Supervisor Localization

A Top-Down Approach to Distributed Control of Discrete-Event Systems

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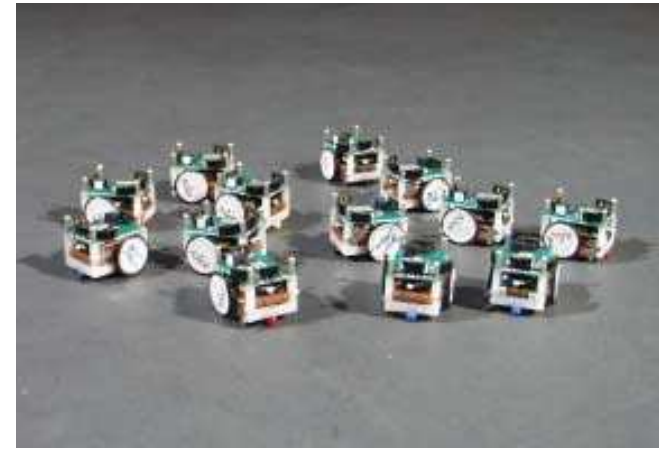
Problem

Distributed Control Design



- Plant: a collection of agents
- Specification: desired collective behavior
- Objective: individual strategy synthesis;
  optimal and *nonblocking*

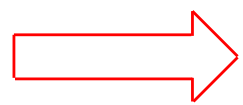
Motivation

Flocking

Schooling

Foraging

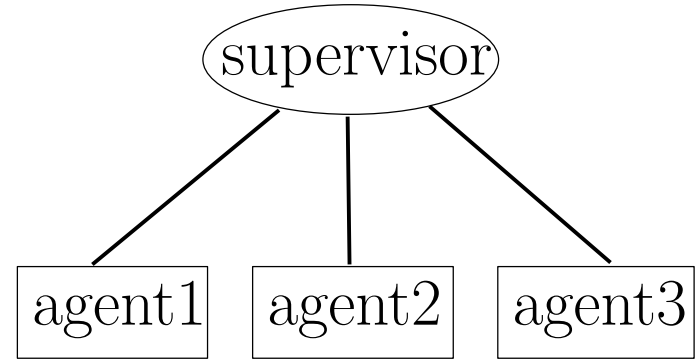
Observations on animal groups



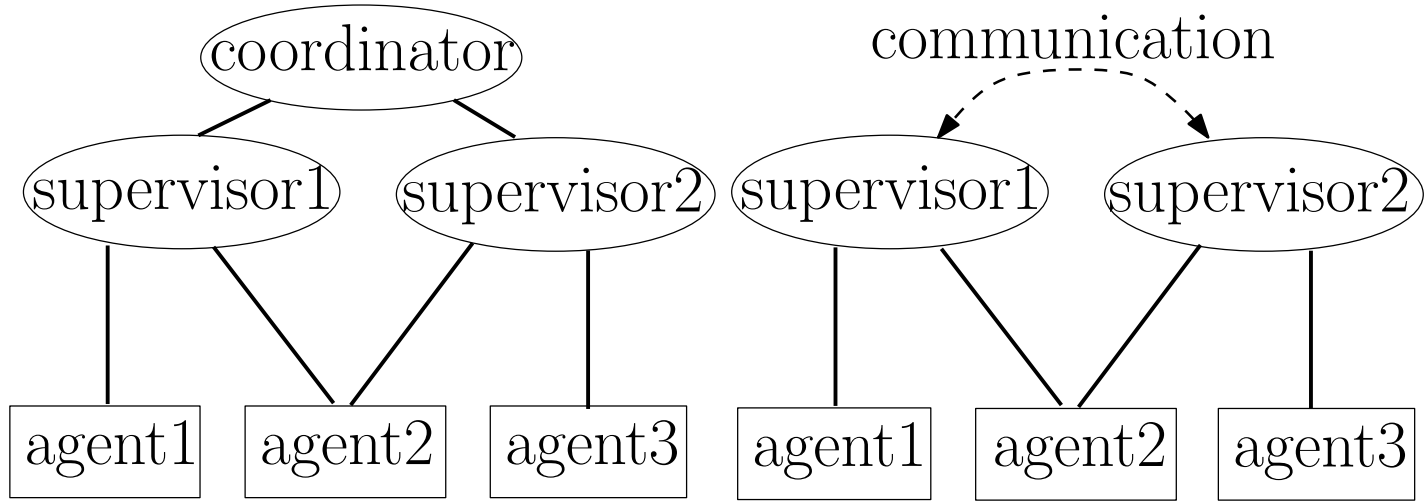
Design of man-made systems

System architectures

- **Monolithic architecture**

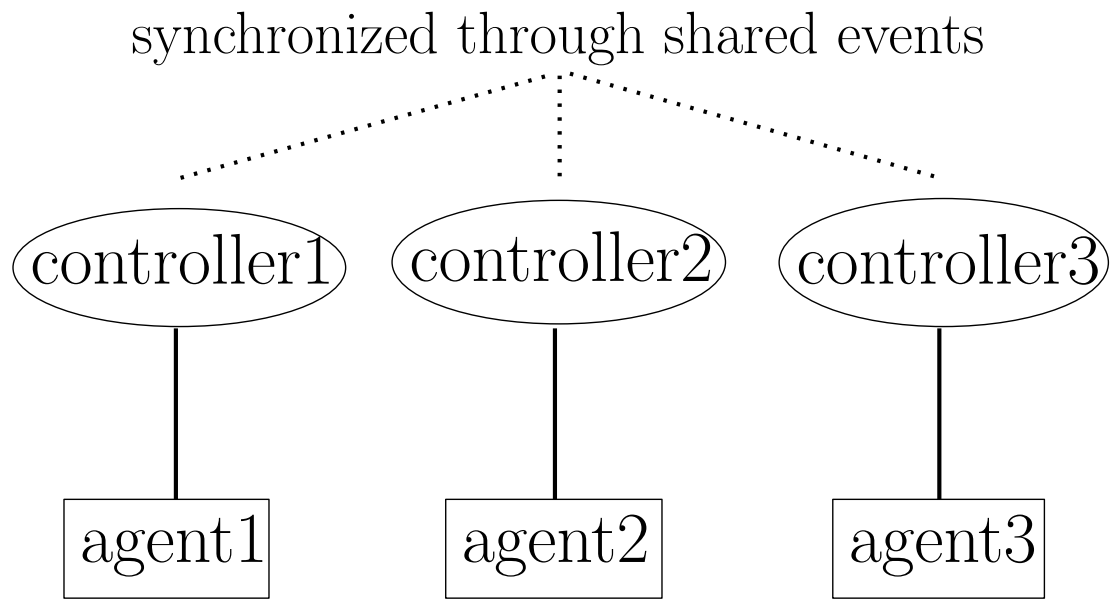


- **Modular arch.**



System architectures

- Distributed architecture



Results

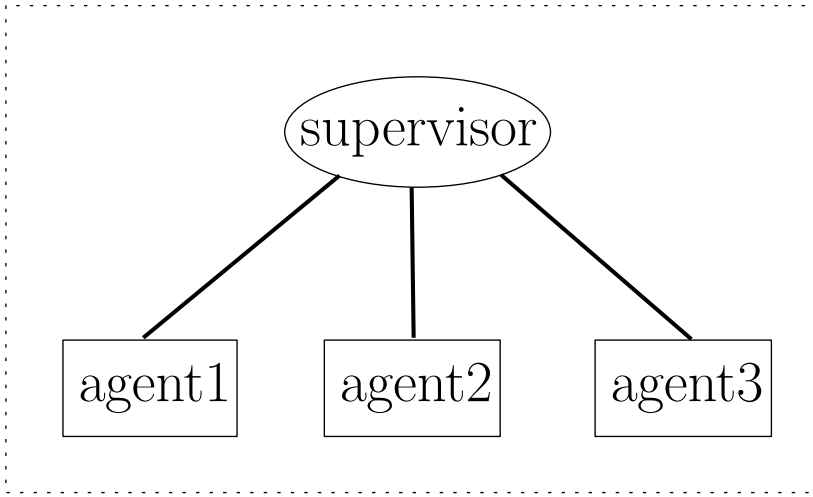
	Small-scale systems	Large-scale systems
Language-based model	Y	Y
State-based model	Y	?

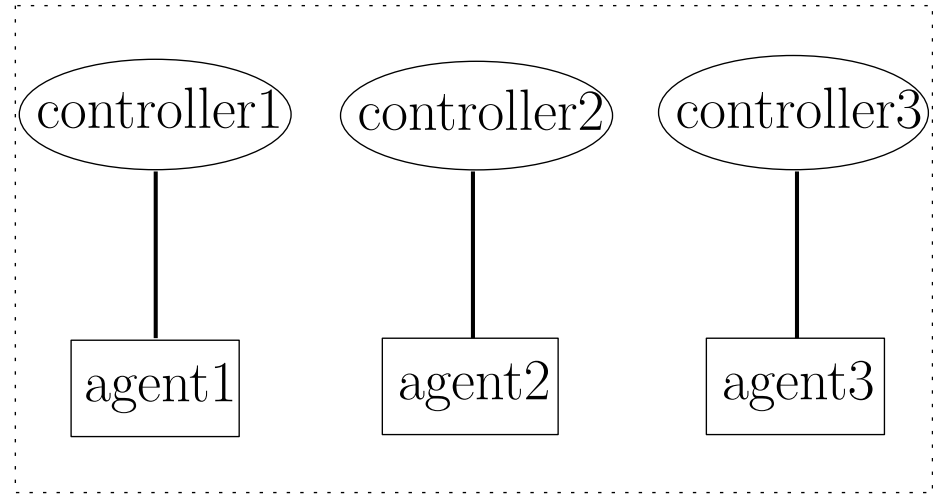
Results

	Small-scale systems	Large-scale systems
Language-based model	Y	Y
State-based model	Y	?

Discuss in detail

Small-scale systems

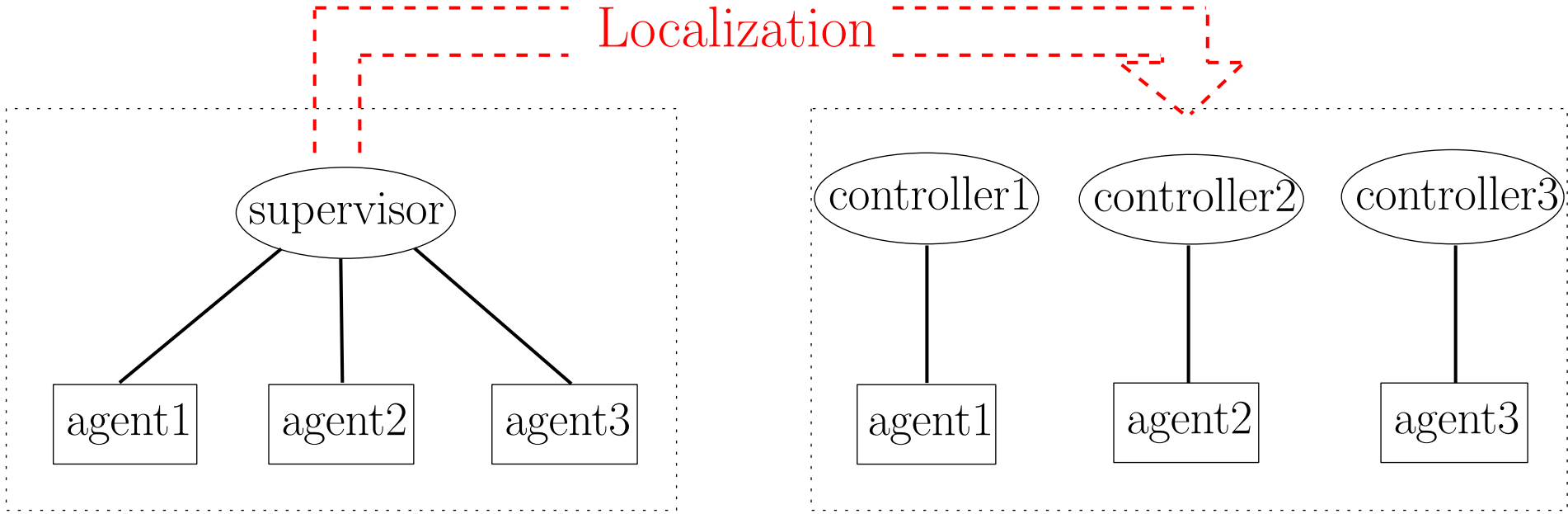




Optimal & Nonblocking

- Assumption: monolithic supervisor computable

Small-scale systems



Optimal & Nonblocking

- Assumption: monolithic supervisor computable
- Question: Can we design a localization algorithm that preserves optimality and nonblockingness?

Supervisor reduction

Plant: $\mathbf{G} = (Y, \Sigma, \eta, y_0, Y_m)$
Supervisor: $\mathbf{SUP} = (X, \Sigma, \xi, x_0, X_m)$

Find a reduced supervisor $\mathbf{SIM} = (I, \Sigma, \kappa, i_0, I_m)$ s.t.

(1) control equivalence:

$$L(\mathbf{G}) \cap L(\mathbf{SIM}) = L(\mathbf{SUP})$$

$$L_m(\mathbf{G}) \cap L_m(\mathbf{SIM}) = L_m(\mathbf{SUP})$$

(2) state reduction (desirable in practice):

$$|I| \ll |X|$$

Supervisor reduction (cont'd)

- Disablement information

$$E : X \rightarrow Pwr(\Sigma)$$
$$x \mapsto \{\sigma \in \Sigma | \xi(x, \sigma)!\}$$
$$D : X \rightarrow Pwr(\Sigma_c)$$
$$x \mapsto \{\sigma \in \Sigma_c | \neg\xi(x, \sigma)! \;\&\; (\exists s \in \Sigma^*)[\xi(x_0, s) = x \;\&\; \eta(y_0, s\sigma)!]\}$$

- Marking information

$$M : X \rightarrow \{0, 1\}$$
$$M(x) = 1 \text{ iff } x \in X_m$$
$$T : X \rightarrow \{0, 1\}$$
$$T(x) = 1 \text{ iff } (\exists s \in \Sigma^*)\xi(x_0, s) = x \;\&\; \eta(y_0, s) \in Y_m$$

Supervisor reduction (cont'd)

- Control consistency relation $\mathcal{R} \subseteq X \times X$
  $(x, x') \in \mathcal{R}$ iff

  (i) $E(x) \cap D(x') = \emptyset = E(x') \cap D(x)$

  (ii) $T(x) = T(x') \Rightarrow M(x) = M(x')$

- Let $\mathcal{C} = \{X_i \subseteq X | i \in I\}$ be a *cover* on X.
  $\mathcal{C}$ is a control cover if

  (i) $(\forall i \in I)(\forall x, x' \in X_i)\ (x, x') \in \mathcal{R}$

  (ii) $(\forall i \in I)(\forall \sigma \in \Sigma)(\exists j \in I)[(\forall x \in X_i)\xi(x, \sigma)! \Rightarrow$
  $\xi(x, \sigma) \in X_j]$

  Further, if $\mathcal{C}$ is a *partition* on X,
  then $\mathcal{C}$ is a control congruence.

Supervisor reduction (cont'd)

- Construct an induced generator **SIM** $= (I, \Sigma, \kappa, i_0, I_m)$

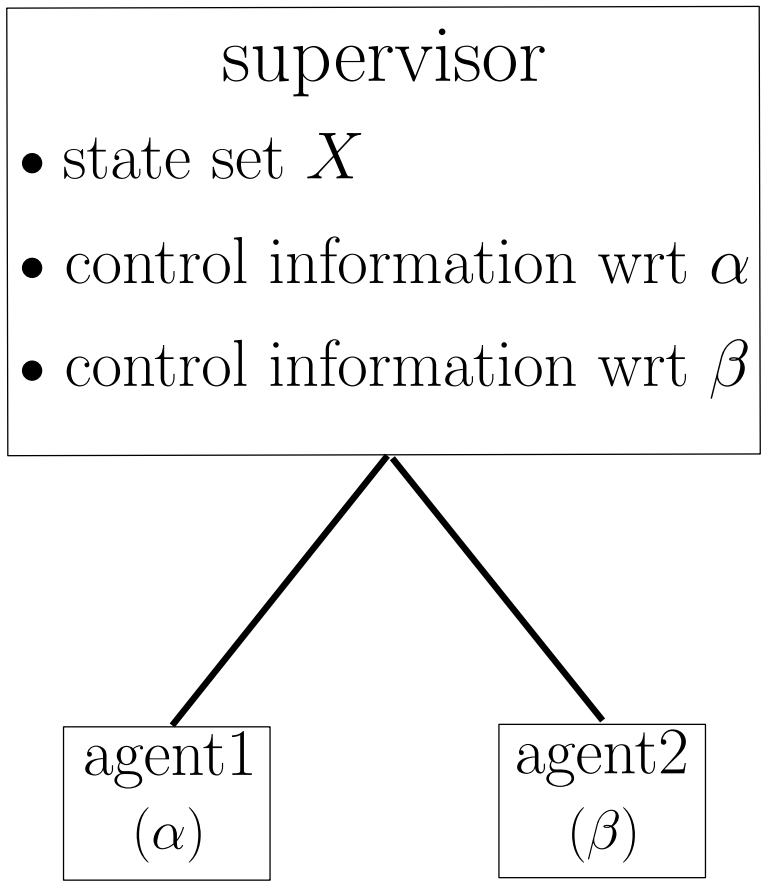
  (i) $i_0 \in I$ s.t. $x_0 \in X_{i_0}$

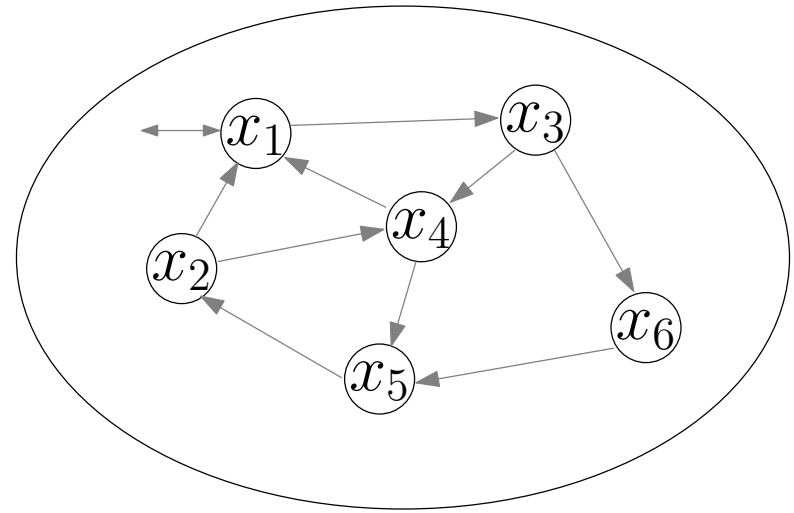
  (ii) $I_m = \{i \in I | X_i \cap X_m \neq \emptyset\}$

  (iii) $\kappa : I \times \Sigma \to I$ (pfn) with $\kappa(i, \sigma) = j$ if $(\exists x \in X_i)\xi(x, \sigma) \in X_j$ & $(\forall x' \in X_i)[\xi(x', \sigma)! \Rightarrow \xi(x', \sigma) \in X_j]$

- **Result**: **SIM** is a reduced supervisor.

