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Application of Robust L_2 Control to Erbium Doped Fiber Amplifier: Input and State Uncertainty

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***Abstract*— We extend the results of [9] to increase the robustness of an L_2 nonlinear controller applied to an erbium doped fiber amplifier (EDFA). We consider the rejection of input and state uncertainties to a full information (FI) extended state space model of the EDFA. The generic robust stabilization problem is solved and the concept of state uncertainty is shown to be associated to parametric uncertainty in the EDFA model. We compare the improved robust L_2 controller to the standard L_2 controller of [9].**

I. INTRODUCTION

Regarded as the greatest innovation since optical fiber, the erbium doped fiber amplifier (EDFA) has revolutionized the optical communications industry in the past decade by enabling amplification of many lightwave channels simultaneously [1]. An EDFA is an optical fiber that is usually a few meters in length that has been doped with erbium ions (Er^{3+})[2]. Erbium ions absorb certain wavelengths of light energy to excite the atoms into higher energy states. Incident signal photons interact with the excited Er^{3+} ions to release their stored energy as photons in the 1550nm band [3]. The resultant released photon propagates with the same phase, polarity and direction as the incident photon, hence, signal amplification is realized. EDFAs enable increased transmission capacity and network transparency in wavelength division multiplexing (WDM) networks by simultaneously amplifying many wavelength channels directly in the optical domain. However, there are a number of challenges that must be overcome as WDM optical networks are transformed from static point-to-point systems into dynamic, reconfigurable networks.

One such challenge arises when information is dynamically routed through fiber links [1]. Data channels are expected to be added and dropped at designated points in the network with minimal power transients occuring[6][7]. Furthermore, communication channels in the optical networks must allow for rerouting around broken connection links or congested traffic routes. In this context, control of the EDFA represents an essential design problem. As channels are added/dropped from the network, the EDFA has been shown to suffer from cross-gain modulation which induces power transients on surviving channels[17]. These transients are detrimental to channel performance and control schemes reduce them [17]-[20]. Though effective, these control schemes lack a systematic approach for control design, virtually all being based on conventional PID and linearization. The EDFA is a nonlinear, multivariable system, whose behaviour is highly dependent on its operating point. Thus, there is a need for more systematic and sophisticated control that results in robust and dynamic EDFA devices that could operate over a wide range of operating conditions. This problem was addressed very recently in [9] where an L_2 nonlinear controller was developed based on an extended, more complete nonlinear EDFA model that includes amplified spontaneous emission (ASE). The L_2 nonlinear control scheme in [9] is based on [11]-[16] and restricts the average inversion level around a predefined operating point. The restriction of the average inversion level ensures that the channel gains will remain constant. Simulation results showed very promising results compared to PID controllers [10] by producing a faster and smoother transient response than its linear counterpart.

Robustness of the control design was not addressed in [9]. This subject is of great importance for the EDFA as uncertainties are always present in modeling. For different optical amplifiers, parameters do not have exactly the same values, but exhibit variations around their nominal values. Physical implementations of controllers on devices are never perfect since parameter data and signal measurements may not be ideal.

In this paper, we specifically address the EDFA robustness aspect. Starting from the extended nonlinear EDFA model with ASE introduced in our previous work [9], we proceed to explicitly consider robustness into the control design. We derive a representation for state uncertainty in a general model to account for unknown factors that cause the state value to deviate from its expected value. We then explicitly derive a relationship between state uncertainty and the absorption and emission coefficients in the EDFA model, α_k and g_k, respectively.

The L_2 nonlinear control framework is amenable to including additional robustness objectives [5]. Robust control for a perturbed system is realized by an L_2 control problem for an augmented system. Based on a derived general extended state space model of an EDFA that includes input and state uncertainty, we apply L_2 nonlinear control. The robust L_2 controller developed in this paper is compared via simulations with the standard L_2 controller [9] showing significant performance improvements. Specifically, we observe a faster response from the robust L_2 controller compared with [9].

The paper is organized as follows. Section II presents the EDFA model with ASE in a state space form from

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[9]. The section also reviews general control and robustness results [5],[21]-[22]. Section III presents the derivation of state uncertainty and its parametric association to the EDFA model. In addition, section III includes the input and state uncertainty in the extended state space model of the EDFA. Section IV presents simulation results for the robust and standard L_2 nonlinear controller. Section V provides the conclusions.

