ECE F410 CONTROL SYSTEMS

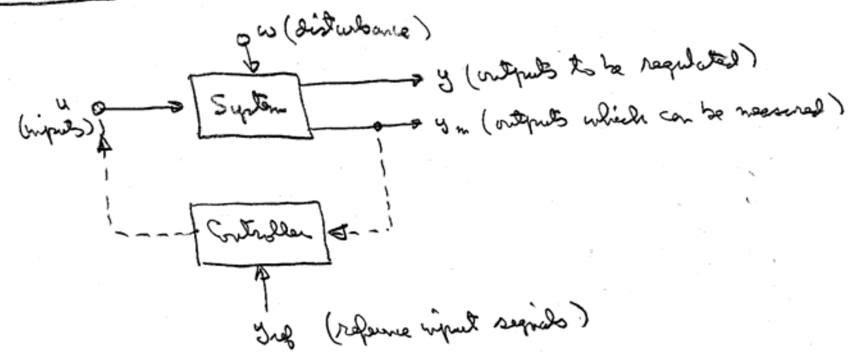
FOCUS OF COURSE

Multivariable control of linear time invariant (LTI) systems

MOTIVATION

MOST INDUSTRIAL CONTROL PROBLEMS FALL INTO THIS CATAGORY

GENERAL PROBLEM TO BE CONSIDERED



Let $e = (y_{ref} - y)$ denote tracking / regulator error in the system

Given a "system" (may be physical, economical, biological, management, financial ...)

it is desired to find a "controller" (may be "centralized" or "decentralized")

so that

(a) $\lim_{t \to \infty} e(t) = 0$, i.e. asymptotic tracking & disturbance rejection occurs

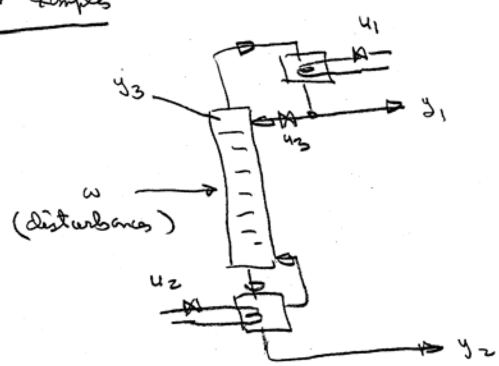
(b) the transient behaviour of the system has desirable properties (which is problem dependent), e.g. may wish controlled system to be stable, with "best" speed of response, with no oscillations etc.

This problem is called the servomechanism problem. Here the system (sometimes called the "plant") is generally multivariable, i.e.

$u \in R^m$ — m inputs
$y \in R^r$ — r outputs
$y_m \in R^{r_m}$ — r_m measurable outputs
$\omega \in R^\Omega$ — Ω disturbances
$y_{ref} \in R^r$ — r reference inputs

Some Typical Industrial Examples

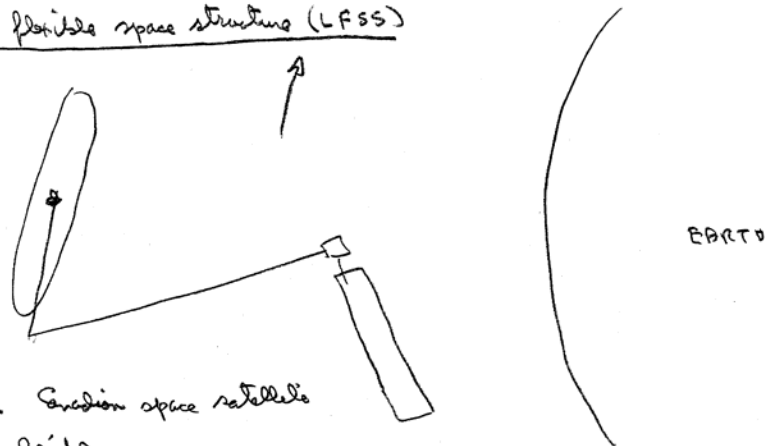
(1) A distillation column



u_1 boiler input
u_2 coolant output
u_3 reflux

y_1 top product composition
y_2 bottom " "
y_3 pressure

(2) A large flexible space structure (LFSS)