Localization – structural analysis

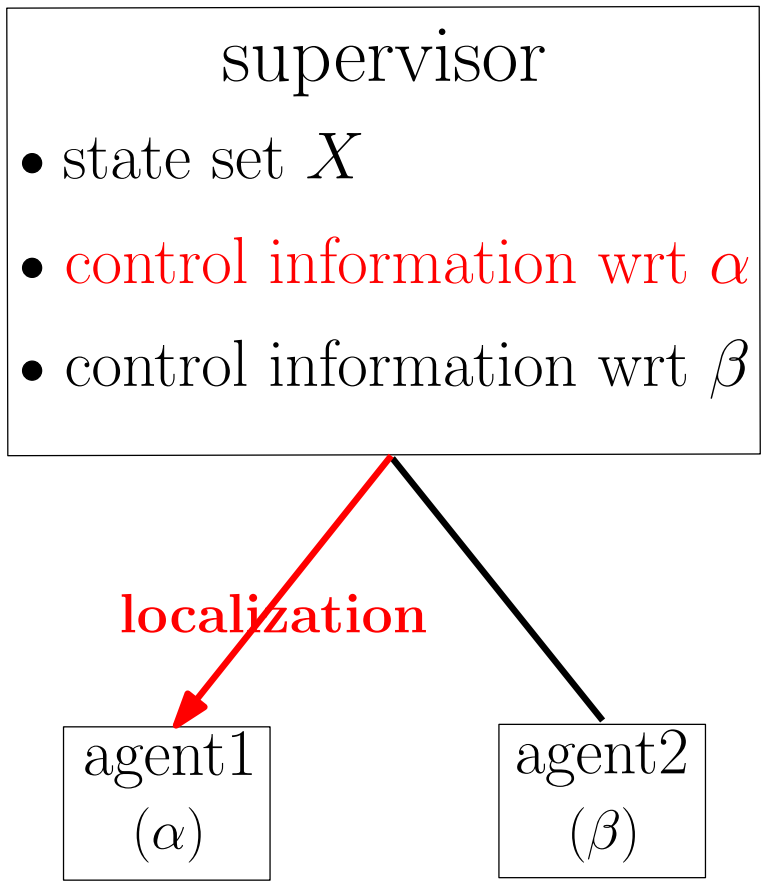


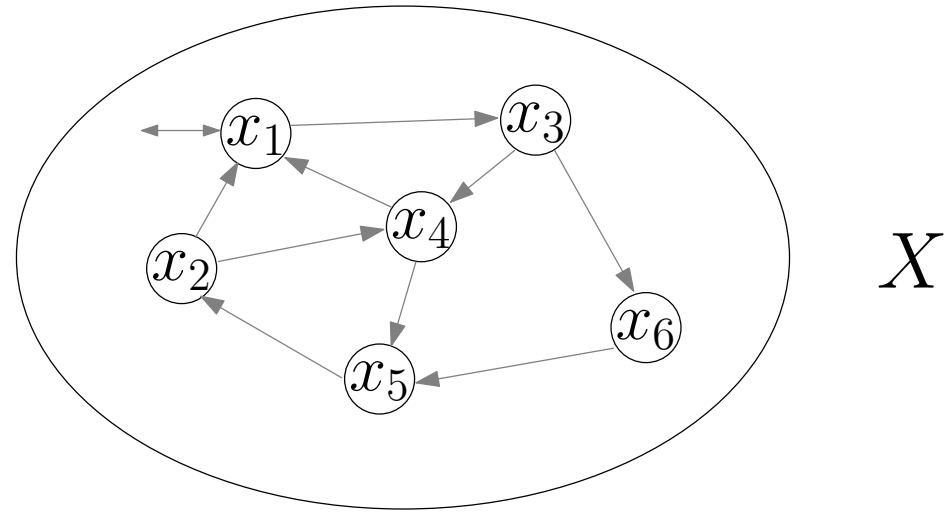


X

Localization – structural analysis

- **control information wrt α**





Localization – structural analysis

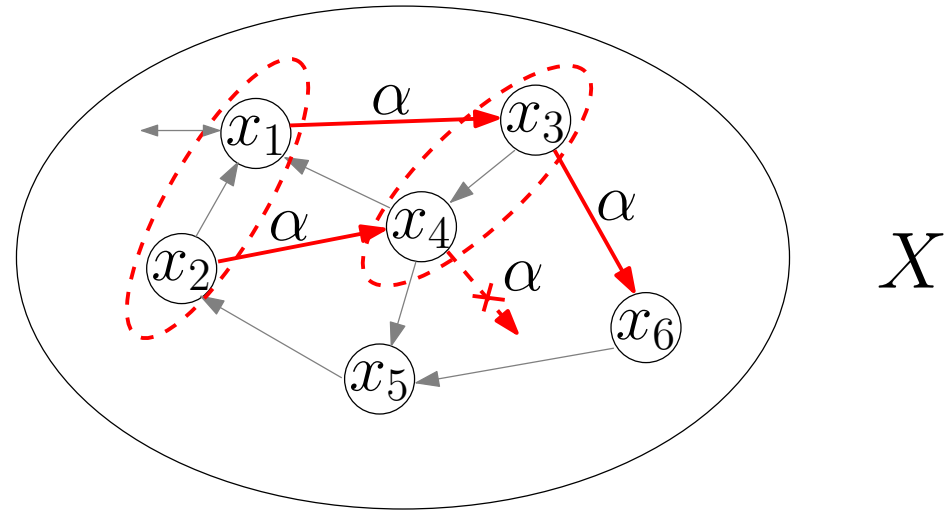
- **control information wrt α**
- **control consistency relation on X**

supervisor
- state set X
- control information wrt α
- control information wrt β

localization

agent1 (α)

agent2 (β)



Localization – structural analysis

supervisor

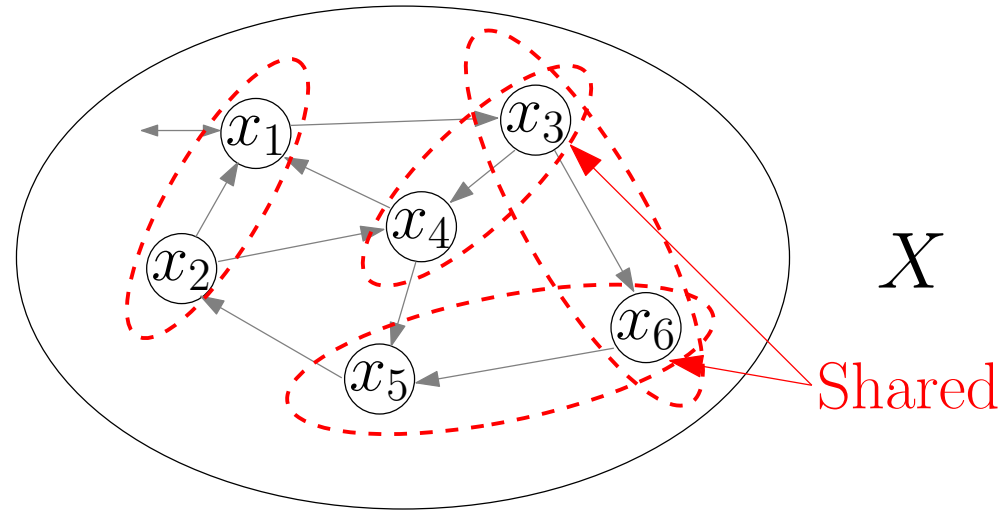
- state set X
- control information wrt α
- control information wrt β

localization

agent1 (α)

agent2 (β)

- **control information wrt α**
- **control consistency relation on X**
  +
  transition consistency
- **control cover on X**



Localization – structural analysis

supervisor

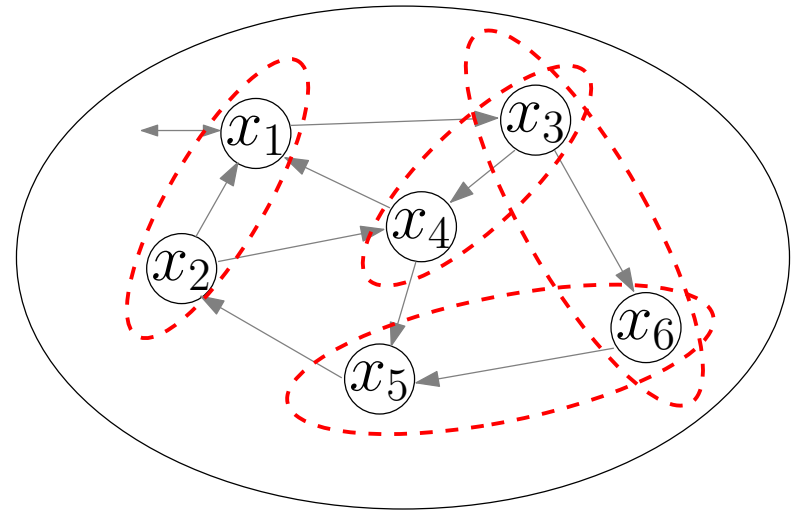
- state set X
- control information wrt α
- control information wrt β

localization

agent1 (α)

agent2 (β)

- **control information wrt α**
- **control consistency relation on X + transition consistency**
- **control cover on X**



X

- **induced transition structure**

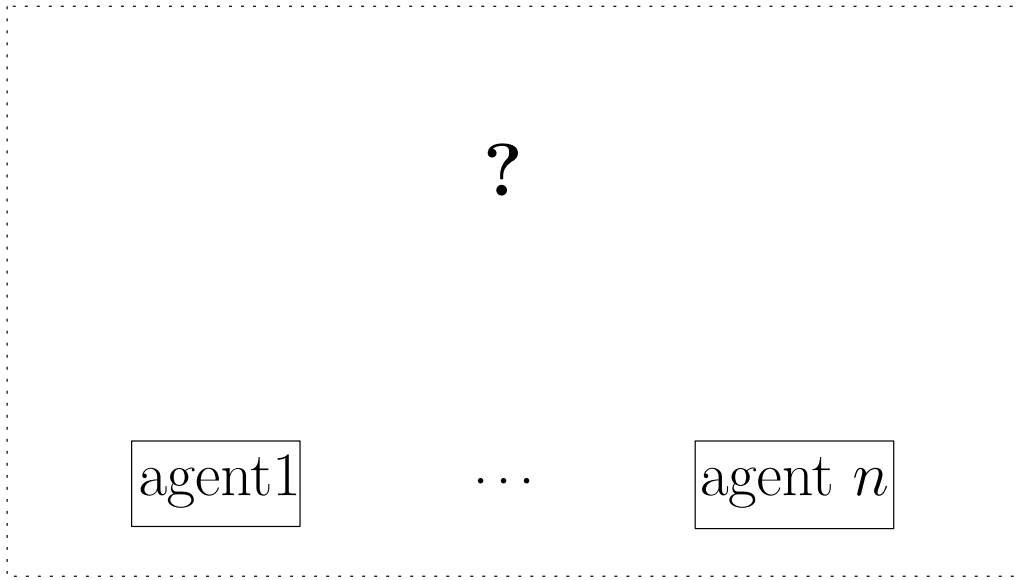
Localization – algorithmic implementation

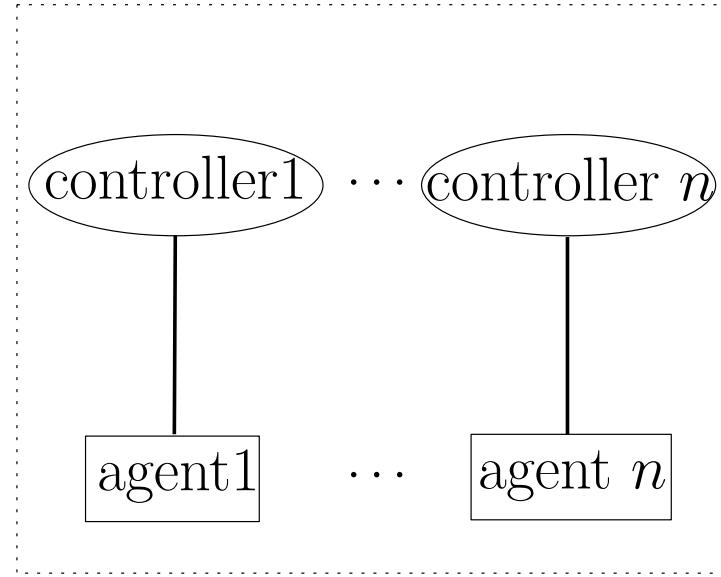
Our algorithm provably

- decomposes monolithic supervisor to local controllers;
- preserves optimality and nonblockingness;
- has time-complexity of $O(n^4)$,
  where n is the state size of monolithic supervisor.

Large-scale systems

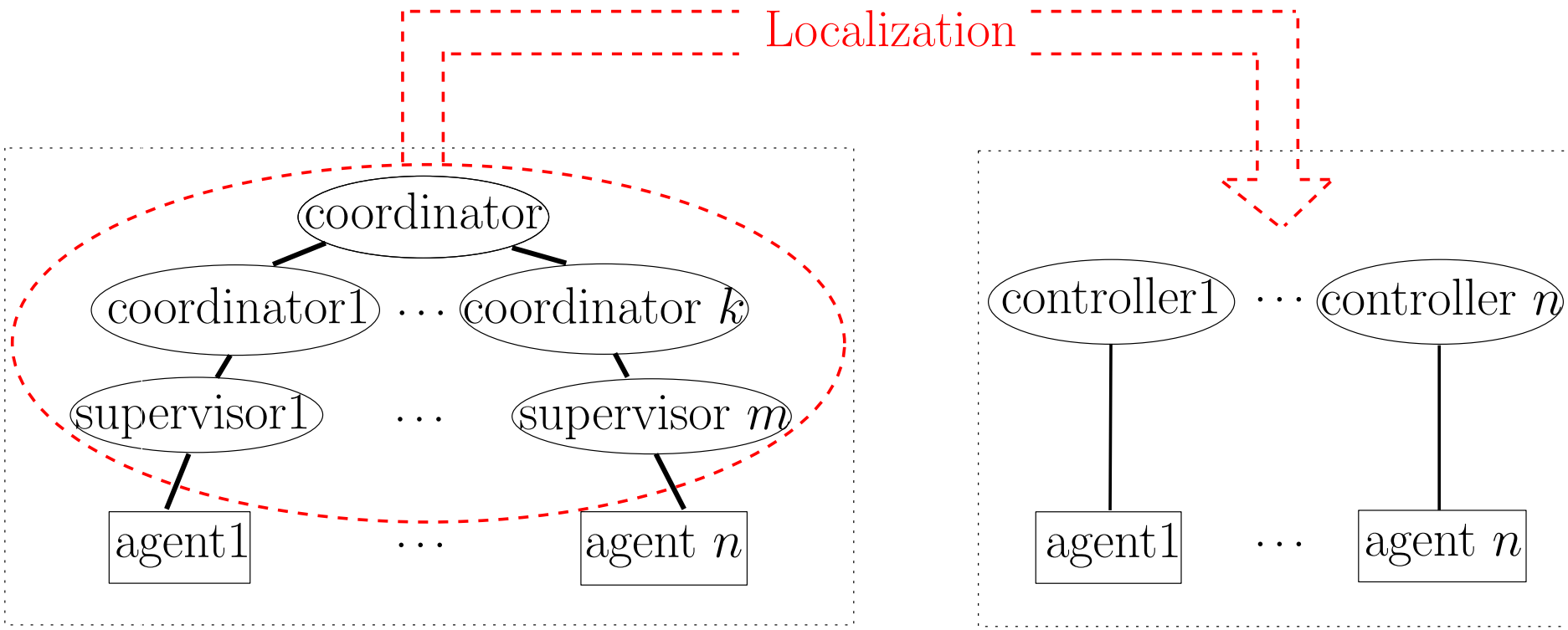
- Problem: state space explosion,
  monolithic supervisor not feasibly computable





Large-scale systems

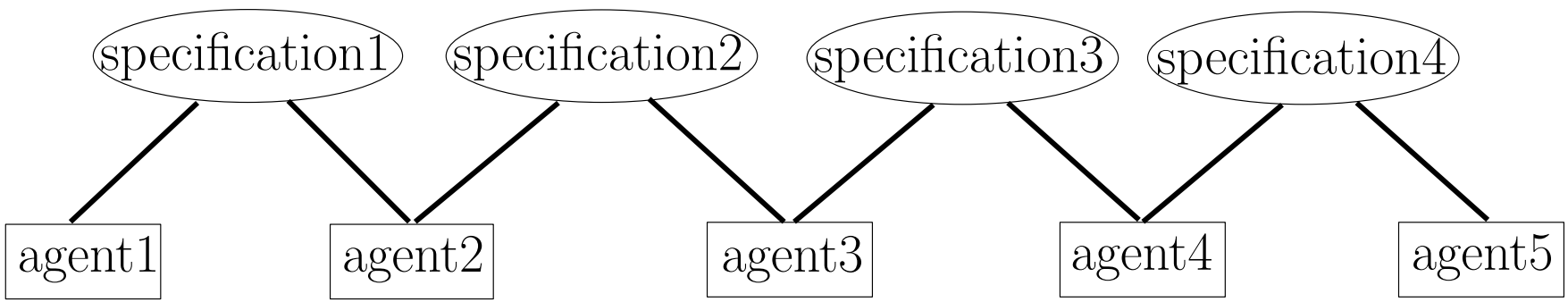
- Problem: state space explosion,
  monolithic supervisor not feasibly computable
- Approach: modular supervisory control + localization



Optimal & Nonblocking

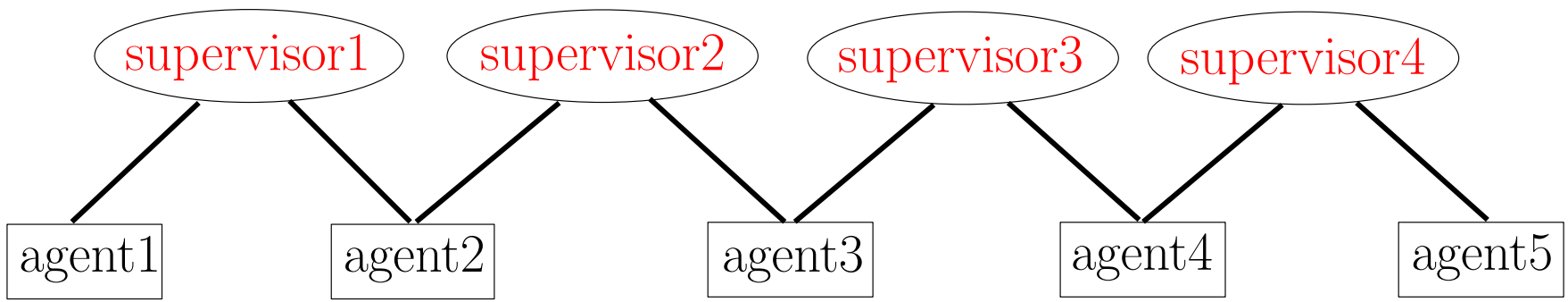
Solution

A decomposition-aggregation procedure



Solution

A decomposition-aggregation procedure



Solution

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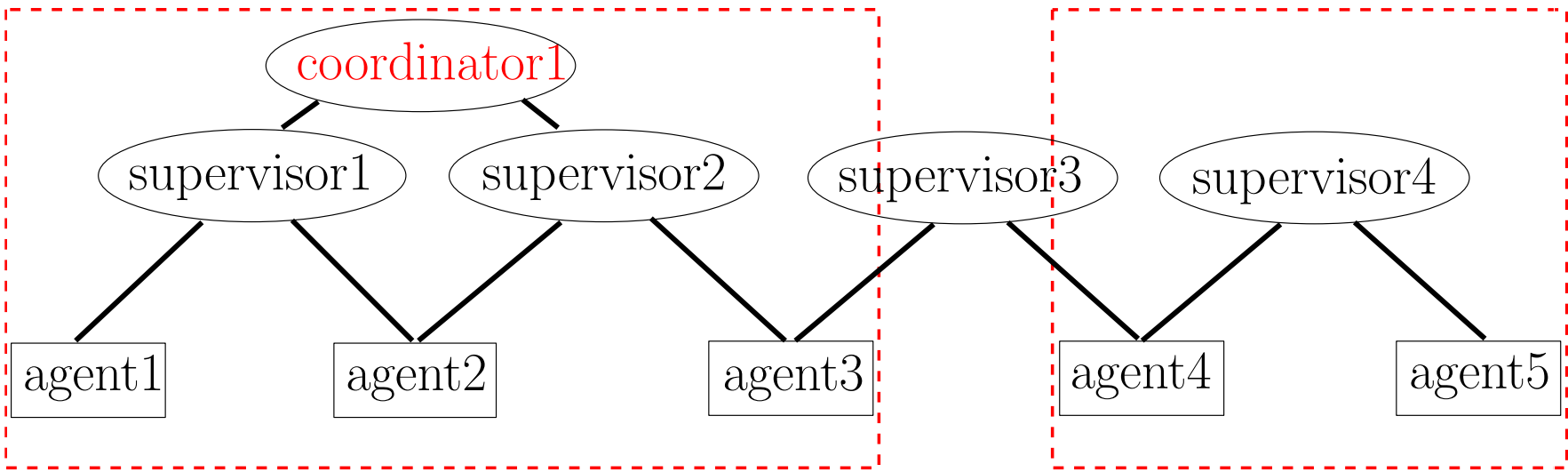