II. BACKGROUND

We outline the Erbium doped fiber amplifier model from [9]. We also review some well known robust control results.

A. EDFA Model

The basic mechanisms in an EDFA are stimulated emission, which amplifies the signal, and spontaneous emission, which causes noise. Each input channel is designated by its own wavelength. This light signal passes through the erbium doped fiber to emerge amplified at the output. The various wavelengths of light within the beam are amplified by different amounts according to the absorption and emission spectrums of the EDFA. The source of amplification energy for the EDFA comes from a pump, which is a laser light at the input operating at 980nm or 1480nm wavelengths[1][3]. Amplified spontaneous emission (ASE) occurs when an excited Er^{3+} ion releases its stored energy without the interaction of an incident photon, resulting in a photon of random phase, polarity and direction[4].

Based on the raw EDFA equations in PDE form in [2], a system EDFA model including ASE has been obtained in [9]. The rate equation and output equations in state space form are

$$
\begin{aligned}
\dot{x} &= -\frac{x}{\tau} - \frac{1}{\zeta \tau L} \sum_k (-g_k m \nu_k L x \\
&+ u_k [\frac{-g_k m \nu_k x}{(\alpha_k + g_k)x - (\alpha_k)} \\
&\quad \cdot (1 - e^{u_k \{(\alpha_k + g_k)x - (\alpha_k)\} L}) \\
&+ (e^{u_k \{(\alpha_k + g_k)x - (\alpha_k)\} L} - 1) Q_k^{in}]) \\
Q_k^{out} &= \frac{-g_k m \nu_k x}{(\alpha_k + g_k)x - (\alpha_k)} \\
&\quad \cdot [1 - e^{u_k \{(\alpha_k + g_k)x - (\alpha_k)\} L}] \\
&+ e^{u_k \{(\alpha_k + g_k)x - (\alpha_k)\} L} Q_k^{in}
\end{aligned} \tag{1}
$$

where x is the average inversion level of the EDFA defined as the proportion of the Erbium atoms that are excited into the higher energy states. In (1), Q_k^{in} and Q_k^{out} represent the normalized input and output powers of the k^{th} EDFA channels, including the pump. They are called photon flux [2][17], and defined as $Q_k = \frac{P_k}{h\nu_k}$, where P_k denotes the power of the k^{th} beam of light in Watts. The absorption and gain spectra of the k^{th} EDFA channels are α_k and g_k, respectively. Various other specified parameters appear in (1) such as L(fiber length); τ(spontaneous lifetime); ρ(active Erbium atom number density); S(fiber cross section); $\zeta = \frac{\rho S}{\tau}$; u_k(propagation direction,$+/-1$);m(fiber modes);ν_k(noise bandwidth).

TABLE I
NONLINEAR EDFA SYSTEM TERMS

Terms	Expression
f(x)	$-\frac{x}{\tau_o} - \frac{1}{\zeta\tau_o L}\sum_k(-g_k m\nu_k Lx + u_k[\frac{-g_k m\nu_k x}{(\alpha_k+g_k)x-(\alpha_k)}(1 - e^{u_k\{(\alpha_k+g_k)x-(\alpha_k)\}L})])$
$g_1(x)$	$-\frac{1}{\zeta\tau_o L}[u_1(e^{u_1\{(\alpha_1+g_1)x-(\alpha_1)\}L} - 1)$ ... $u_N(e^{u_N\{(\alpha_N+g_N)x-(\alpha_N)\}L} - 1)]$
$g_2(x)$	$-\frac{1}{\zeta\tau_o L}u_p(e^{u_p\{(\alpha_p+g_p)x-(\alpha_p)\}L} - 1)$

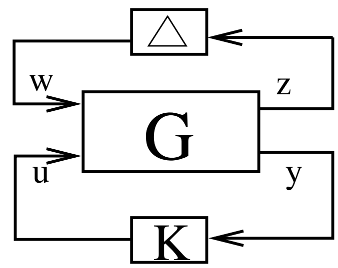


Fig. 1. Unstructured Uncertainty and Robust Controller

Model (1) has been derived assuming that the erbium ion distribution and the average inversion level are uniformly distributed in the EDFA. Furthermore, the state, x, is directly measurable by inverting the output equations.