MSAT II Canadian space satellite

highly flexible

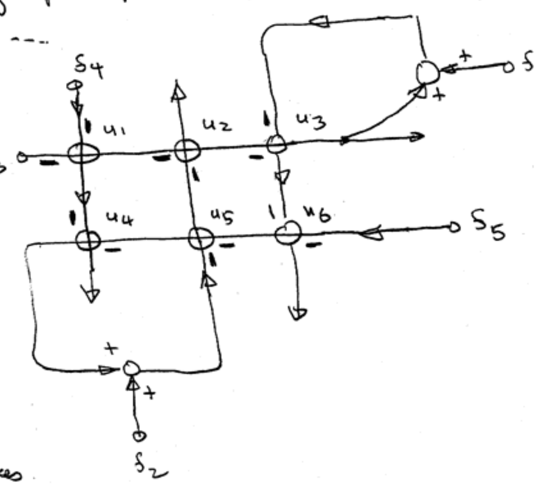
want to bring up position control

attitude "

shape "

system has 11 outputs to be controlled & 9 control inputs with disturbances arising from particle impact, solar flux, non-symmetry of earth ---

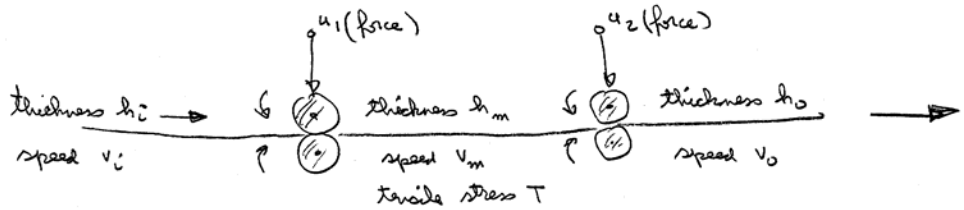
(3) Traffic light control



Want to control queue lengths of travel in NS & EW direction. In this figure there are 6 inputs 12 outputs & 5 disturbances.

In general case, there are 100's of inputs & outputs

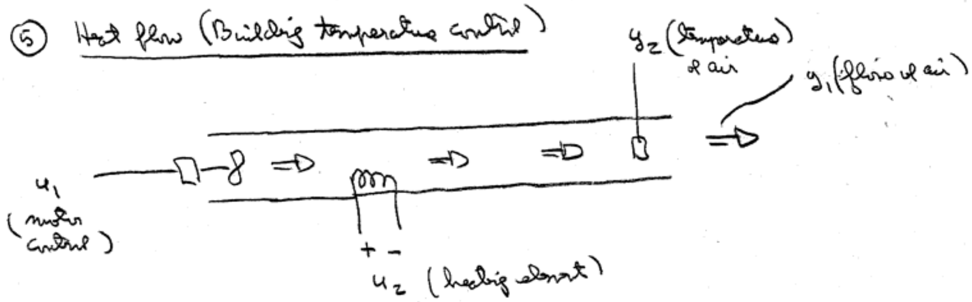
4 Rolling Mill (2 stand)



Inputs $u = \begin{pmatrix} u_1 \\ u_2 \\ v_i \end{pmatrix}$ Outputs $y = \begin{pmatrix} h_o \\ T \\ h_m \end{pmatrix}$

disturbance h_i

5 Heat flow (Building temperature control)



All of these systems have property

- Multivariable systems
- In order to make system work, it is essential to control the system
- It is highly desirable to apply a "good" controller to make system work, so outputs are regulated to desired accuracy & so control inputs are optimized to minimize energy consumption (to minimize operating costs)

⇓

leads to a study of optimal MV controller design

Modelling of Systems

In order to develop a theory of controller design, it is necessary to describe the 'plant' mathematically (called modelling of the system). This can be done by a number of different ways:

- Diff Eq^{n}s [time domain]
- Transfer functions (Laplace, z-transform, Mellin-Transform, ...) [frequency domain]
- (*) State space eq^{n}s [time-domain]
- Algebraic methods rings of matrices ...
- Geometric methods subspaces ...