Solution

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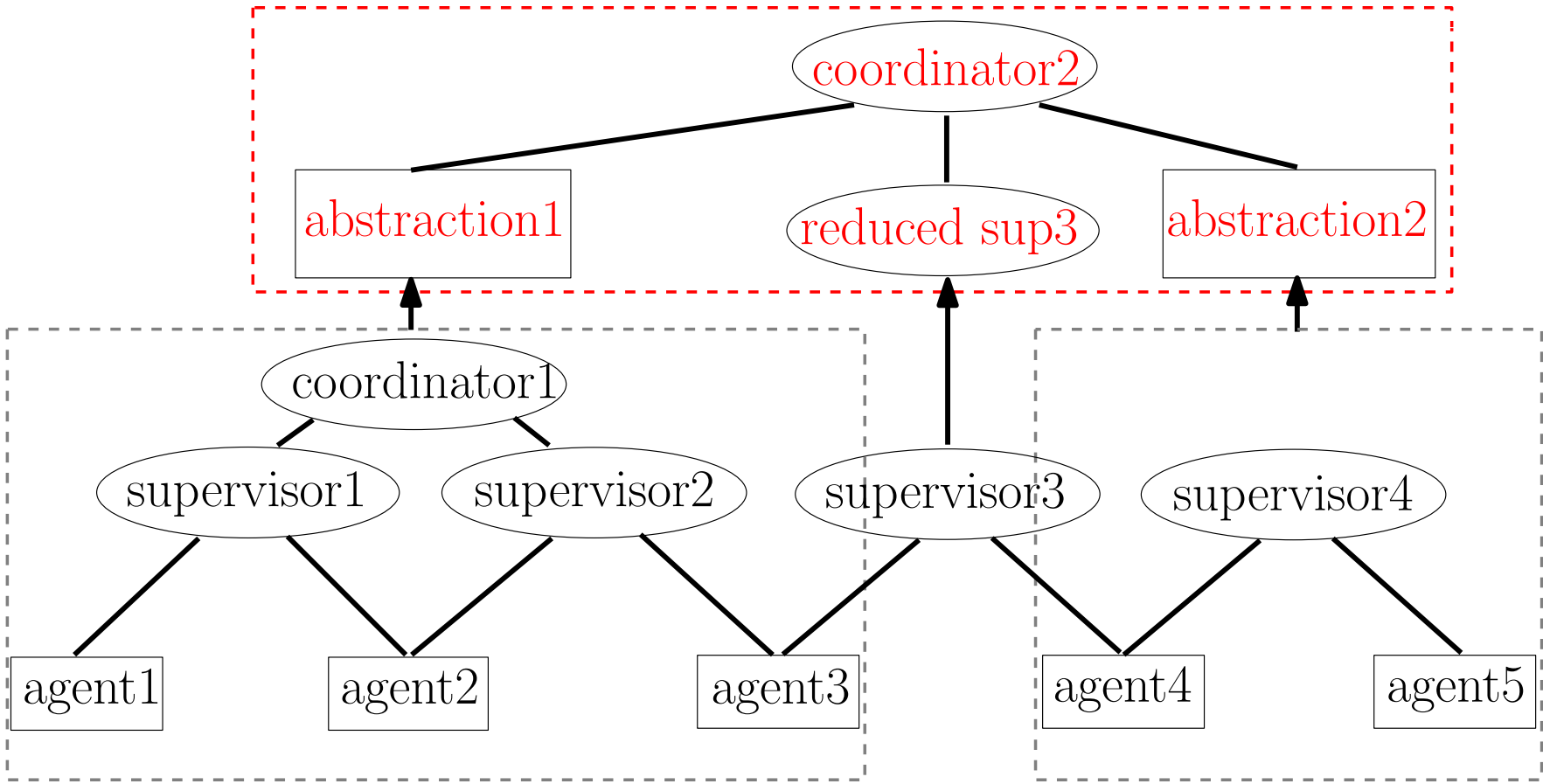

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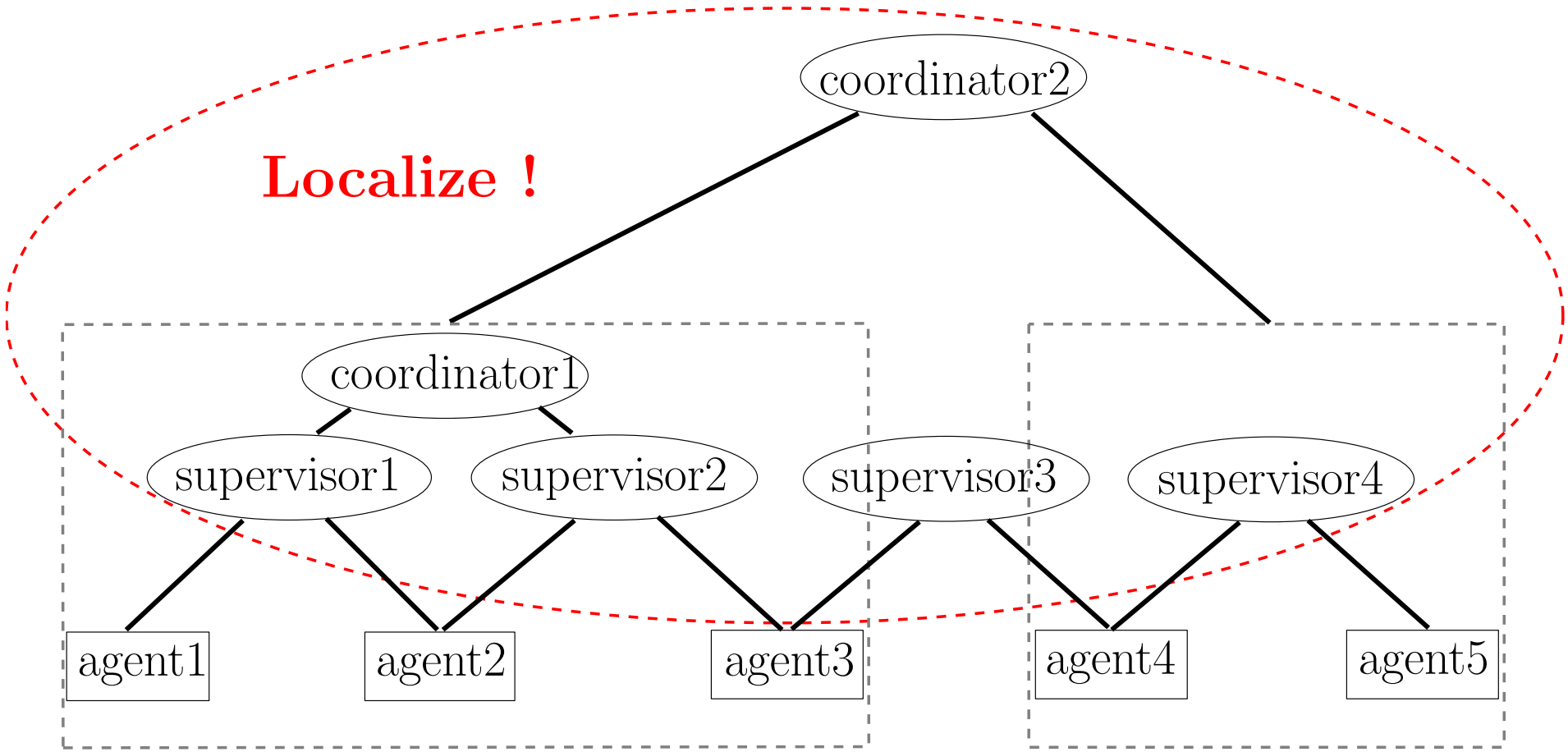
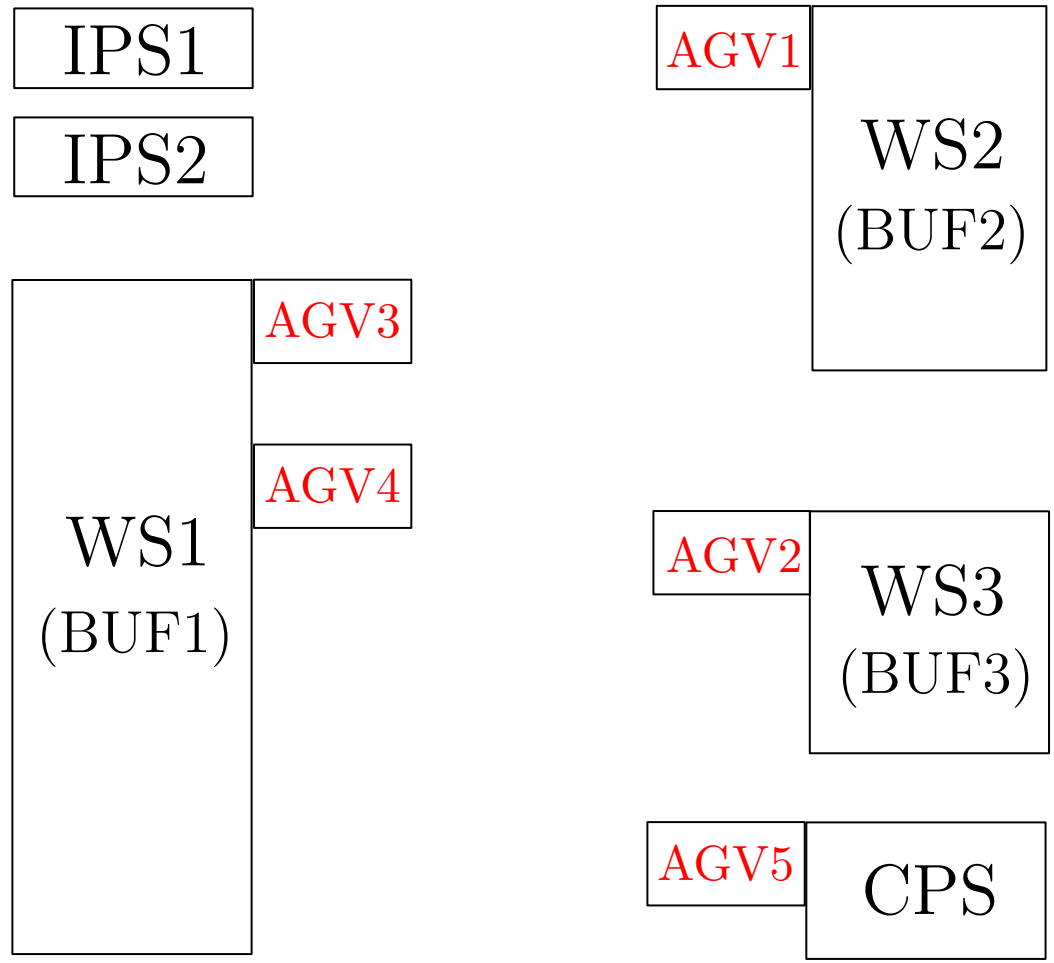


Example 1: Automated Guided Vehicles



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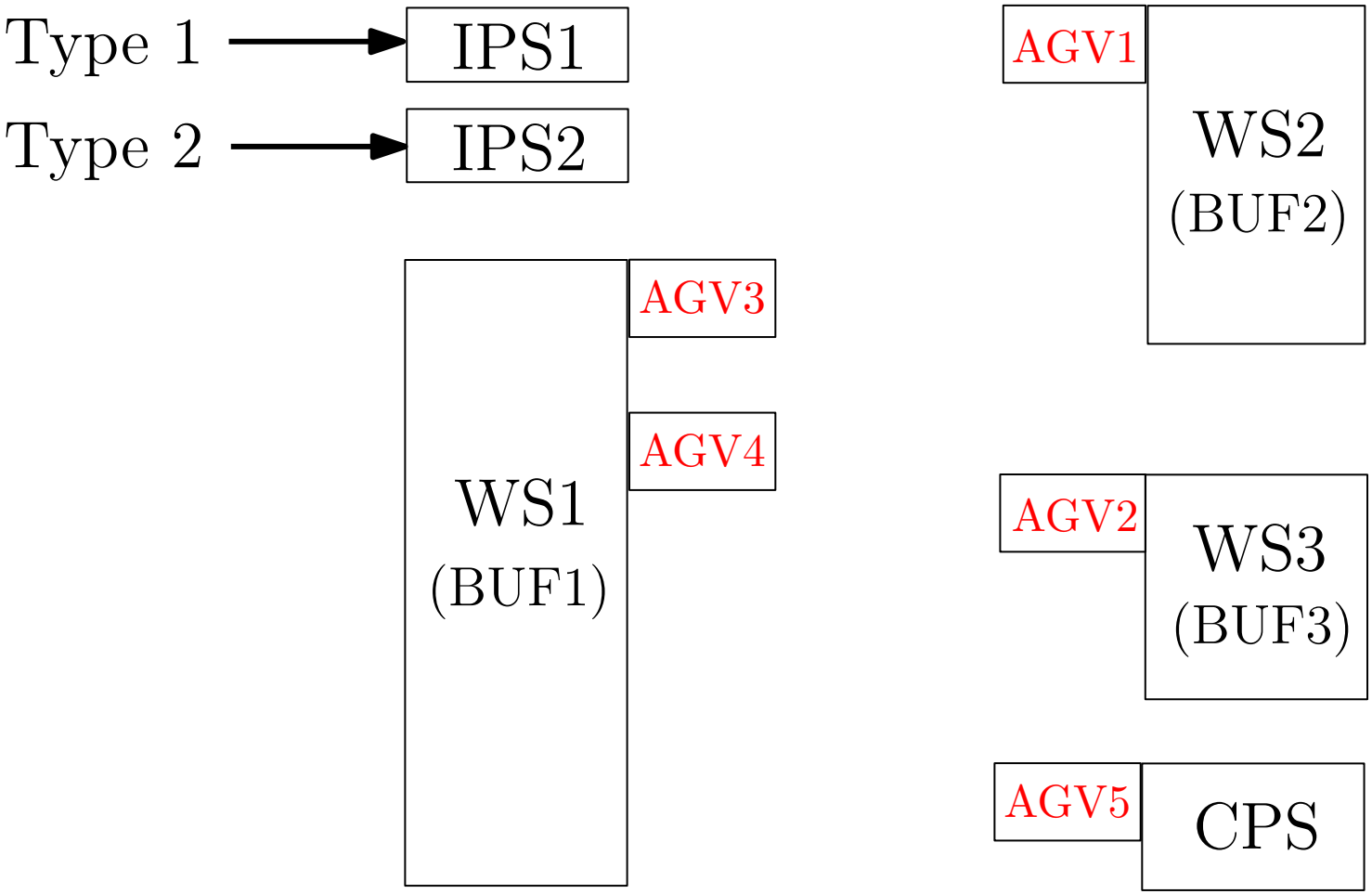

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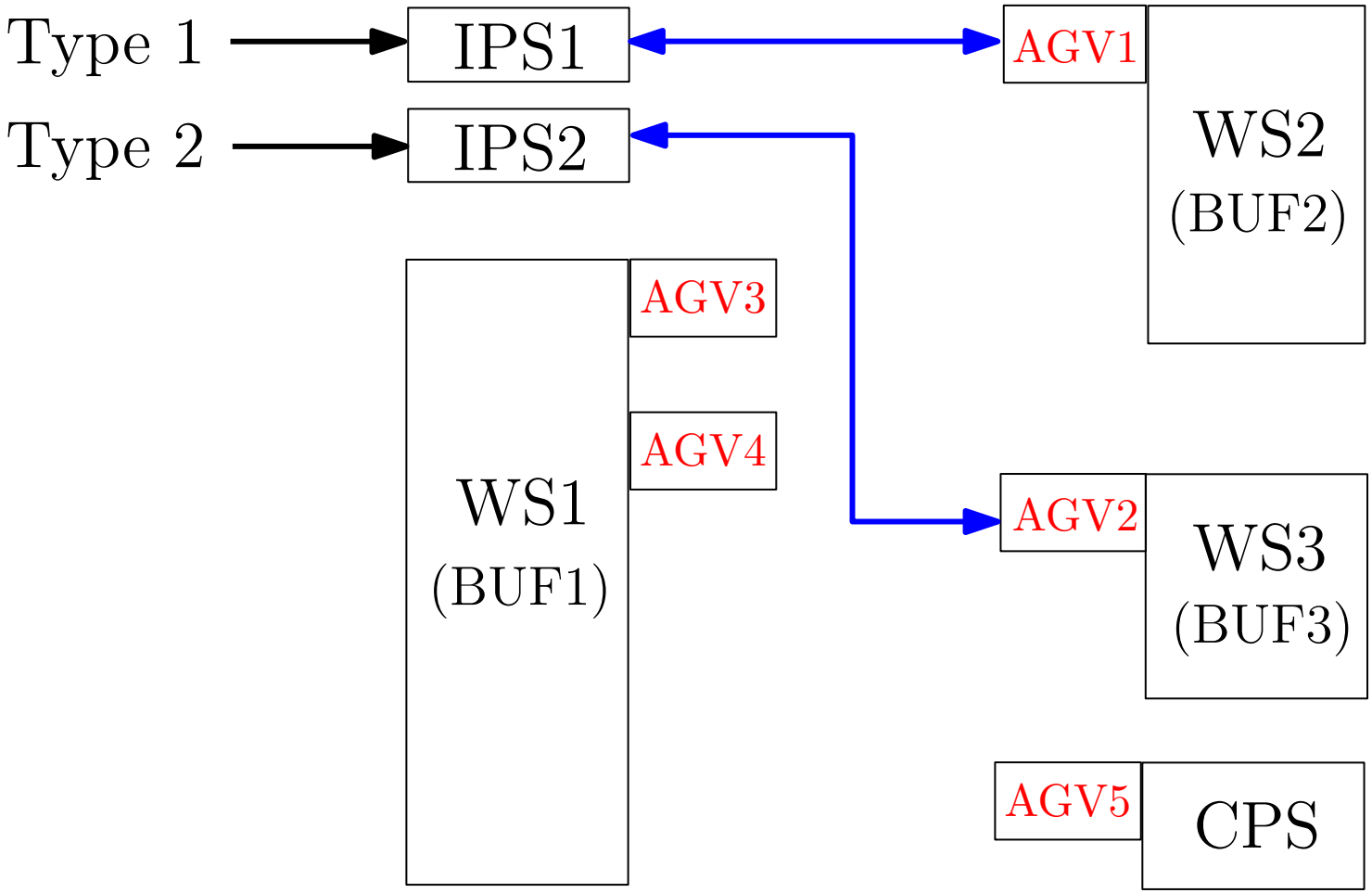

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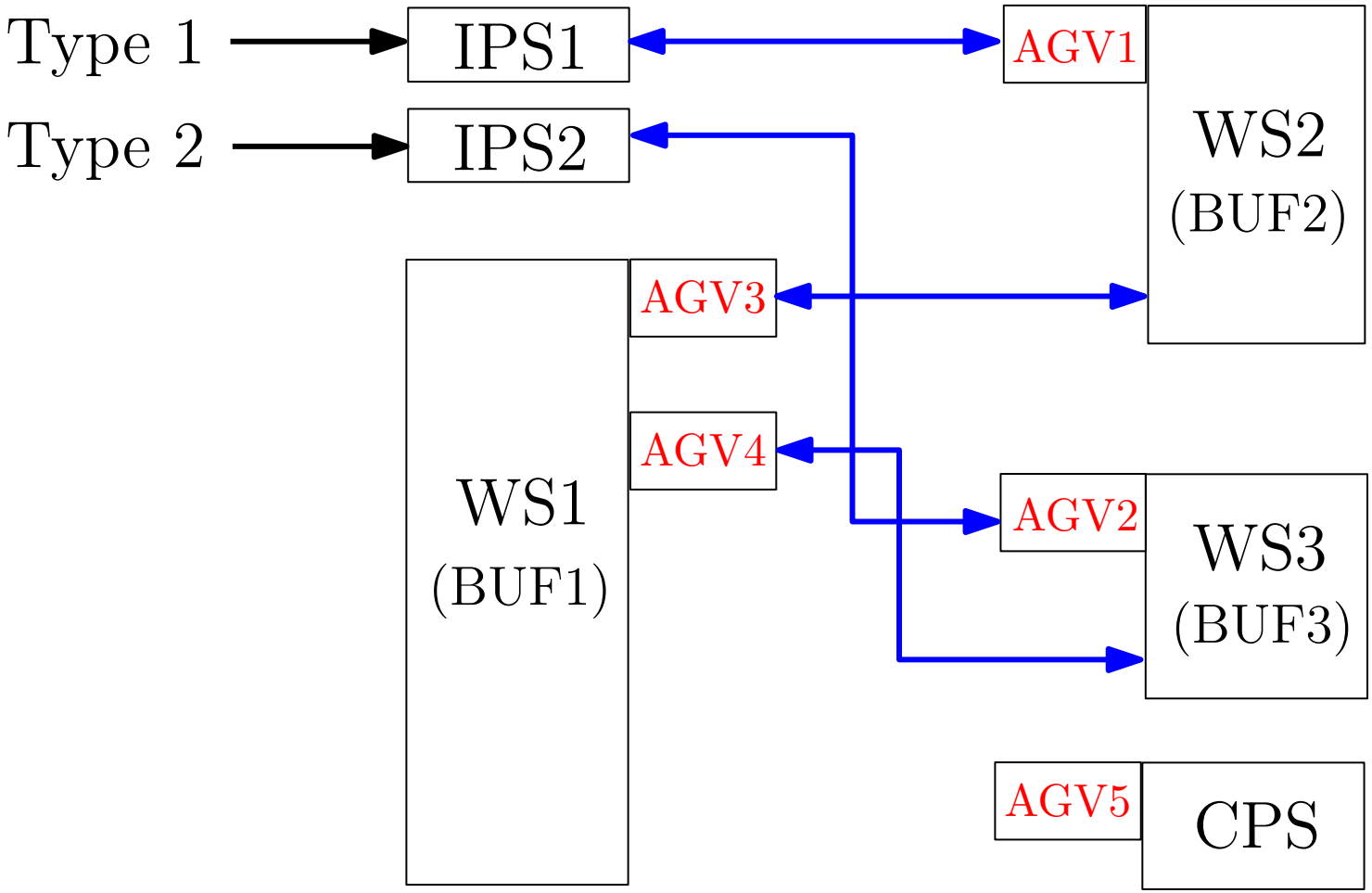