The model (1) can be represented in the full information (FI) [5][14][16] extended state space form, (2), which is amenable to applying L_2 nonlinear control. The representation, (2) consists of a state equation, an output equation, y, and an output performance equation, z, which is to be designed. The purpose of the performance output is to choose which variables to attenuate in the model.

$$
G : \left\{ \begin{array}{ccccccc}
\dot{x} & = & f(x) & + & g_1(x)w & + & g_2(x)u \\
z & = & h_1(x) & + & k_{11}(x)w & + & k_{12}(x)u \\
y & = & \begin{bmatrix} x \\ 0 \end{bmatrix} & + & \begin{bmatrix} 0 \\ I \end{bmatrix} w & &
\end{array} \right. \tag{2}
$$

We treat the input signal channels, Q_k, as disturbances to the system, denoted w, while one of the channels represents the pump signal, denoted as u. We associate the terms in (1) to $f(x)$, $g_1(x)$ and $g_2(x)$ in (2) as in Table I. A p subscript refers to a pump parameter.

B. Robustness Review

This section briefly reviews some well known control and robust theory concepts.

The first step when applying robust control theory is to quantify plant uncertainty, which can be structured or unstructured. Structured uncertainty refers to uncertainty in a parameter, while unstructured uncertainty refers to an unknown part in the system without explicit form[21].

Consider a plant, G, where all the uncertainty has been included in the unknown part Δ, as in Figure 1. A controller, K, that compensates the uncertainty and controls the system, can be designed by using L_2 nonlinear control theory

[5],[11]-[16]. The system structure of Δ will be defined later in the section.

Consider the nominal system, G, in Figure 1, when $\Delta = 0$. We say that a system, G, described generically as in (2), has an L_2 gain, γ, if the following holds:

$$\int_0^T \|z(t)\|^2 dt \leq \gamma^2 \int_0^T \|w(t)\|^2 dt \tag{3}$$

where γ represents the amount of attenuation from the disturbance $w(t)$ to the performance output $z(t)$.

In Figure 1, the closed loop system from w to z is defined via a lower fractional transform [21] on G by K without uncertainty Δ:

$$z = F_\ell(G, K)w \tag{4}$$

where, with some abuse of notation

$$\begin{bmatrix} z \\ y \end{bmatrix} = G \begin{bmatrix} w \\ u \end{bmatrix}, \qquad u = Ky \tag{5}$$

Here, (5) is an input-output mapping representation of the nonlinear system (2).

The L_2 control objective is to find a controller K applied to G as in Figure 1 such that

1) $F_\ell(G, K)$ is asymptotically stable for $w = 0$
2) $F_\ell(G, K)$ has L_2 gain from w to z less than or equal to a specified number γ.

When unstructured uncertainty Δ is present the upper fractional transform on G by Δ (without controller K) is defined as the following system:

$$y = G_\Delta u \tag{6}$$

where,

$$\begin{bmatrix} z \\ y \end{bmatrix} = G \begin{bmatrix} w \\ u \end{bmatrix}, \qquad w = \Delta z \tag{7}$$

Typically, only Δ is assumed in the class of asymptotically stable systems with L_2 gain bounded by some ϵ.

The Generic Robust Stabilization Problem[5]: *Assuming that G_Δ is well posed, [22], find a controller, K, from y to u such that the composite uncertain system, G_Δ, is internally asymptotically stabilized by K for every Δ in the class of stable, bounded L_2 gain systems.*

Under this assumption, it was shown in [5] via a small gain theorem argument [11] that the general robust stabilization problem can be achieved with an L_2 nonlinear controller for the nominal plant G.

Specifically, the following result [5] gives sufficient conditions to solve the generic robust stabilization problem.

Theorem 1: Under reachability and well posedness assumptions [22], a controller, K, asymptotically stabilizes G_Δ if the following three conditions hold.

1) K internally stabilizes G
2) the nominal closed loop system $F_\ell(G, K)$ has L_2 gain $\leq \gamma$.
3) $0 < \gamma < 1/\epsilon$

The third condition specifies the extent of the robustness.

