). Channels readjust to new levels and after a few link iterations, the total power satisfies the constraint, while the resulting OSNR levels are around 20dB, slightly below than during the first interval.

VI. CONCLUSIONS

We considered a game theory approach for OSNR optimization. We extended [14] to consider explicitly the optical link power capacity constraints into the game formulation, as required in optical WDM networks. We formulated an OSNR Nash game with coupled utilities and coupled power constraints. Solving directly such a coupled Nash game requires coordination among possibly all channels and is impractical. Instead we used an approach based on the recent duality extension for Nash games. This allowed us to naturally decompose the coupled game into a lower-level Nash game, with no coupled constraints, and a higher-level link problem. We proposed an iterative hierarchical algorithm to solve it. At the lower-level, for a given price received from the link the algorithm is decentralized with respect to channels. At the link level, the link pricing parameter is adjusted based on the measured total channel power, and acts as a coordination so that channels satisfy the total power constraint. Future work will address extension to the multi-link case, as well as incorporation into this game framework of other optical network constraints, in addition to total power constraint.

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