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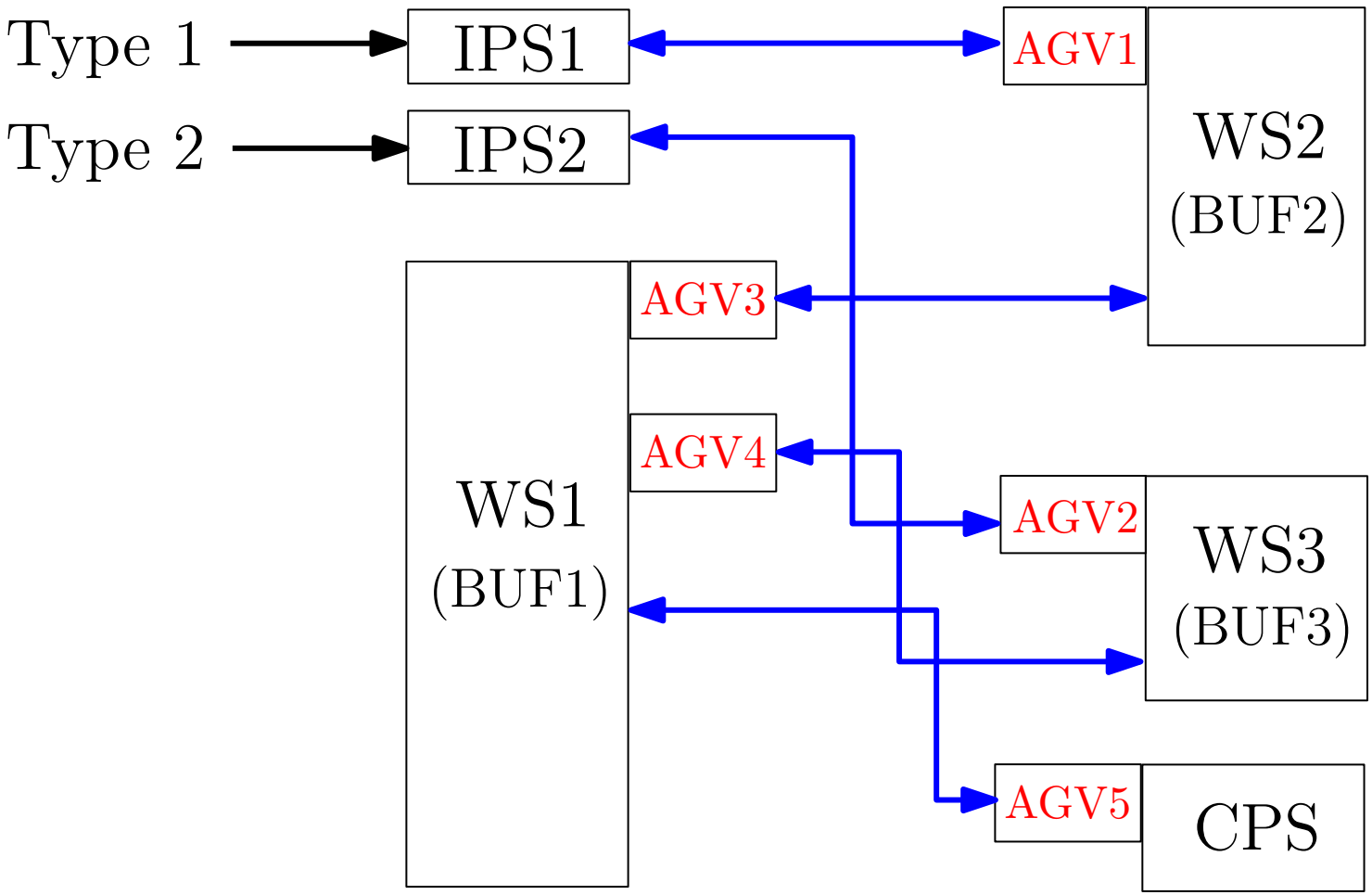

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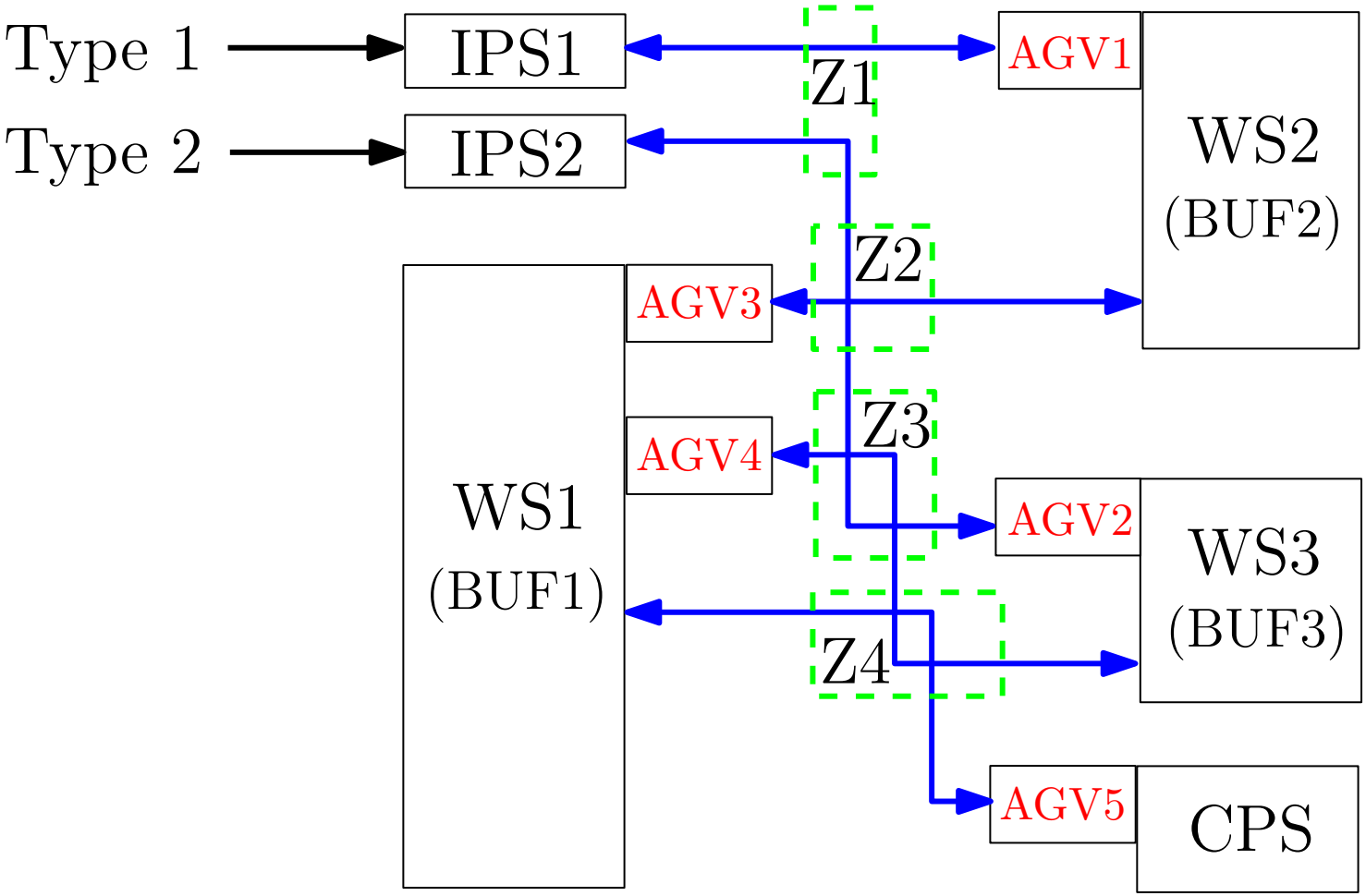

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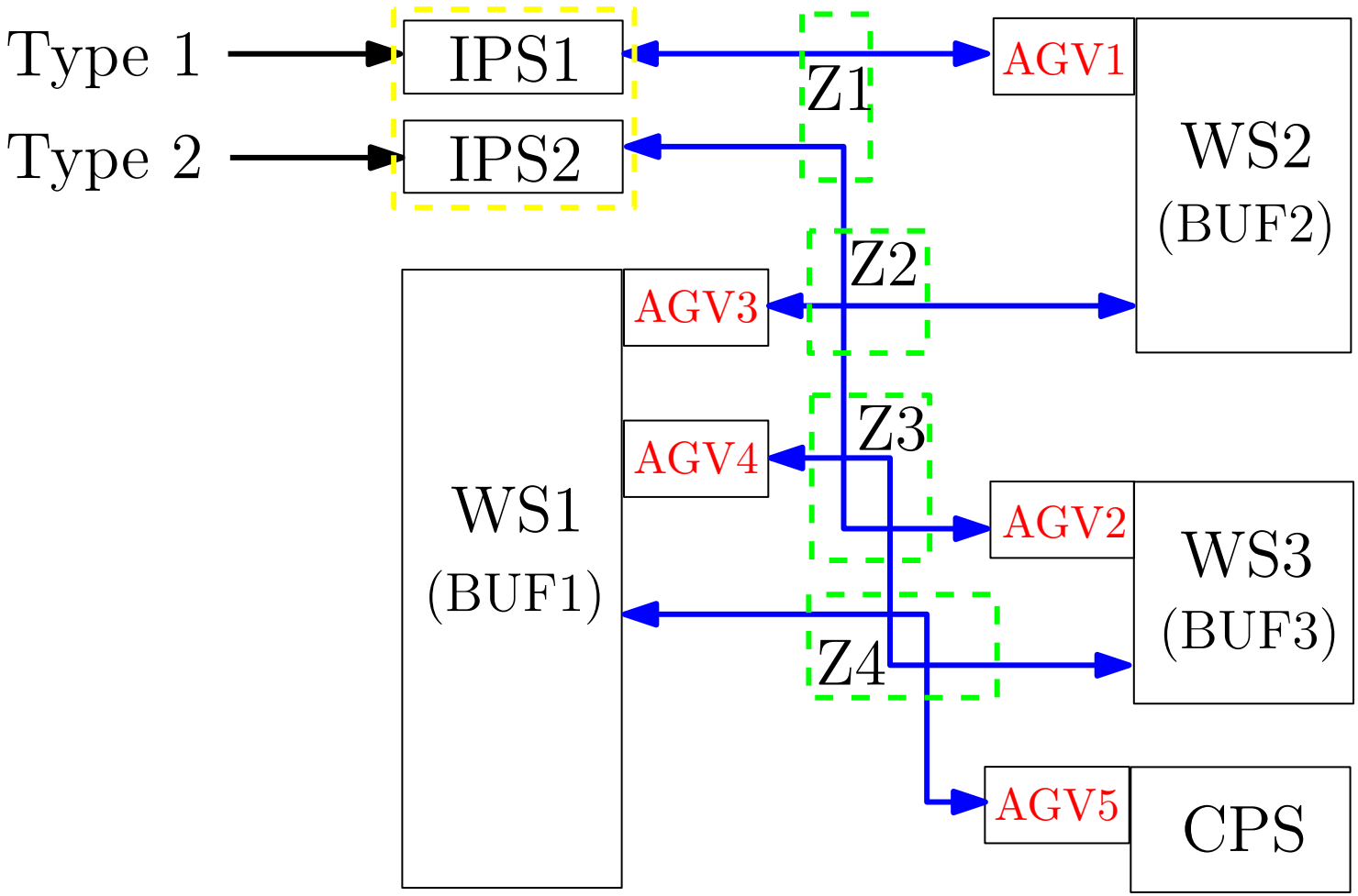

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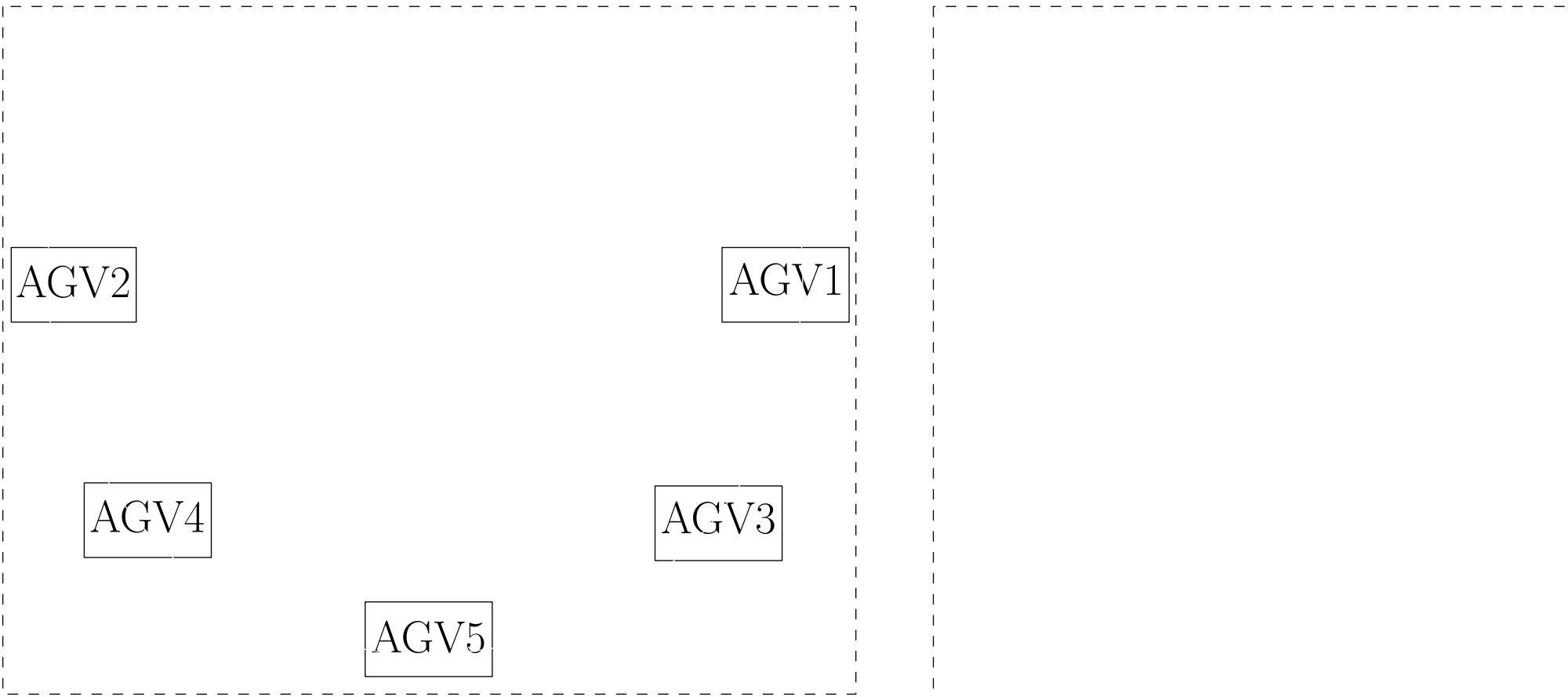

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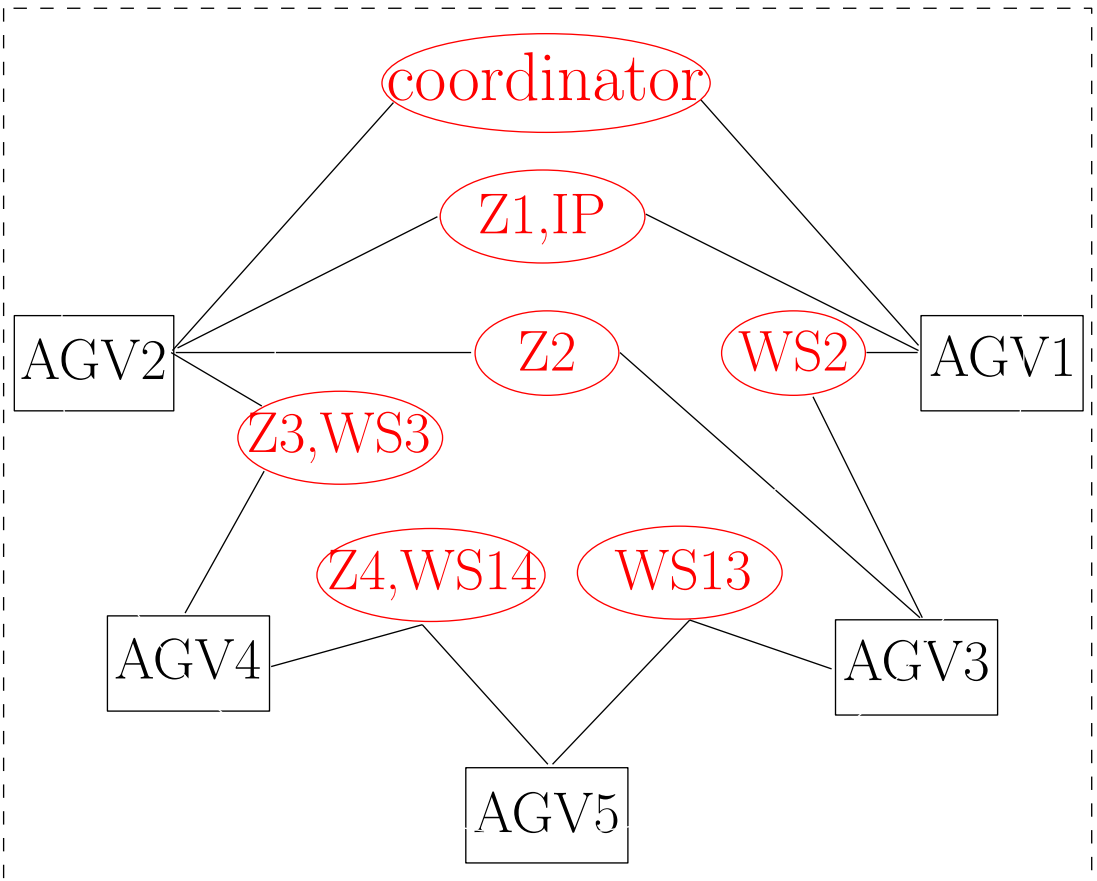

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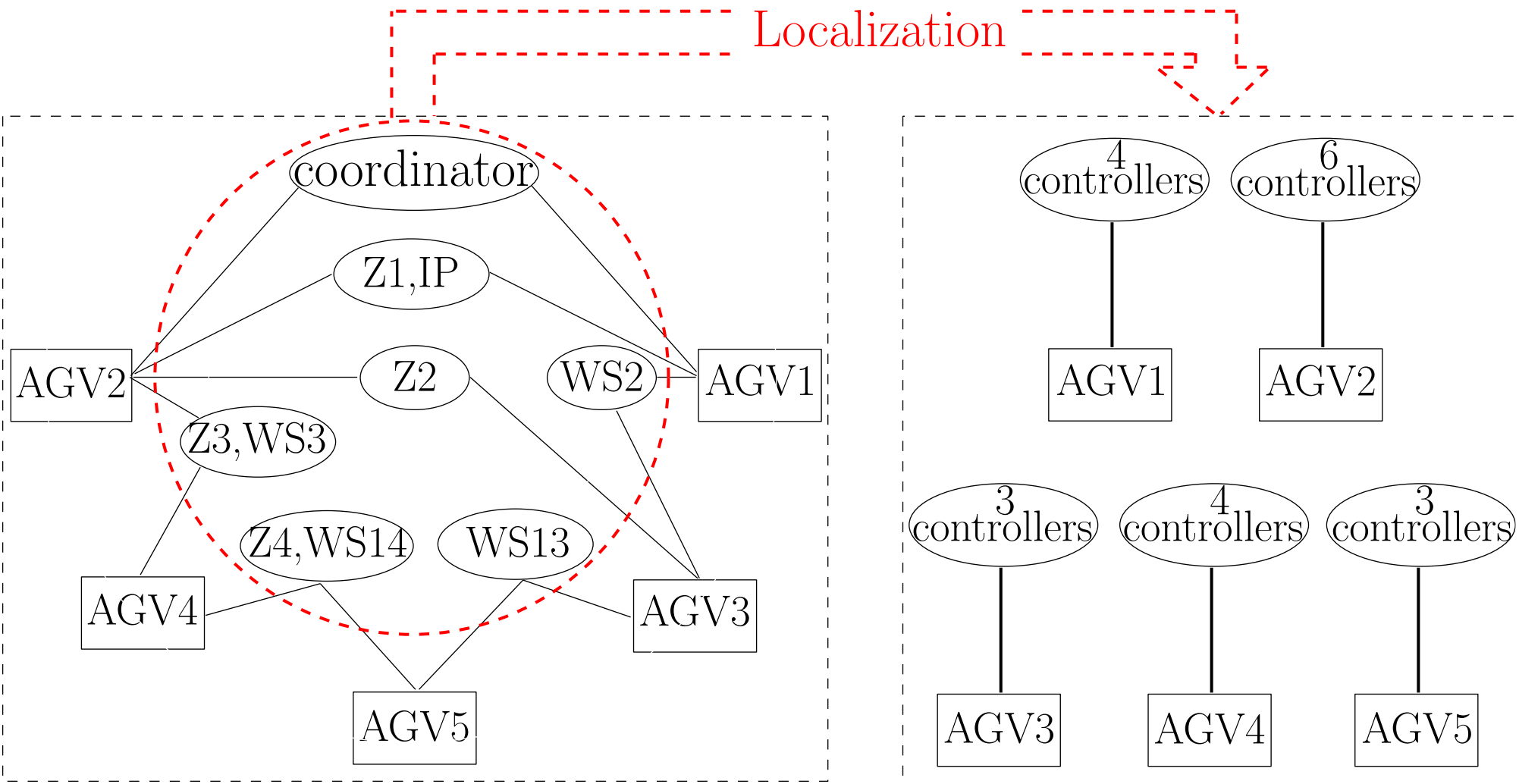