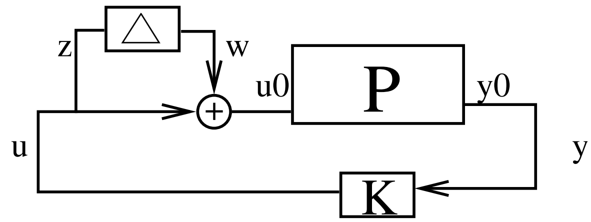


Fig. 2. Input Multiplicative Uncertainty

Various types of uncertainty can be dealt with such as input multiplicative uncertainty and output multiplicative uncertainty [5]. Input multiplicative uncertainty is an unknown deviation on the inputs of the system. Figure 2 depicts an example system, P, to illustrate input multiplication. Notice that the uncertainty is multiplied by the input signal, and added back to the input, which characterizes its name.

III. STATE AND INPUT UNCERTAINTY ROBUSTNESS APPLIED TO THE EDFA MODEL

By applying L_2 nonlinear control to the EDFA system, we can restrict the state variable, x, to guarantee all of the channel gains remain constant, despite input power variations. The state must be calculated in real-time. This requires precise knowledge of the input and output power of at least one channel, along with the precise values of the EDFA parameters.

We begin by introducing the concept of state uncertainty for EDFAs. We derive a mathematical framework to apply a linear approximation of state uncertainty to (2). Next, we represent state uncertainty as parametric uncertainty in (1) and (2) for g_k and α_k. Finally, we apply both input and state uncertainty robustness to (2) by augmenting the disturbance vector.

A. State Uncertainty

The first step is to associate state deviation in any term (2). Let uncertainty in the state variable, x, be denoted as x_Δ. We denote a general, nonlinear term of the EDFA as $f_{NL}(x)$. So, with state uncertainty included, we obtain $f_{NL}(x + x_\Delta)$. However, this representation of uncertainty is not input affine. We can approximate the term using a Taylor expansion about a point, x, and if we neglect the higher order terms we obtain

$$f_{NL}(x + x_\Delta) = f_{NL}(x) + f'_{NL}(x)x_\Delta \tag{8}$$

From (8), we now have a method to represent nonlinear state uncertainty in the form of an input affine structure.

B. Parametric Uncertainty As State Uncertainty

One important reason for including x_Δ as a disturbance comes from the idea that we can associate uncertainty in the parameter values of the EDFA with an uncertainty in x. The intuition is clear as a deviant value of one of the parameters in the EDFA system equations would produce a change in x. To prove the structural link between parametric uncertainty and state uncertainty, we first write out every term in (1)

TABLE II

ENUMERATED EDFA SYSTEM TERMS

Terms	Expression
term 1	$\frac{1}{\zeta\tau_o L} g_k m\nu_k Lx$
term 2	$-\frac{1}{\zeta\tau_o L} u_k [\frac{-g_k m\nu_k x}{(\alpha_k+g_k)x-(\alpha_k)} \cdot(1-e^{u_k\{(\alpha_k+g_k)x-(\alpha_k)\}L})]$
term 3	$-\frac{1}{\zeta\tau_o L} u_k (e^{u_k\{(\alpha_k+g_k)x-(\alpha_k)\}L}-1)$
term 4	$-\frac{x}{\tau_o}$

TABLE III

SUBDIVISION OF TERM 2

Terms	Expression
term 2.1	$-g_k m\nu_k x$
term 2.2	$(\alpha_k+g_k)x-(\alpha_k)$
term 2.3	$1-e^{u_k\{(\alpha_k+g_k)x-(\alpha_k)\}L}$

with parametric uncertainty included, and separately, write out the terms in (1) with state uncertainty. We then compare the parametric uncertainty terms with the state uncertainty terms to see the mathematical relationship.

We begin with the absorption and emission coefficients, α_k and g_k, respectively. Upon inspection with (1), they almost always have a direct multiplicative association with the state, x. We enumerate the terms in the state equation of the EDFA model with ASE, as shown in Table II. Notice specifically that g_k directly multiplies x in every term except term 4. Also notice that terms 3 and 4 are usually very small for high average inversion levels[9]. We choose to neglect term 4 since it is very small with respect to the other terms with high average inversion.

We represent the parametric uncertainties as $g_{k\Delta}$ and $\alpha_{k\Delta}$. Furthermore, let LS represent the term with parametric uncertainty applied and let RS represent the term with state uncertainty applied.