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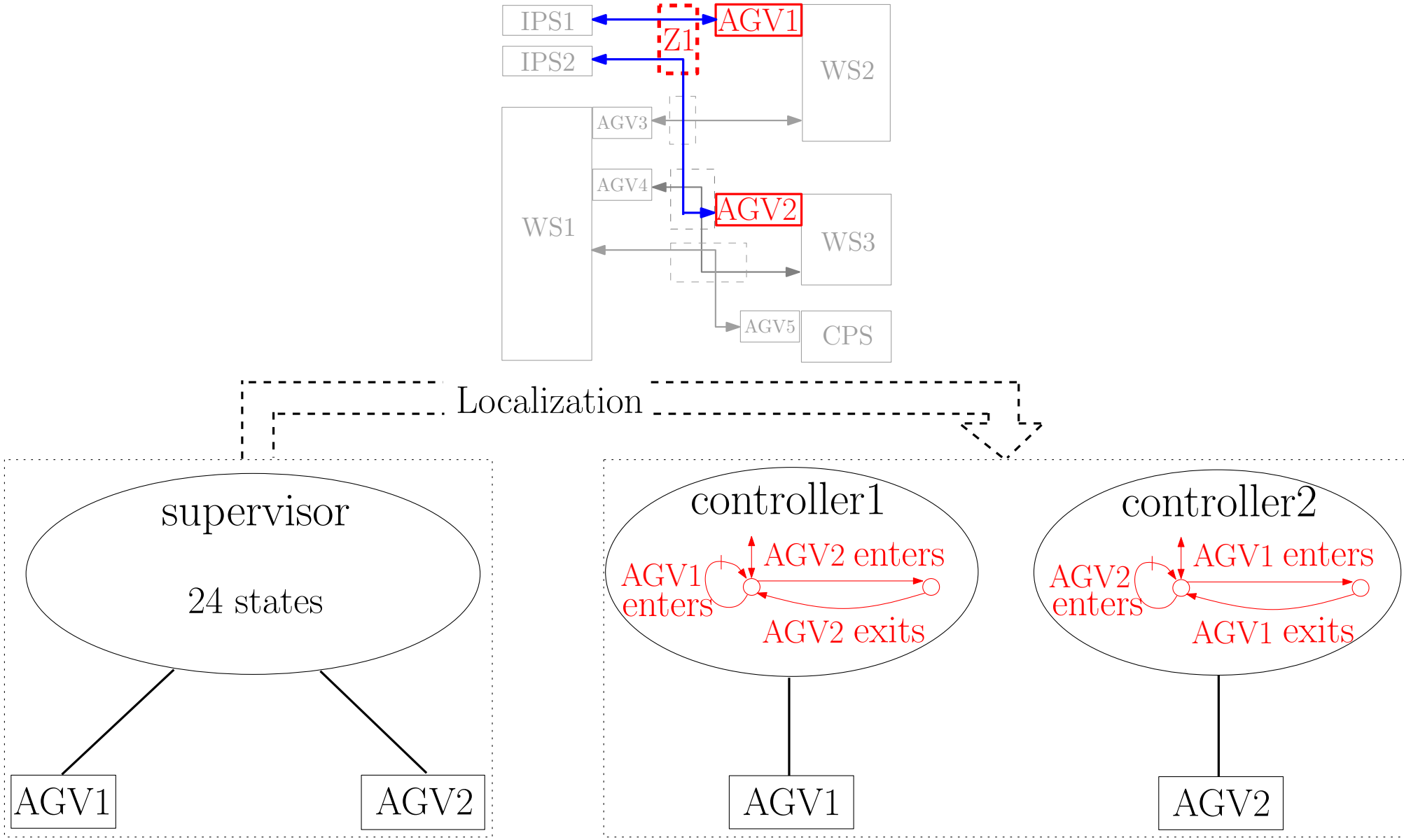
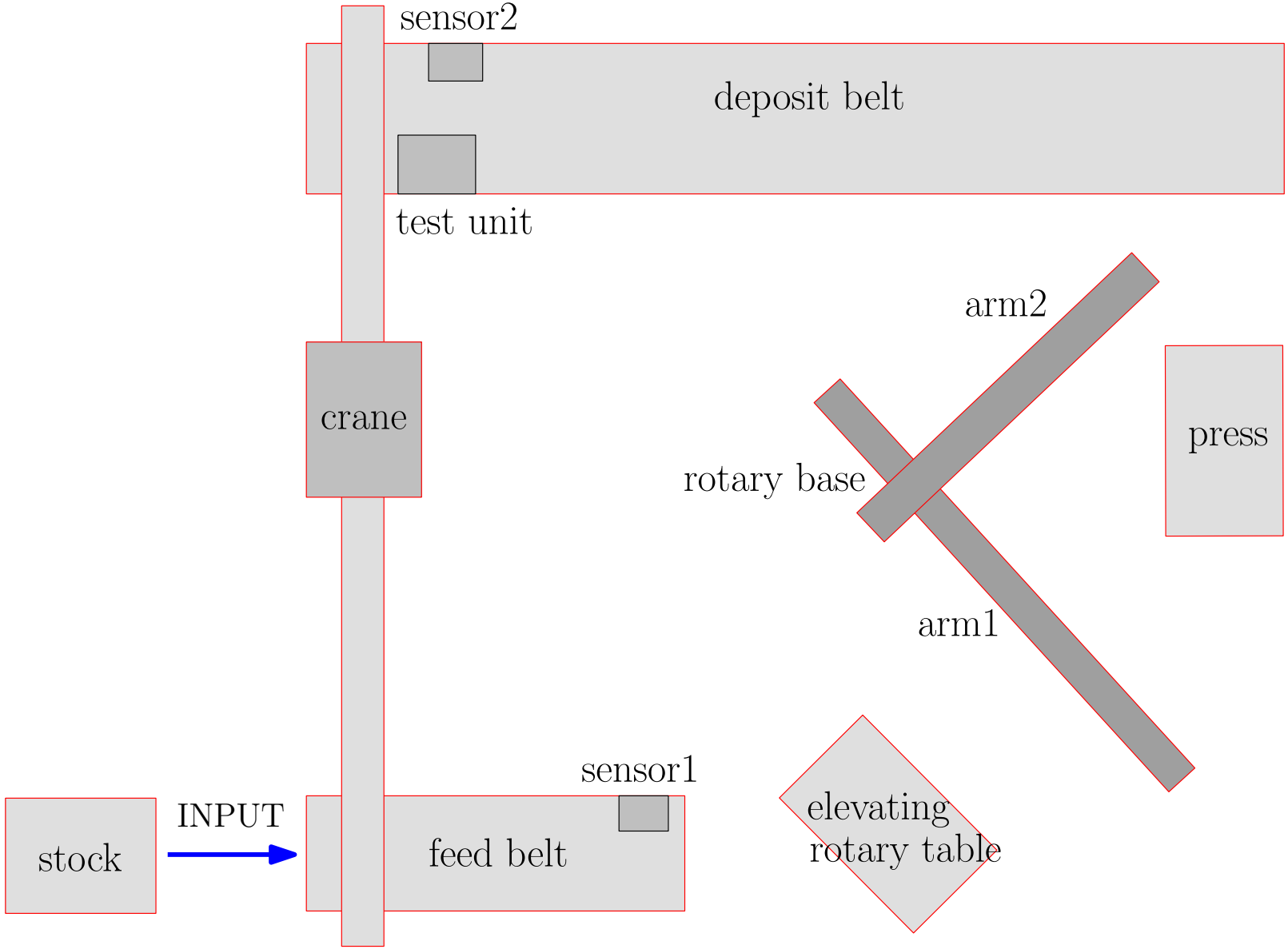


Example 2: Production Cell



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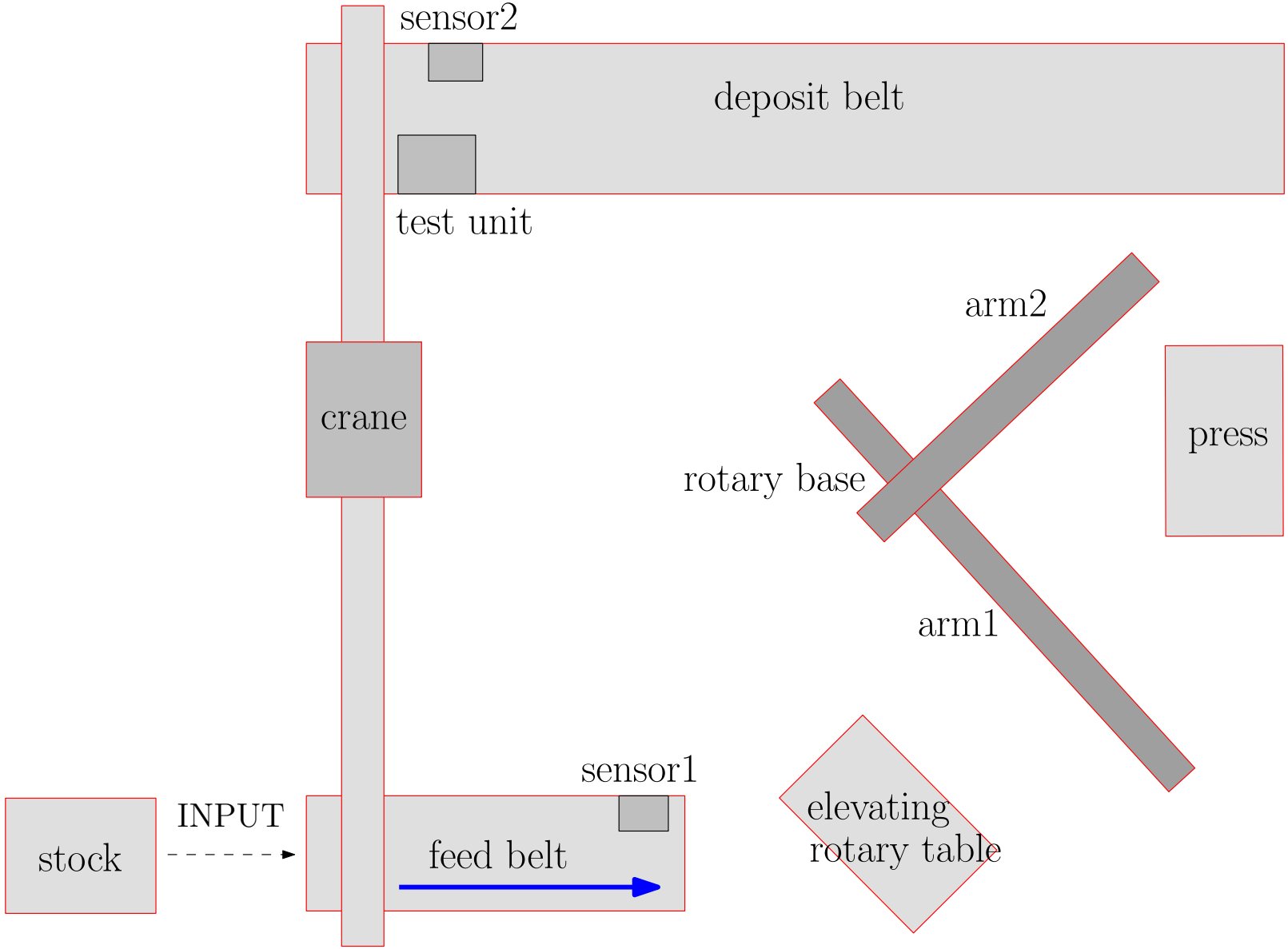

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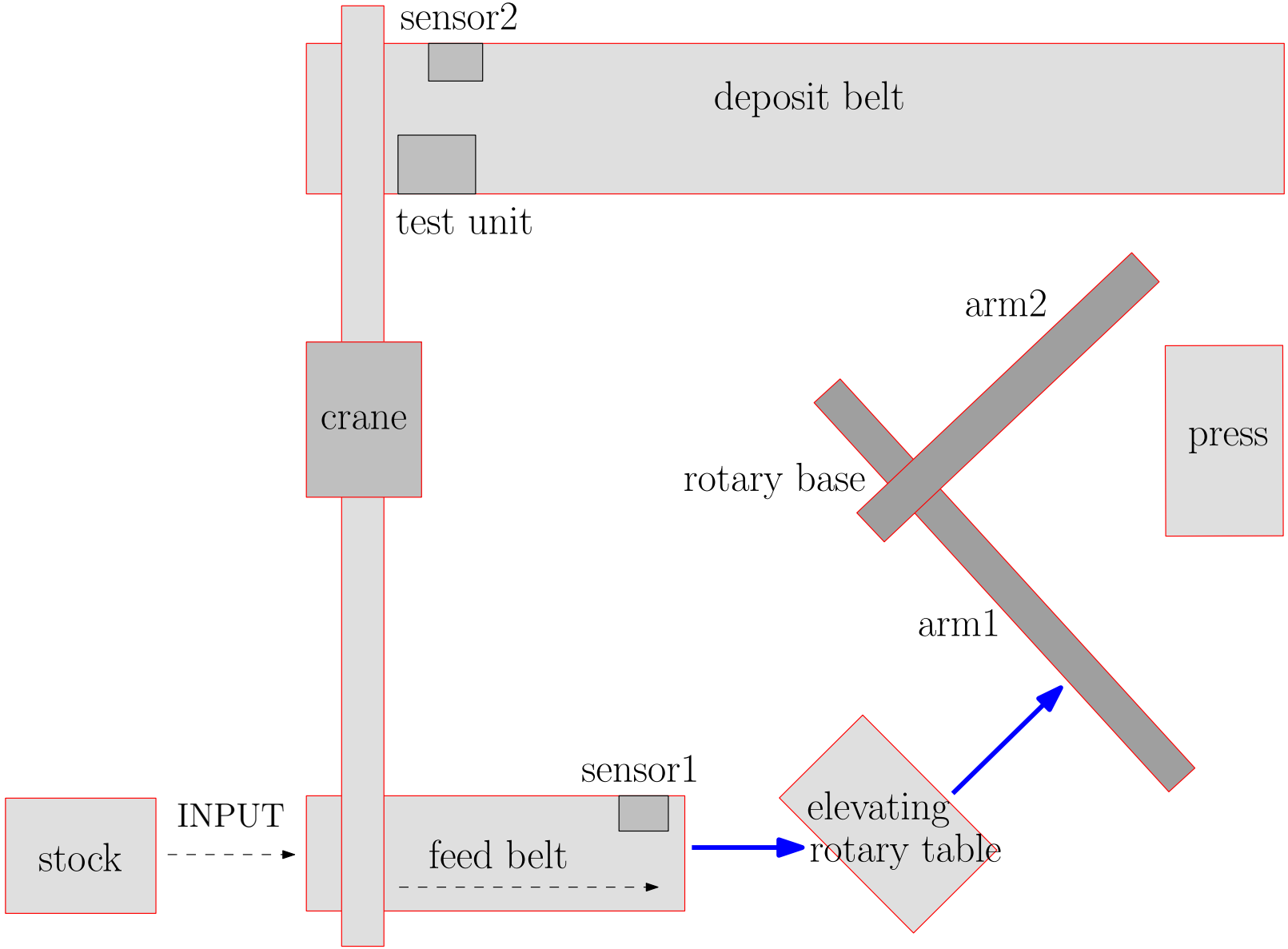

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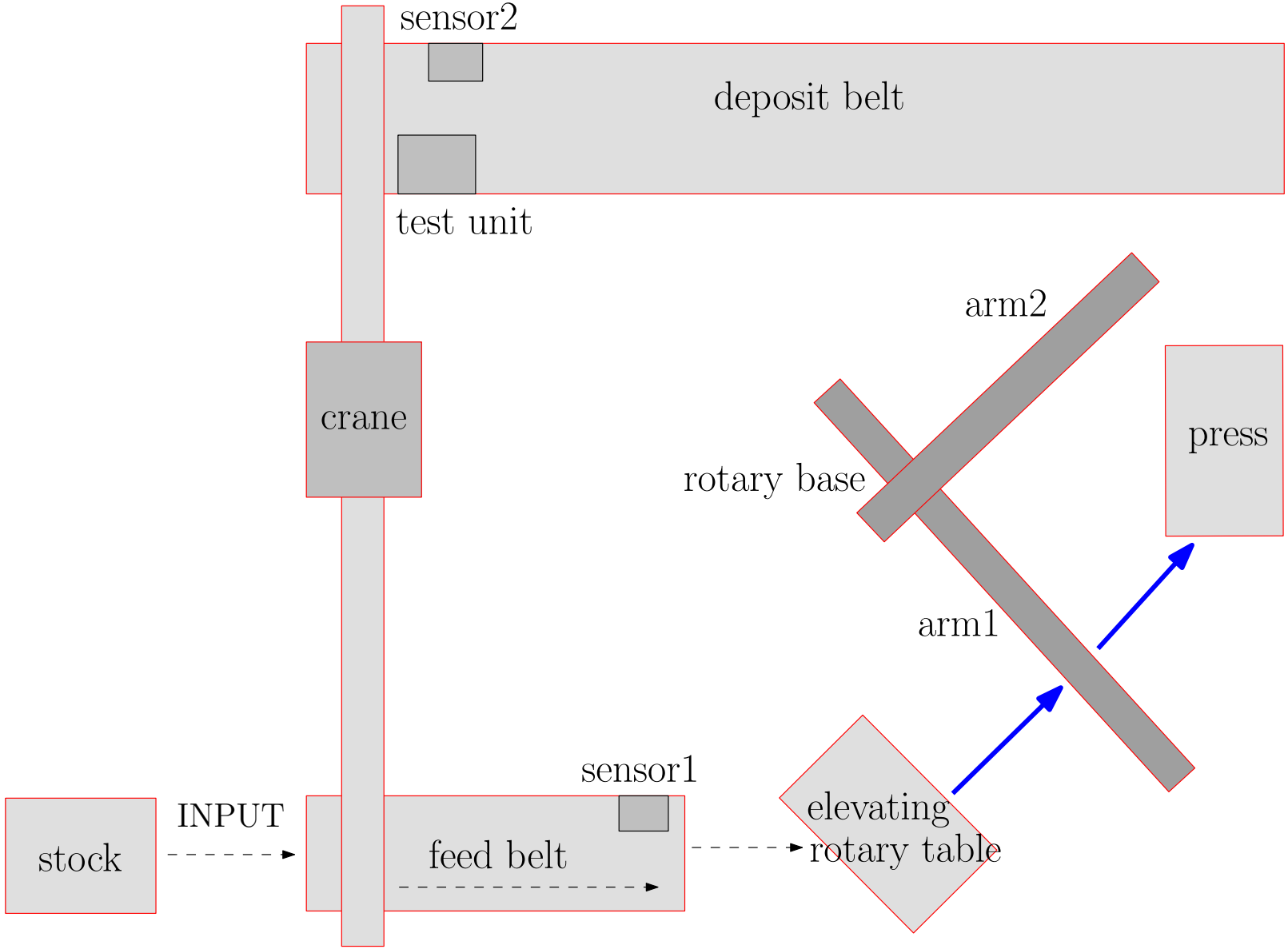

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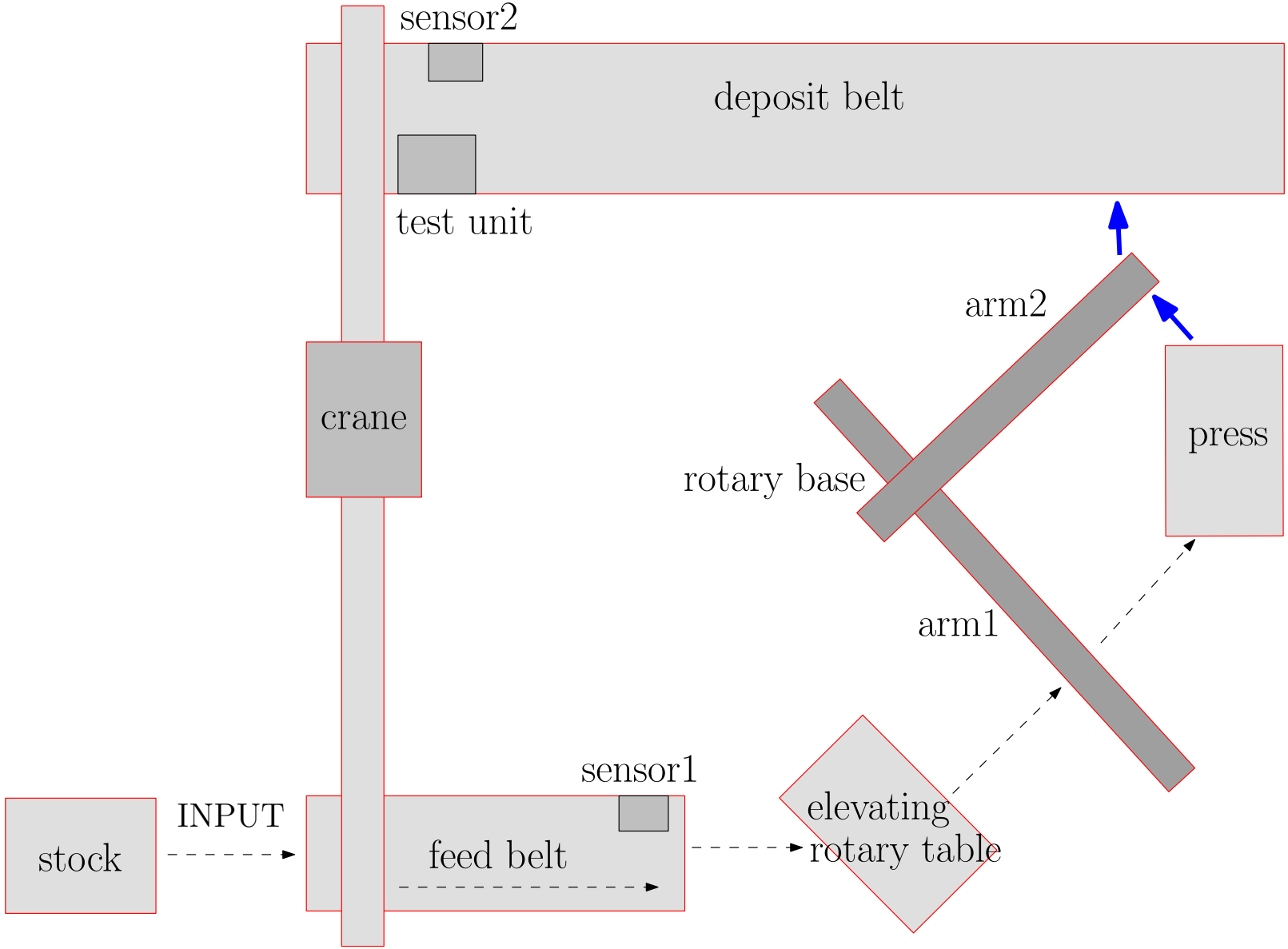

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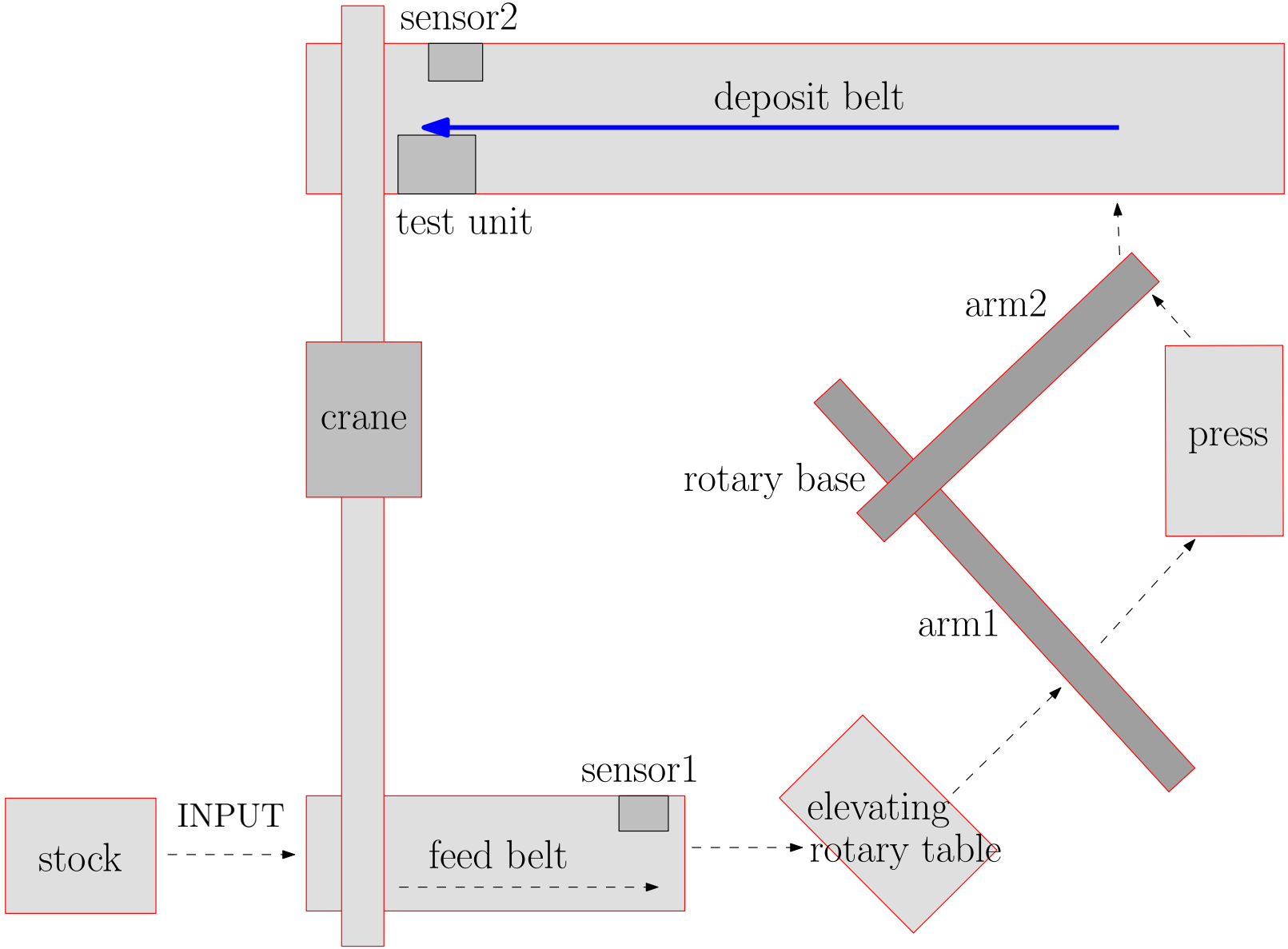

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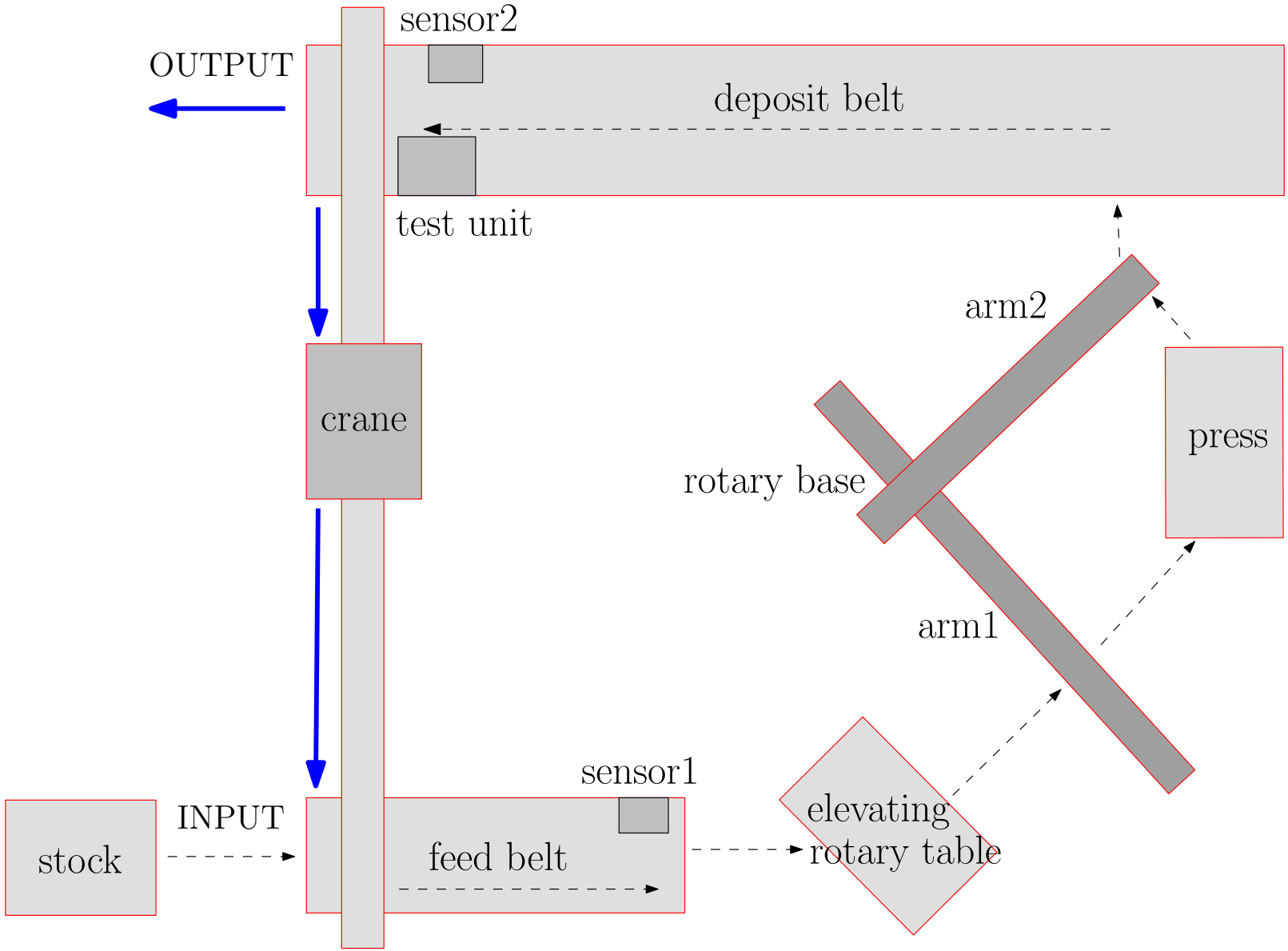

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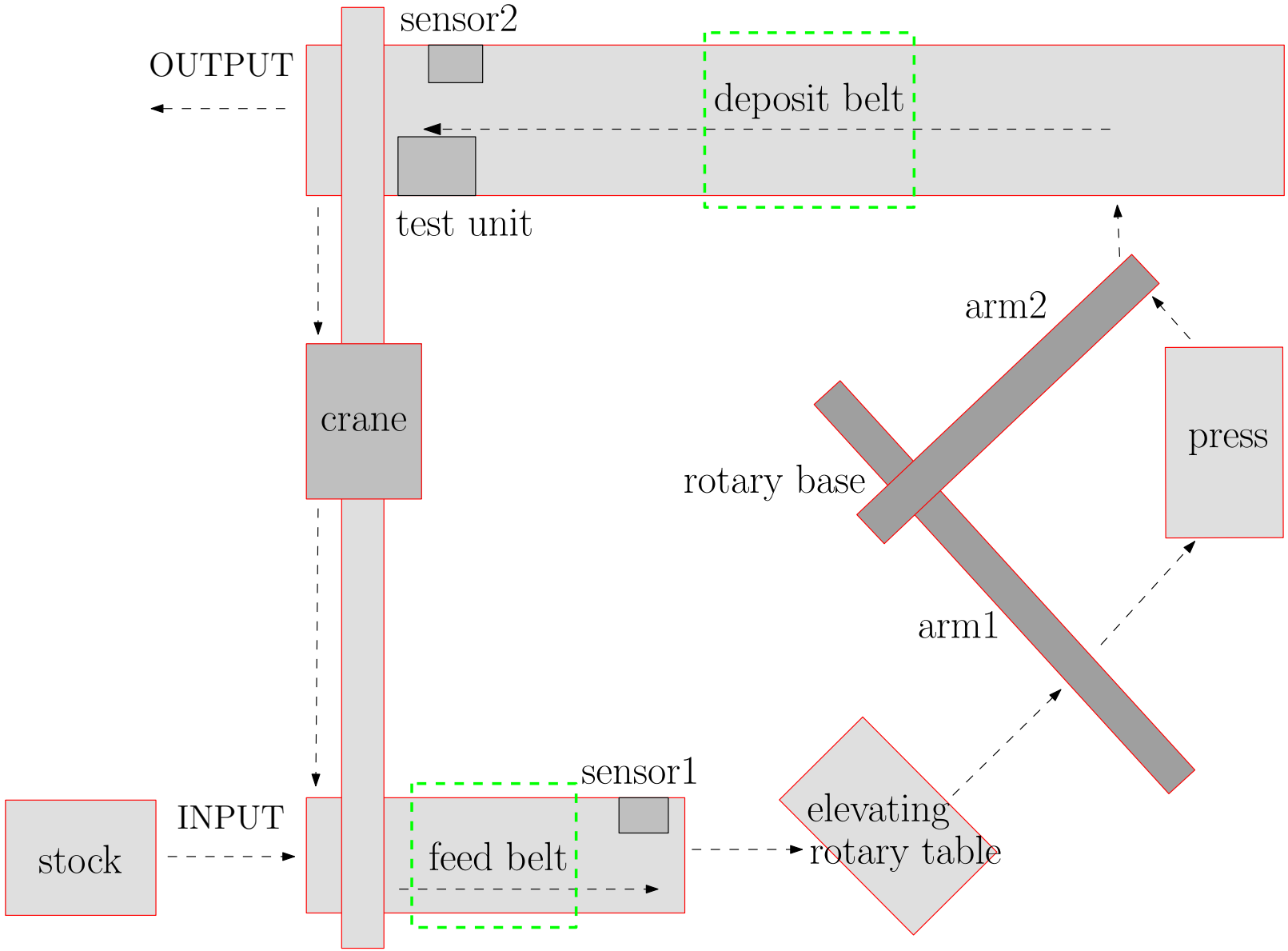

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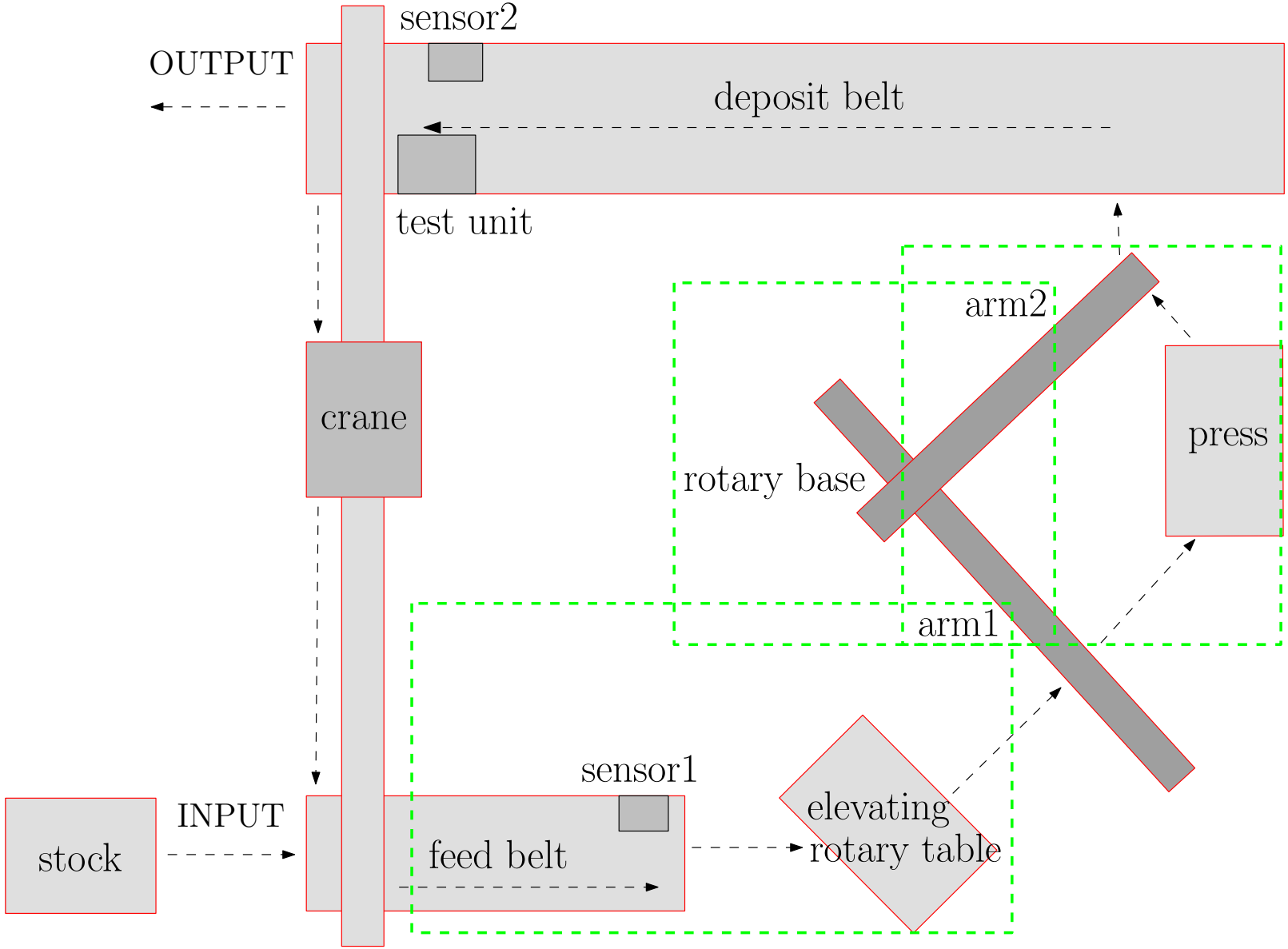

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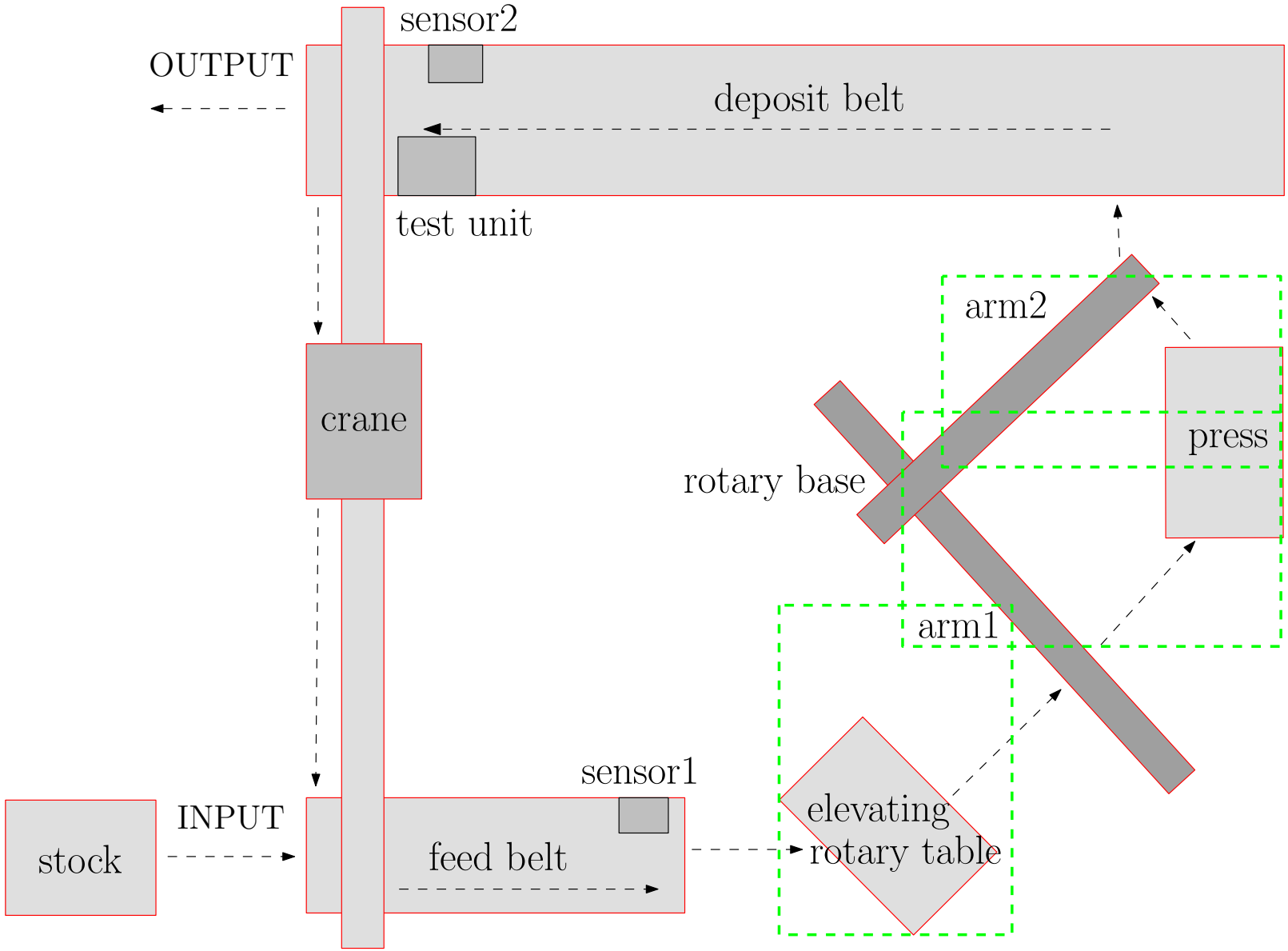