For term 1,

$$\begin{aligned} LS &= \frac{1}{\zeta\tau_0 L}(g_k+g_{k\Delta})m\nu_k Lx \\ RS &= \frac{1}{\zeta\tau_0 L} g_k m\nu_k L(x+x_\Delta) \end{aligned}$$

and imposing $LS = RS$, we get

$$\frac{x_\Delta}{x} = \frac{g_{k\Delta}}{g_k} \tag{9}$$

For term 2, the equation is much more complex, but we can break it down into 3 smaller terms. The subdivided terms are given in Table III.

For term 2.1, we get the same result as (9).

For term 2.2, we have

$$\begin{aligned} LS &= (\alpha_k+g_k+\alpha_{k\Delta}+g_{k\Delta})x-(\alpha_k+\alpha_{k\Delta}+\ell_k) \\ RS &= (\alpha_k+g_k)(x+x_\Delta)-(\alpha_k+\ell_k) \end{aligned}$$

so that for $LS = RS$, we get

$$\frac{x_\Delta}{x} = \frac{\alpha_{k\Delta}+g_{k\Delta}}{\alpha_k+g_k} - \frac{\alpha_{k\Delta}}{x(\alpha_k+g_k)} \tag{10}$$

It can be shown that for term 2.3, and term 3, we get the same result as (10).

Notice that the second term in the right half side of (10) can also be neglected if we assume that the absorption uncertainty, $\alpha_{k\Delta}$, is much smaller than the absorption and emission variables, α_k and g_k. This in fact should be true since the channel wavelengths have higher emission coefficients in general[3]. With this assumption, (9) and (10) are representations of percentage deviations from the parameter values. We make the assumption that all of the parameters are deviant by the same percentage. Thus, we associate all of the parameter uncertainties to one variable, the state uncertainty.

C. Robustness Applied To EDFA Model with ASE

In this section we consider both input multiplicative uncertainty and state uncertainty to create an EDFA system model that rejects unwanted physical effects. All uncertainties are combined into one augmented disturbance vector. In addition to the usual L_2 performance output, z, robust performance outputs are also included.

We begin by including the input multiplicative uncertainty into the EDFA model. We represent uncertainty on every individual signal input power as well as the pump power. Let $w^u = w + w_\Delta$ and $u^u = u + u_\Delta$, where w is the nominal input vector of channel powers, w_Δ is the uncertainty on the input channels, u is the nominal pump input power, and u_Δ is the uncertainty on the pump input. We substitute w^u and u^u into the state equation of (2) and we apply the state uncertainty as in (8). Also, we substitute $x^u = x + x_\Delta$, where x is the nominal state and x_Δ is the state uncertainty, and let $g_1^*(x) = \begin{bmatrix} g_1(x) & g_1(x) & g_2(x) \end{bmatrix}$. After some manipulation, we obtain

$$\dot{x} = f(x) + \hat{g}_1(x)\hat{w} + g_2(x)u \tag{11}$$

where

$$\hat{g}_1(x) = \begin{bmatrix} g_1^*(x) & g_1^{*\prime}(x) & f'(x) & g_2'(x) \end{bmatrix} \tag{12}$$

and,

$$\hat{w} = \begin{bmatrix} w & w_\Delta & u_\Delta & wx_\Delta & w_\Delta x_\Delta & \cdots \\ & & u_\Delta x_\Delta & x_\Delta & ux_\Delta \end{bmatrix}^T \tag{13}$$

In examination of the final system structure, (11), we note that $w_\Delta x_\Delta$ and $u_\Delta x_\Delta$ are actually uncertainties multiplying uncertainties, meaning that their influence would be very small. As such, these are neglected in the simulations.

Assumption 1: *$w_\Delta x_\Delta$ and $u_\Delta x_\Delta$ are neglected.*

Furthermore, it is important to note that wx_Δ and ux_Δ represent two multiplied signal values. However, these disturbances are treated as just one signal each, mixed together.

We design the output performance equation, z', as:

$$z' = \begin{bmatrix} \alpha_x x \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha_w I \\ 0 \end{bmatrix} \hat{w} + \begin{bmatrix} 0 \\ 0 \\ \alpha_u \end{bmatrix} u \tag{14}$$

where α_x, α_w and α_u are all scalars in the performance output. It will be shown that α_w will be taken as 0 later in the section. For now, we will treat (14) in its general form.

The first observation to make is that we chose to scale the disturbance vector, $\hat{w}$, by a scalar and not a matrix. In fact we could choose to scale this by a matrix, but this would unnecessarily complicate matters. The designer would usually scale the disturbances according to a worst case disturbance that they wish to attenuate. Hence, only one scalar is necessary. Secondly, we notice that the state also has a scalar multiplying it. Although this is valid, this also introduces unnecessary complication since the impact of the state is relative to the sizes of the $\hat{w}$ element and the u element. Since we are scaling $\hat{w}$ and u, the impact of x changes relative to these values, and we do not need to explicitly scale this term. Hence, here we consider $\alpha_x = 1$, and without loss of generality, we assume $\alpha_u = 1$.

With (11) and (14) written in the form of (2), we get:

$$\begin{aligned} \dot{x} &= f(x) + \hat{g}_1\hat{w} + g_2(x)u \\ z' &= \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha_w I \\ 0 \end{bmatrix}\hat{w} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u \\ y &= \begin{bmatrix} x \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix}\hat{w} \end{aligned} \tag{15}$$

Robust stability for EDFA is solved if we design an L_2 controller for (15) as in Theorem 1. The disturbance term in (15) has been significantly augmented. Also, we can see that $k_{11} \neq 0$ which could potentially cause computational problems.

Given careful scrutiny, it is apparent that so long as k_{11} has a simplified structure like that chosen above, $k_{11} = \alpha_w I$, we can actually rewrite system (15) into a simpler system where $k_{11} = 0$. Starting from

$$||z'||^2 \leq \gamma^2||\hat{w}||^2 \tag{16}$$

Next, we can expand z' according to its representation in terms of x, $\hat{w}$ and u:

$$||x||^2 + \alpha_w^2||\hat{w}||^2 + ||u||^2 \leq \gamma^2||\hat{w}||^2 \tag{17}$$

and if we rewrite equation (17), we get

$$||x||^2 + ||u||^2 \leq (\gamma^2 - \alpha_w^2)||\hat{w}||^2 \tag{18}$$

which is exactly the relationship that we would get for our system when $k_{11} = 0$, but with a lower L_2 gain, $\gamma'^2 = \gamma^2 - \alpha_w^2$. This makes sense since including $\hat{w}$ in the output performance, z', does not translate into a physical reduction on $\hat{w}$ because $\hat{w}$ is an external signal. The effect of wanting to restrict the $\hat{w}$ results in a more restrictive L_2 gain on the other variables. So in effect, we can actually neglect α_w by setting it equal to 0. This is a very nice and intuitive result.

IV. RESULTS AND DISCUSSION

We derive both a robust L_2 controller and a standard L_2 controller from [9] for simulation on an EDFA. We consider both input uncertainty and state uncertainty simultaneously in simulations, and evaluate the response of the closed-loop systems under the two controllers.

TABLE IV
EDFA INPUT VALUES

Channel Number, N	32	
Channel Powers, $P_{in,k}$	42.6	μW
Pump Power, P_{pump}	150	mW

TABLE V
EDFA PARAMETERS

Core Radius	1.8	μm
Er Radius	1.7	μm
Er ion Density,ρ	4.33E+24	$ions/m^3$
Metastable Lifetime,τ	10	ms
Channel Separation (for ν_k)	0.75	nm
Fiber Length, L	13	m

A. Controller Derivation

We follow the procedure as outlined in [9] to derive the standard L_2 nonlinear controller. We simulate our EDFA system with 32 input channels. We apply the data in Tables IV, V and VI. Given these parameters, we calculate the operating point, $x_0 = 0.60725$. For the robustness problem herein, we derive a robust L_2 nonlinear controller to satisfy the L_2 control objective for (15).

We use the linear theory of H_∞ control as a first step in the design process. We choose the scaling factors X_w and X_u on the disturbance and pump parameters, respectively. Here, we design for a worst case scenario, so we scale the norm of the disturbance for a 100% channel drop to equal unity. For the system (15), we calculate $\|w\| = 1.8716 * 10^{15}$ for 32 channels. We calculate $X_w = \frac{1}{\|w\|}$. Thus, $\Delta w = 1$, so our x is limited directly by the L_2 gain.