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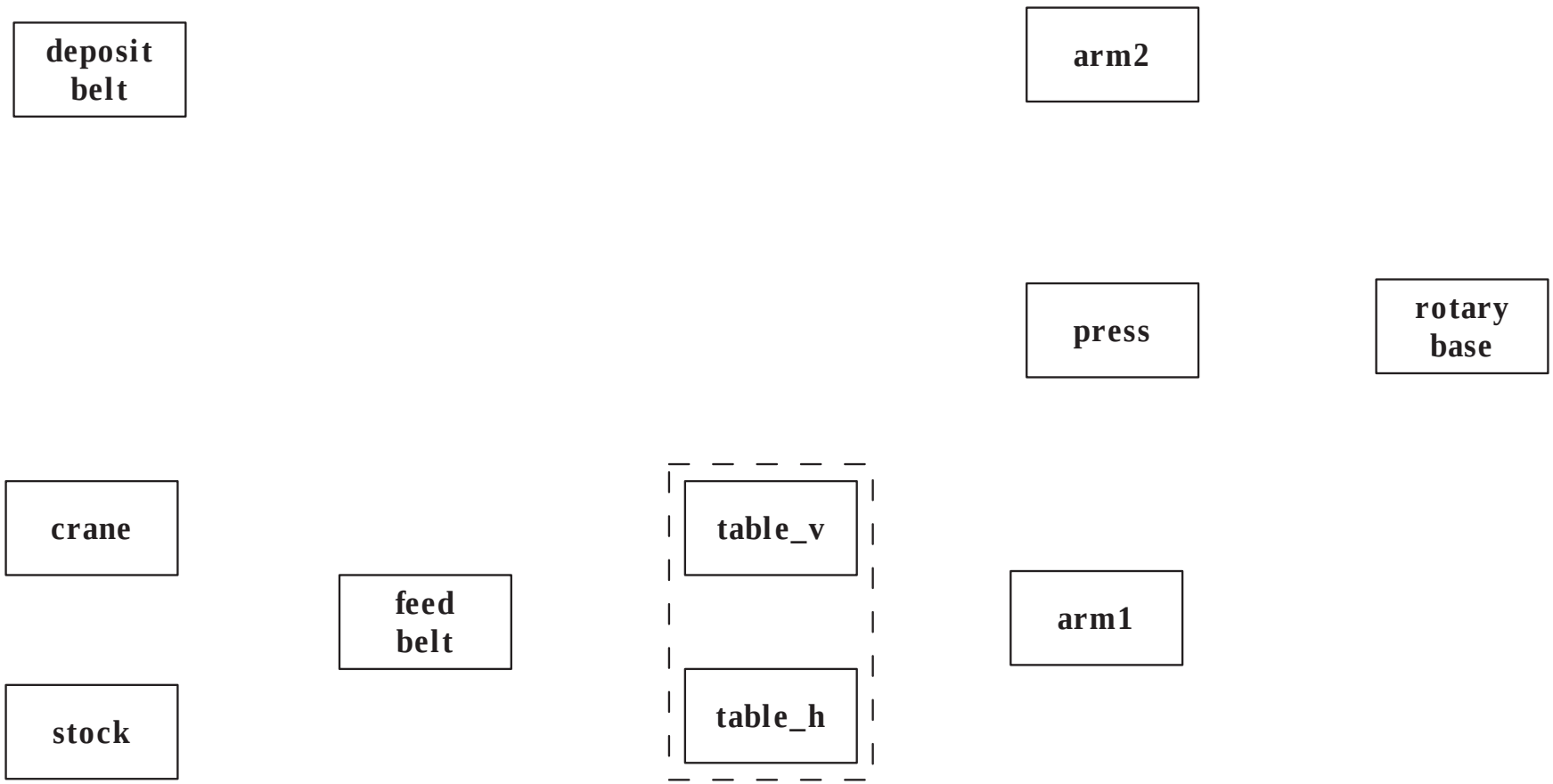

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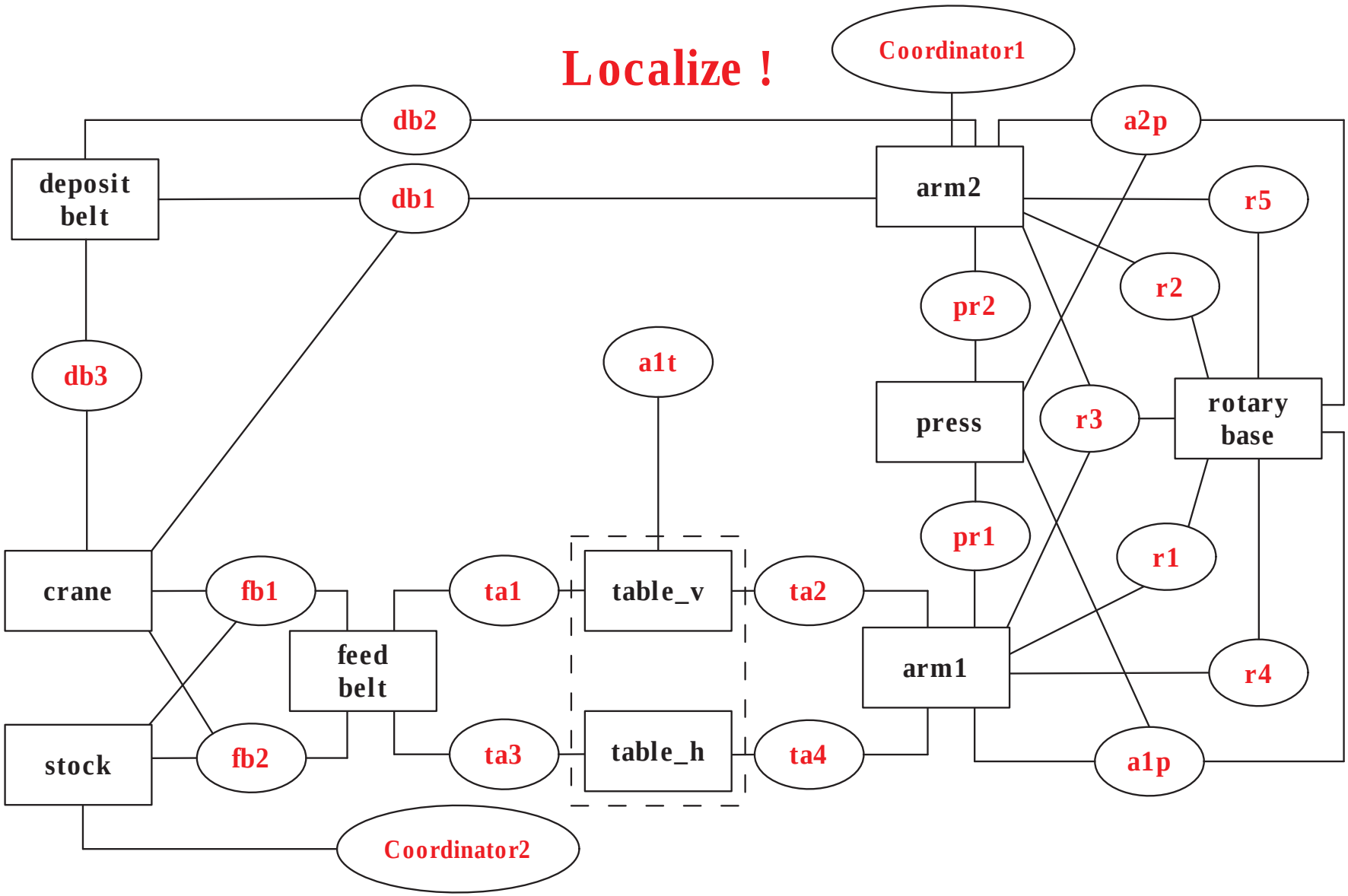

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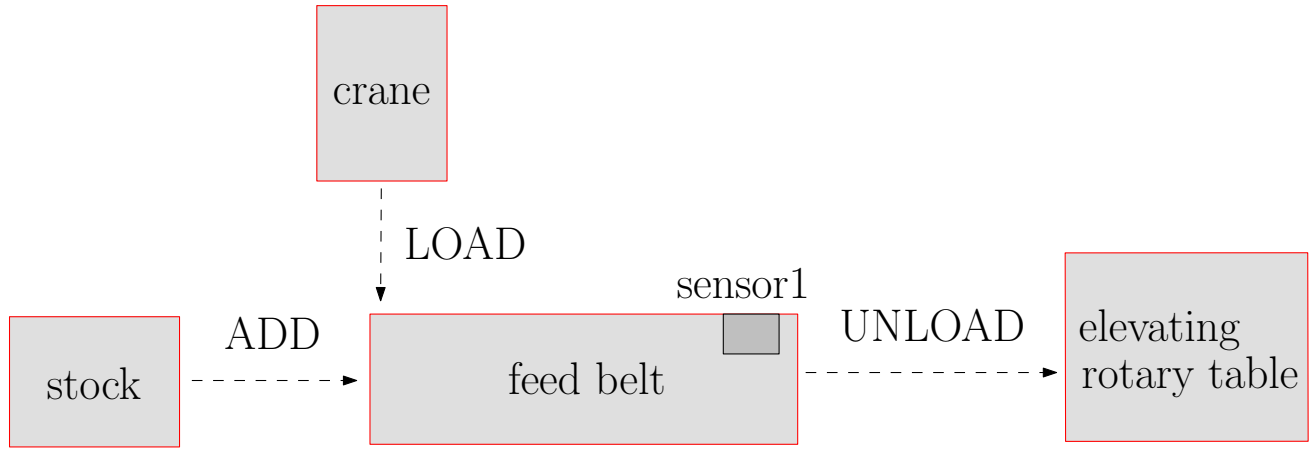
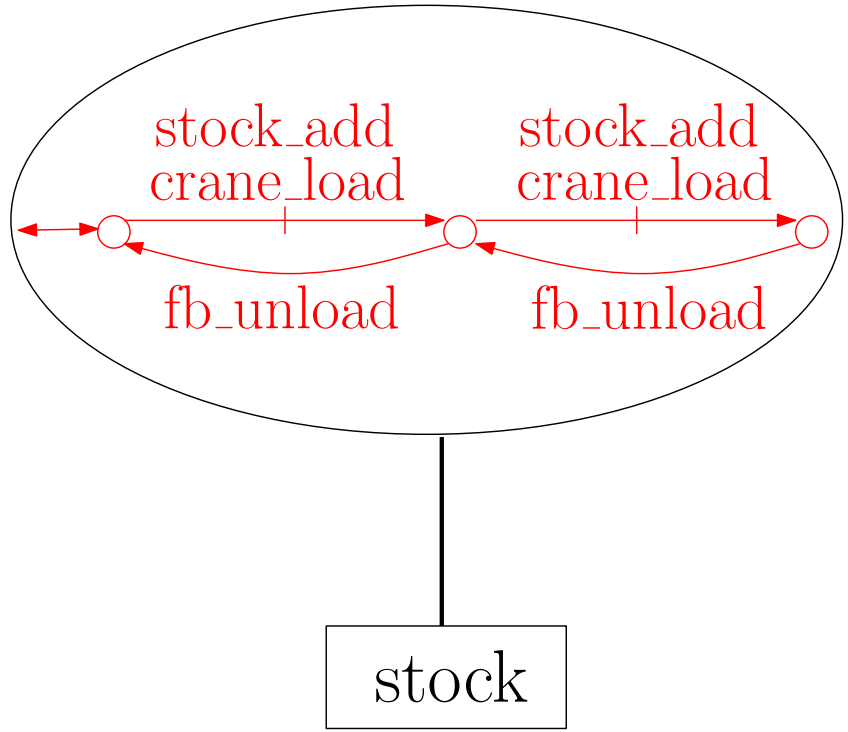
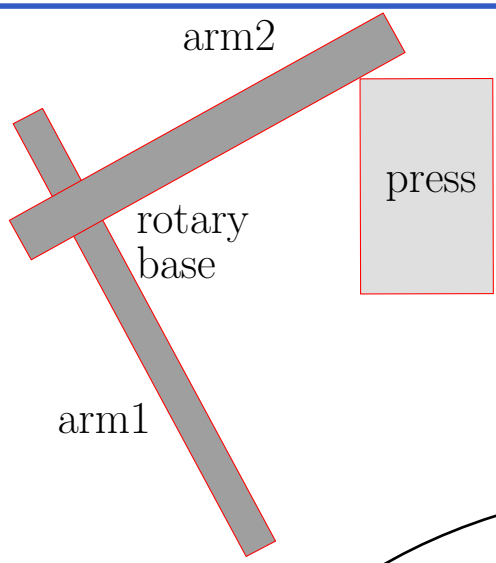




Example 2: Production Cell

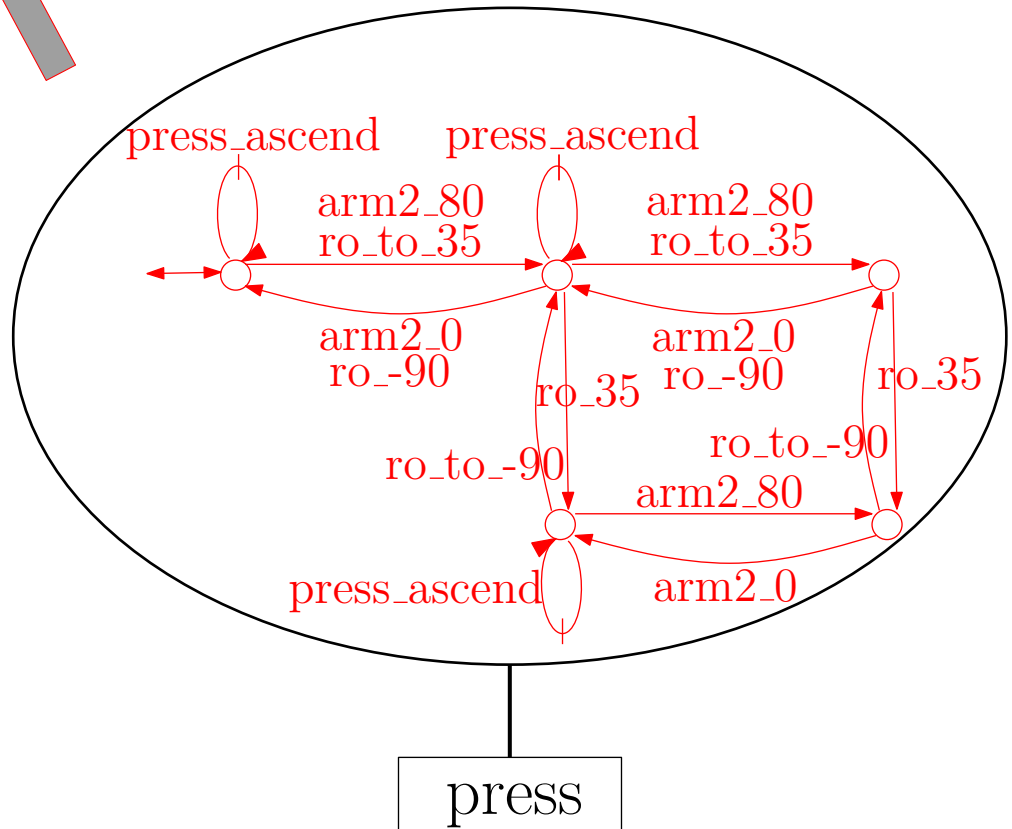


Collision iff

(1) press is not at bottom

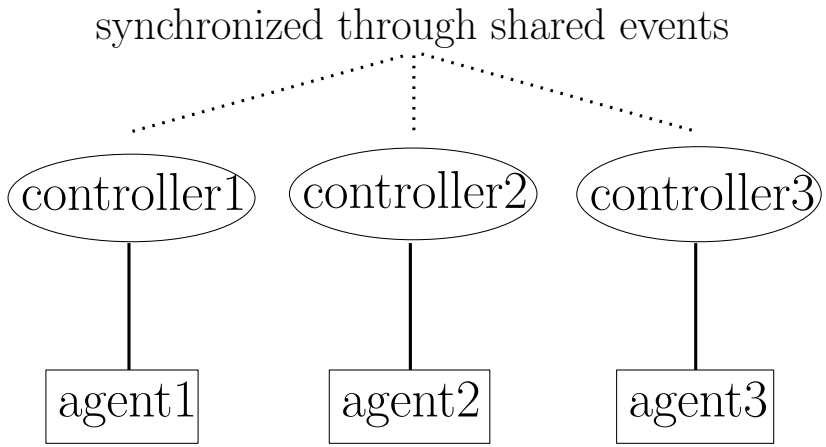
& (2) arm2 is not of length 0

& (3) rotary base is at 35 deg

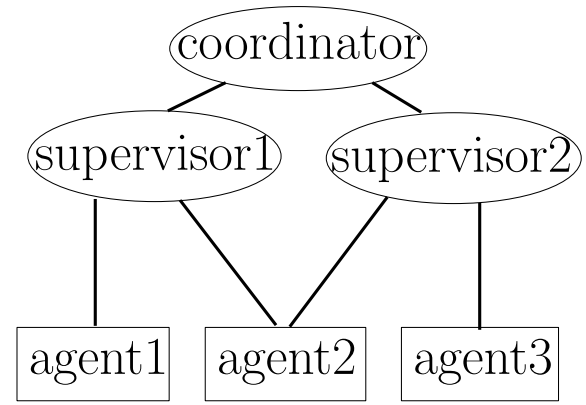


Architectural comparison

- Distributed vs. decentralized:



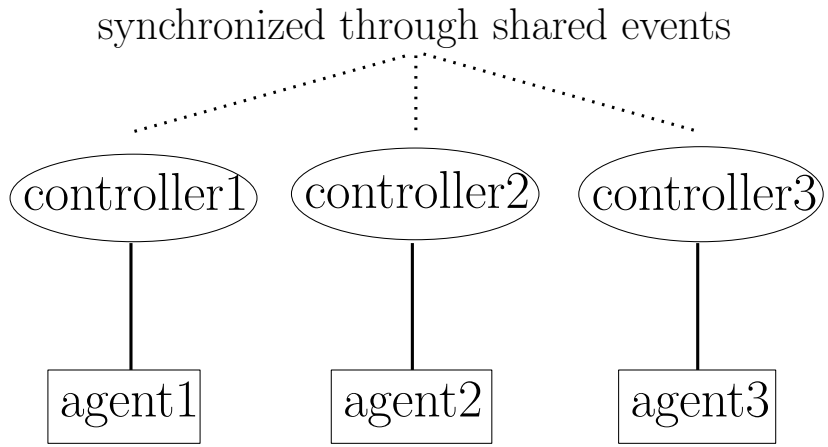
Distributed



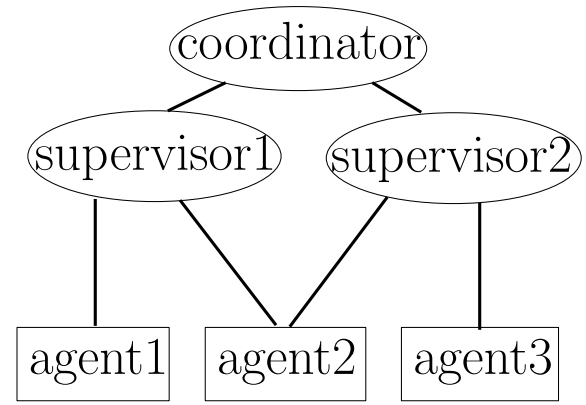
Decentralized

Architectural comparison

- Distributed vs. decentralized:



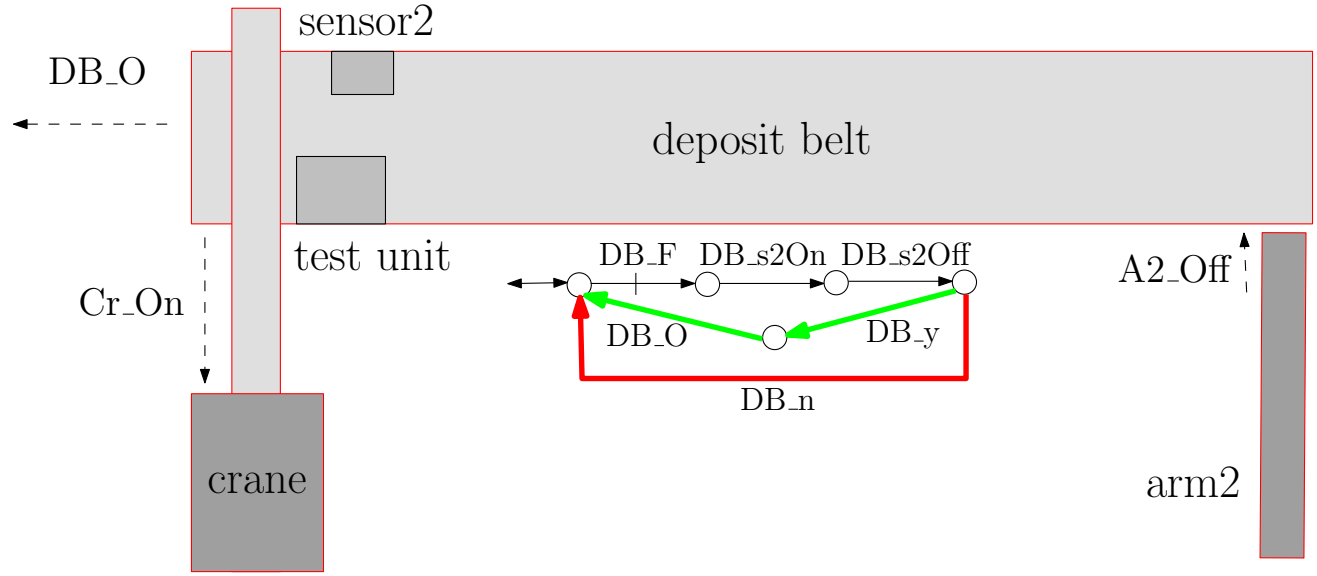
Distributed



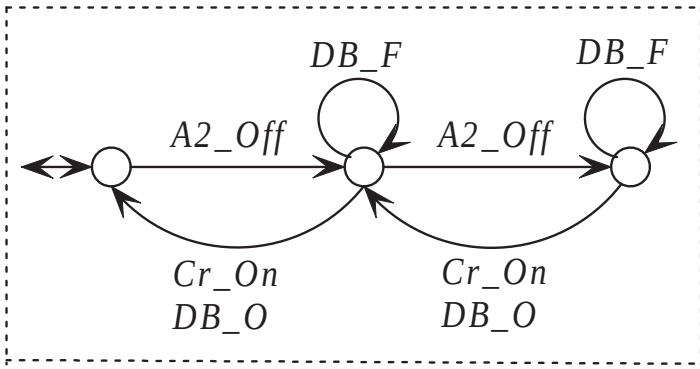
Decentralized

- Key factors: state size, computing load, observation scope
- Key issues: cost-benefit tradeoffs, system robustness, criteria for architectural choice

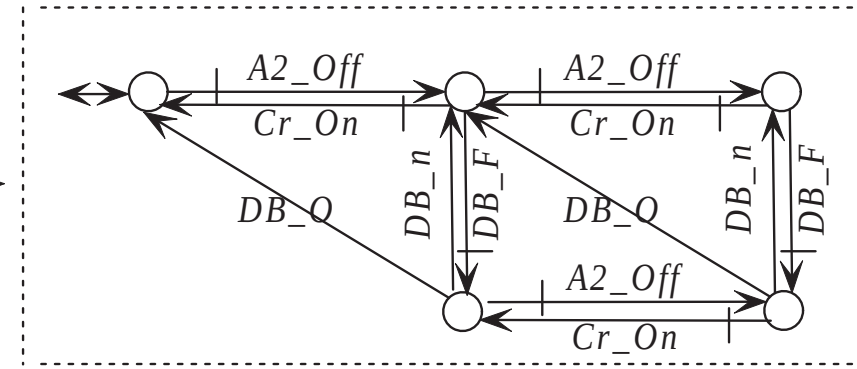
Comparison 1



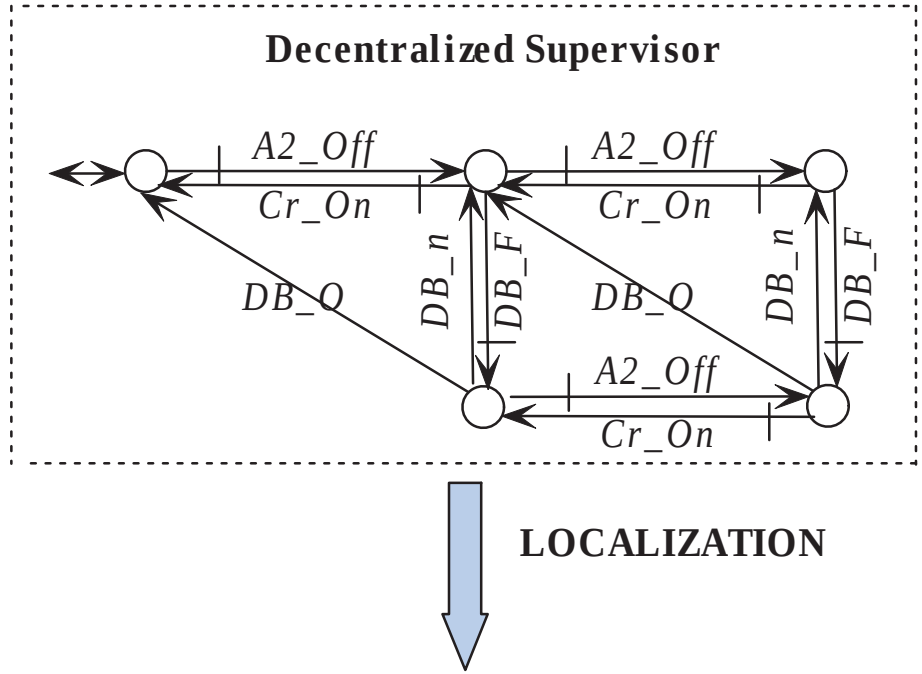
Specification



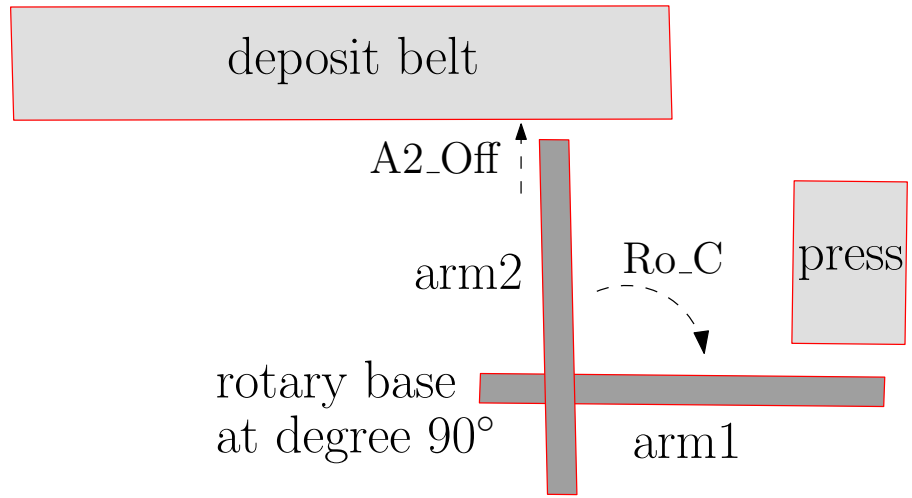
Decentralized Supervisor



Comparison 1



Comparison 2



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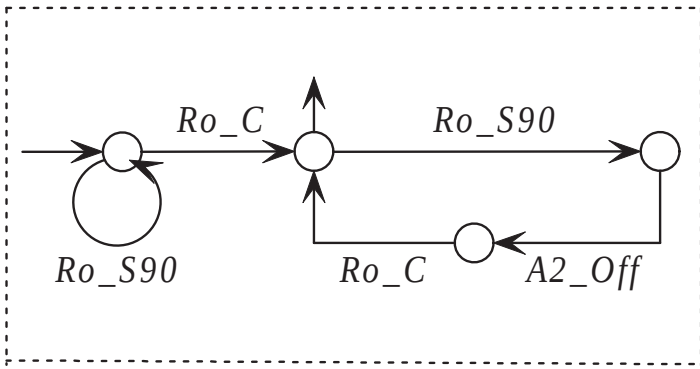
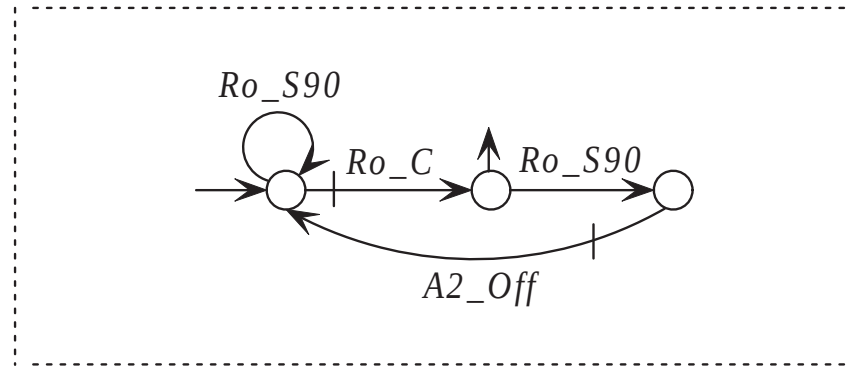
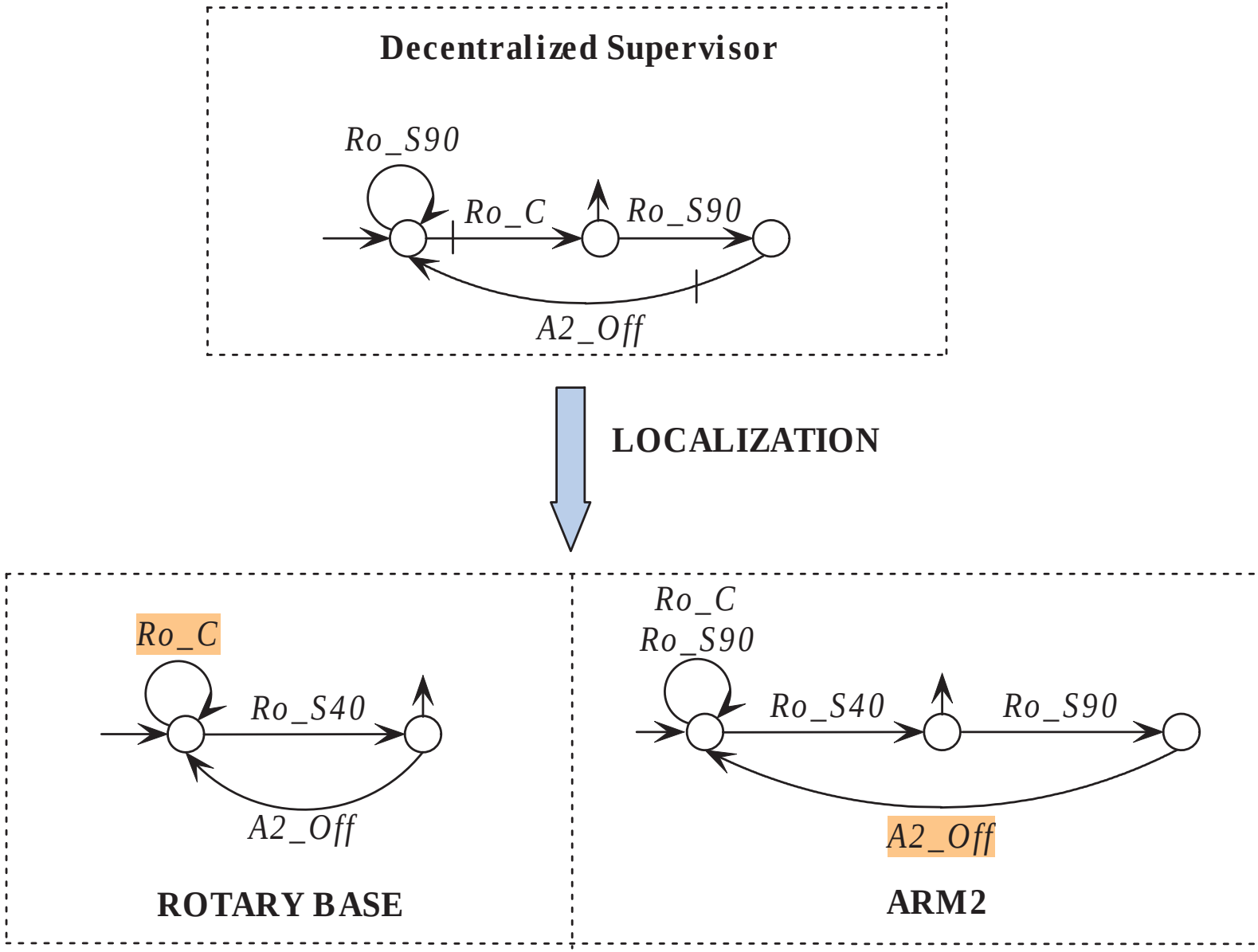


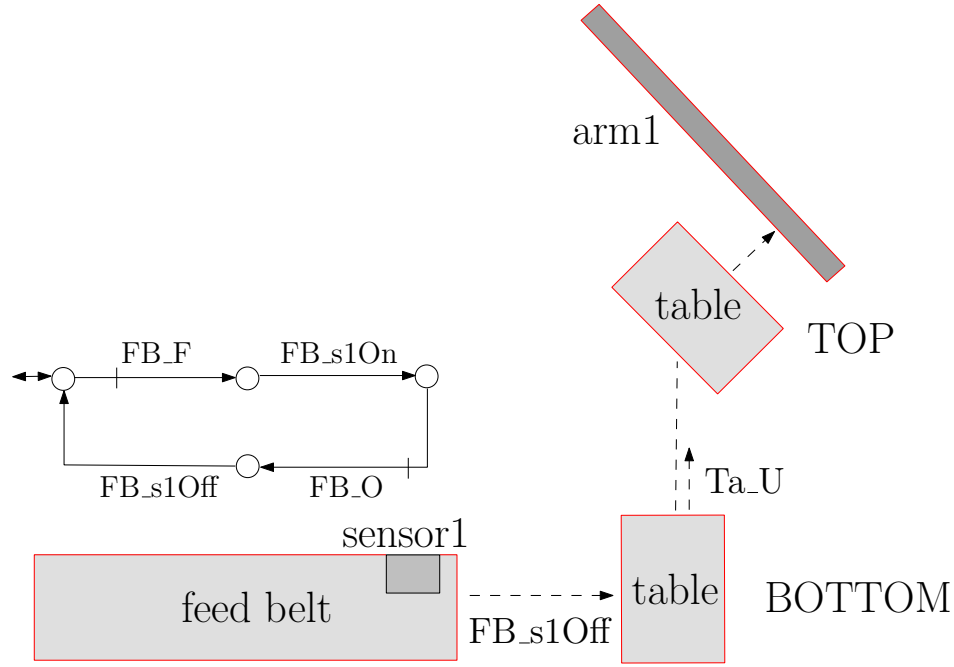
Decentralized Supervisor



Comparison 2

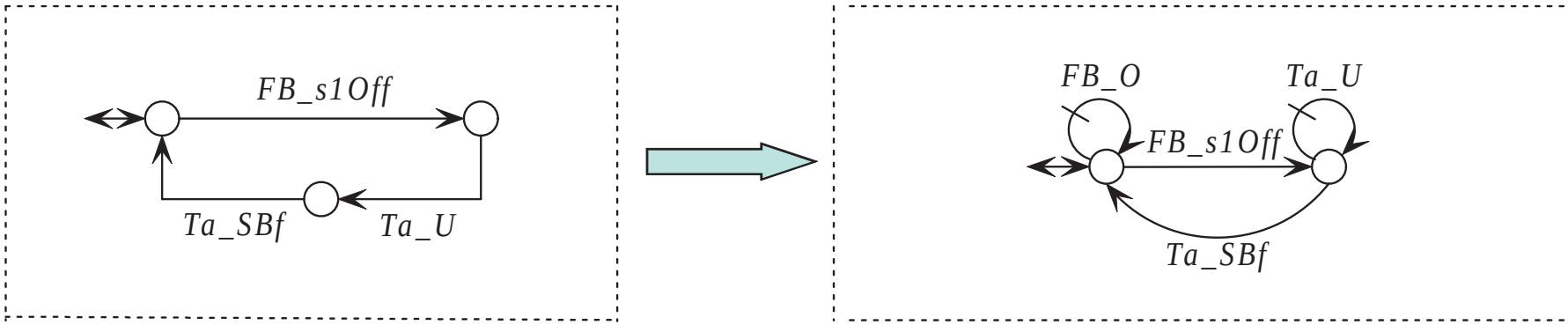


Comparison 3

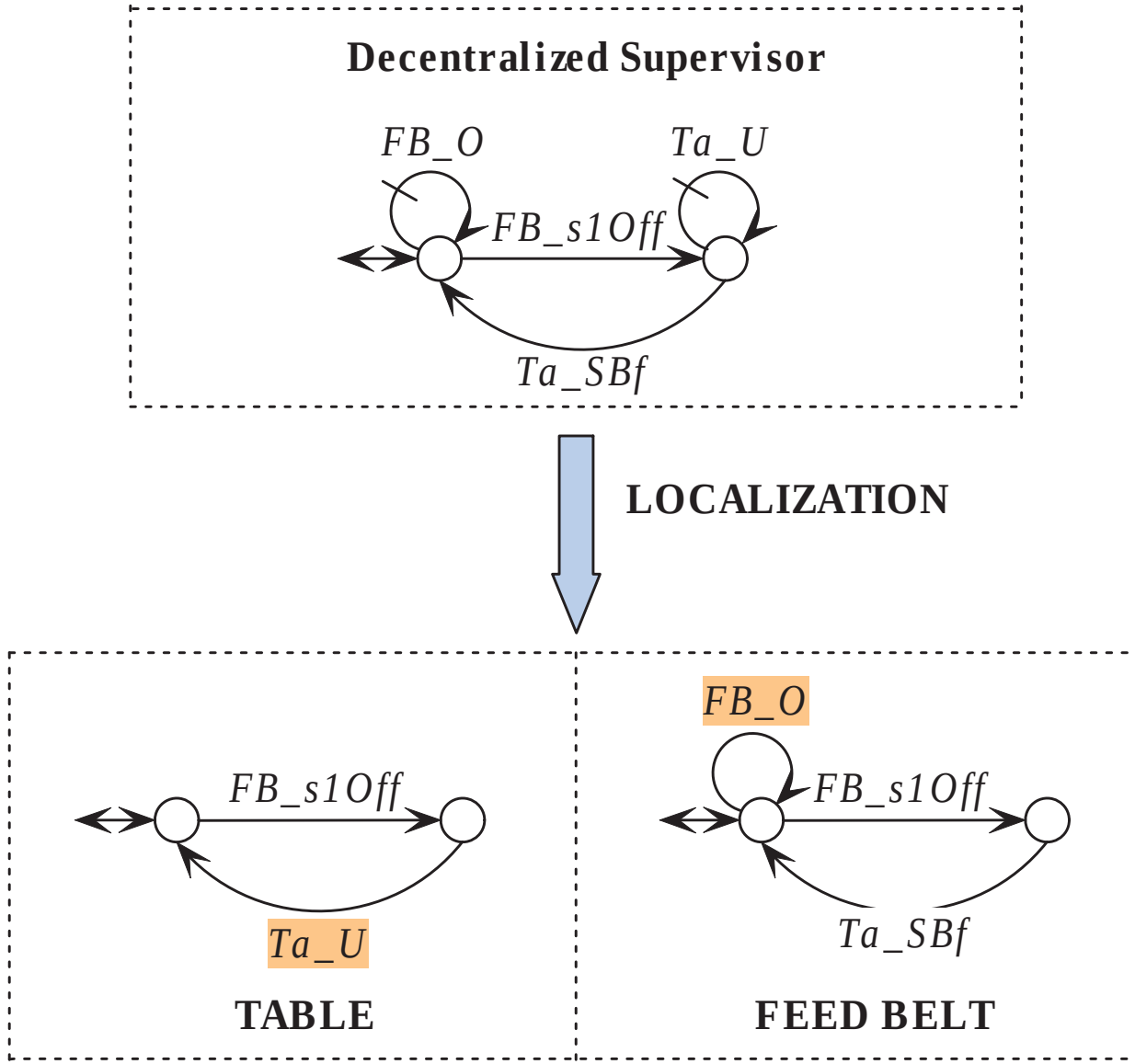


Specification

Decentralized Supervisor



Comparison 3



Conclusions

- Language-based model:
  - supervisor localization algorithm (small-scale systems)
  - decomposition-aggregation procedure (large-scale systems)
- State-based model: state tree structure (STS), efficient monolithic supervisor synthesis
  - STS-based supervisor localization algorithm (small-scale systems)
- Trade-offs between decentralized control and distributed control

References

Five main references to date:

- K. Cai MASc thesis: "Supervisor Localization: A Top-Down Approach to Distributed Control of Discrete-Event Systems", Univ. of Toronto, 2008.
- CISA'09: K. Cai and W.M. Wonham, "Supervisor localization: a top-down approach to distributed control of discrete-event systems", Zarzis, Tunisia.
- CDC'09: K. Cai and W.M. Wonham, "Supervisor localization for large-scale discrete-event systems", Shanghai, China.
- IEEE Trans. Autom. Control: K. Cai and W.M. Wonham, "Supervisor localization: a top-down approach to distributed control of discrete-event systems", to appear March 2010.
- Submitted October 2009: K. Cai and W.M. Wonham, "Supervisor localization for large discrete- event systems – case study production cell".

Future work

- STS-based supervisor localization algorithms for very (!) large-scale systems
- Extensive quantitative trade-offs between decentralized control and distributed control
- Application to fault tolerance via localized controller redundancy
- Application to collective behavior of many agents: formation control

The Distributed Control Principle

- ANYTHING YOU CAN DO GLOBALLY, YOU CAN DO LOCALLY!
- 凡 全 局 可 解，
  则 局 部 亦 可 解！