Similarly, we can obtain X_u. We choose the initial L_2 gain, $\gamma = 0.003$. We are interested in both input multiplicative and state uncertainty as a test case. Moreover, each simulation is compared to the baseline reference of the standard L_2 controller with no robustness as in [9]. We use the same γ values for each test case. This allows us to interpret the performance differences between the simulations

TABLE VI
EDFA SIGNAL AND PUMP BANDS

PUMP Band		
	absorption	emission
Wavelength (nm)	alpha (dB/m)	g* (dB/m)
980	3.229	0
Signal Band		
	absorption	emission
Wavelength (nm)	alpha (dB/m)	g* (dB/m)
1530.072	6.48847101	6.34602815
1530.822	6.46745262	6.42559327
1531.572	6.39438961	6.45346687
	⋮	
1552.572	2.74515587	4.27367610
1553.322	2.68572360	4.24545752

according to this γ value. The X_u scaling for the standard and the robust L_2 controllers are $X_u = 4.1168(10^{-21})$ and $X_u = 8.3177(10^{-38})$, respectively.

If we increase the value of γ, which decreases the amount of attenuation, we can achieve an L_2 gain over a larger neighborhood. A good final value for both standard and robust L_2 control is $\gamma = 0.03$. We do not need to obtain a global solution to the L_2 control problem, since simulations show that the controllers function properly in the worst case channel drop. The final controllers used are:

$$\begin{aligned} u_{32} &= -1.0237x + 7.8867(10^{-1})x^2 \\ &+ 3.1106x^3 - 9.95(10^1)x^4 \\ &- 6.8735(10^2)x^5 + 2.6259(10^2)x^6 \\ &+ 7.586(10^2)x^7 \\ u_{32robust} &= -1.1010x - 3.1001(10^{-1})x^2 \\ &- 9.0484x^3 - 1.3708(10^2)x^4 \\ &- 1.2159(10^3)x^5 + 4.2958(10^2)x^6 \\ &+ 1.3501(10^3)x^7 \end{aligned} \quad (19)$$

where u_{32} and $u_{32robust}$ are the standard L_2 controller and the robust L_2 controller incorporating both state and input multiplicative uncertainty, respectively.

B. Simulation and Discussion

We simulate the EDFA system with 32 input channels, for data as in Tables IV, V and VI.

We introduce a 5% and 10% signal reduction at $t = 0$ seconds. We choose to use state uncertainties of 0.01 and 0.001 for an extreme and modest case, respectively, at $t = 0$ seconds. The uncertainty is chosen to subtract from the equilibrium state value. We permute 0.01 and 0.001 state uncertainty with 5% and 10% input uncertainty. The simulations are depicted in figures 3 - 4.

If we first examine the average inversion plot in Figure 3(a) with robustness, we immediately notice that the state moves by at most $x = 4(10^{-20})$. This is extremely small as compared with the standard L_2 controller in Figure 3(b). The angular responses of the robust L_2 controller are due to the extremely small timescale plotted over a much larger timescale. The reason for this extreme reaction is that the state uncertainty term, $f'(x)$, in the disturbance vector from (11) dominates the remaining terms substantially. This response would clearly not be feasible for a physical implementation but it serves the purpose of showing the significant impact state uncertainty has on the system using the chosen gain of $\gamma = 0.03$. A physical realization would involve increasing the L_2 gain up until a reasonable performance can be generated by the pump.

If we examine the standard L_2 controlled system with no robustness we see a much slower response time for the state as shown in Figure 3(b). The effect of modeling robustness causes the system to restrict the state variable much more tightly. Uncertainties in x have a very significant impact on the system response.

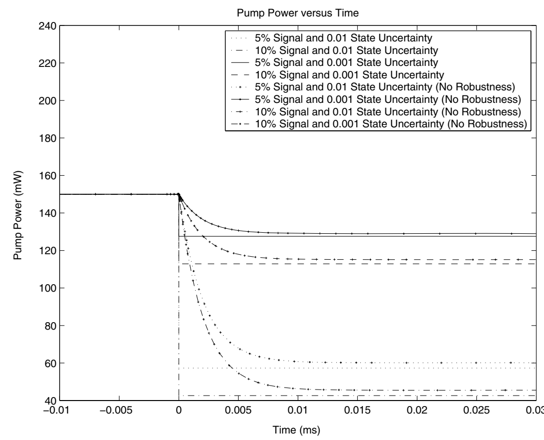


Fig. 4. State Uncertainty and Multiplicative Uncertainty Test Case for 32 Channels with 0.01 and 0.001 State Uncertainty and 5% and 10% Input Uncertainty

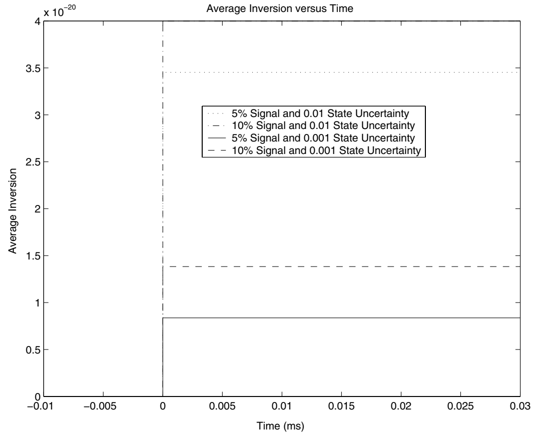
In Figure 4 we see that the pump power must change more rapidly in the robust controller. The final power values of the test cases show a few milliwatts difference between the robust and non-robust controllers. This means that the robust control pump must be able to vary more power in a shorter period of time. The small difference in pump power compensation compared with the large impact on the state variable restrictions offers encouraging results about the usefulness of this control scheme. The 10% input uncertainty with 0.01 state uncertainty case needs a much larger drop in pump power as compared with the 5% input uncertainty with 0.001 state uncertainty case. Once again, this is due to the larger uncertainty requiring a larger pump power variation response.

Finally, it is also important to note the trivial case of extreme robustness when x is made to be extremely restricted. Imagine that using L_2 control, we restrict x very tightly by giving the system a very small L_2 gain. Clearly, given any disturbance, our controller will be able to minimize variation in x to an extremely small range. The smaller the restriction on x, the more robust the system. Unfortunately, controllers are always restricted to physical constraints that prevent an unlimited restriction. Never the less, given a controller that can operate on a faster time scale to dispense more power, we achieve greater robustness.

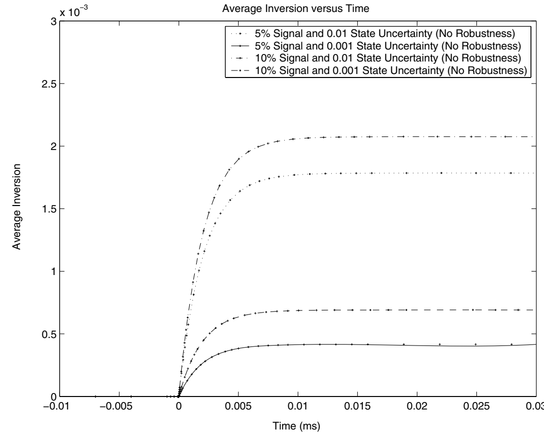
V. CONCLUSION

In this paper, we derived a methodology to incorporate a linear approximation of state uncertainty in an EDFA system with amplified spontaneous emission. We further derived a relationship between state uncertainty and parametric uncertainty. In the EDFA model, variations in the variables g_k and α_k were represented as variations in the state, x. Moreover, we applied input uncertainty robustness to the EDFA model as well to account for small variations in input power.

We devised a robust L_2 nonlinear controller that outperformed the standard L_2 nonlinear controller from [9] such that it was less sensitive to input power uncertainty and



(a) Average Inversion versus Time for 32 Channels



(b) Average Inversion versus Time for 32 Channels

Fig. 3. State Uncertainty and Multiplicative Uncertainty Test Case for 32 Channels with 0.01 and 0.001 State Uncertainty and 5% and 10% Input Uncertainty

parametric uncertainty. We extended the EDFA system model from [9] to incorporate uncertainties in the state variable and input powers. We then derived a robust L_2 nonlinear controller from the extended EDFA system model. We compared and contrasted the robust L_2 controller with the standard L_2 controller from [9]. Through simulation, we revealed that that the state uncertainty plays a significant role in the robustness of the EDFA since state uncertainty terms dominate the disturbance vector. With a fixed L_2 gain, $\gamma = 0.03$, for the robust L_2 control law and the standard L_2 control law, there is a stark difference in response time. The robust L_2 controller significantly outperforms the standard L_2 control law by many orders of magnitude in transient time. The robust controller can be easily utilized by increasing the L_2 gain to generate a reasonable pump response.

Future work could involve eliminating the linear approximation of the state uncertainty.

VI. ACKNOWLEDGEMENTS

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