⋮

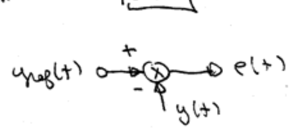
In this course, we shall concentrate on (*) since

- it is valid for both linear time-invariant (LTI) systems & also for linear time-varying (LTV) & nonlinear (NL) systems

For simplicity of presentation, only the case when $y_m \equiv y$ will now be considered

Notation

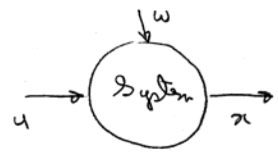
block diagram notation $\xrightarrow{u} \boxed{G(s)} \longrightarrow y$ $\Rightarrow$ $y(s) = G(s)\,u(s)$

summing junction  $\Rightarrow$ $e(t) = y_{ref}(t) - y(t)$

State Space Modelling of Systems

Definition

Given a dynamic system, the <u>state of the system</u> are those variables of the system (called <u>state-variables</u>) which contain sufficient information about the history of the system to allow us to compute the future behaviour of the system given ~~that~~ we know the inputs to the system & the <u>initial state of the system</u>.



; $x(t_0)$ = <u>initial state of system at time $t = t_0$</u>.

Here u (control inputs), w (disturbances) are the inputs to the System & $x \in \mathbb{R}^n$ denote the <u>state of the system</u>

<u>Remark</u> 99.99 % of physical systems can be described by the state space model:

nth order NL differential eq^n

$$\dot{x} = f(x, u, w) \quad ; \; x \in \mathbb{R}^n, \; u \in \mathbb{R}^m, \; y \in \mathbb{R}^r, \; w \in \mathbb{R}^\Omega$$

$$y = g(x, u, w) \quad ; \; x(t_0) \text{ given} \quad ; \; t \in [t_0, \infty)$$

NL algebraic eq^n for y

here $$\dot{x} = \frac{dx}{dt} = \begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \vdots \\ \frac{dx_n}{dt} \end{pmatrix}$$

<u>Remark</u>: The state of a system is <u>not unique</u>!

Existence of a sol^n to this DE is given by:

Theorem

The system $\dot{x} = f(x,u)$, $x(t_0)$ given has a unique solution for $\|x - x(t_0)\| < a$, $t \in [t_0, b]$ for a given input $u(\cdot)$, if

(i) $f(\cdot,\cdot)$ obeys the Lipschitz condition, i.e. if $\exists$ constant scalar k so

$$\|f(x_1,u) - f(x_2,u)\| \le k\|x_1 - x_2\|$$

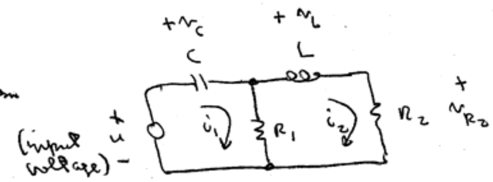
$$\forall\ x_1, x_2 \in \{x \mid \|x - x(t_0)\| < a\},\ \forall\ t \in [t_0, b]$$

if (ii) $f(\cdot,\cdot)$ is bounded, i.e. $\exists$ constant M so

$$\|f(x,u)\| \le M \quad, \forall\ x \in \mathbb{R}^n \ , \forall\ t \in [t_0, b]$$

In practice, these conditions almost always hold $\Rightarrow$ unique sol^n to DE always true in practice.

Example consider system



here

(i_1, i_2) are states of the system

or (v_C, v_L)

or (v_C, v_{R_2})

or (v_{R_1}, v_{R_2})

but (i_2, v_{R_2}) is not a state of the system

<u>Example</u> Given a <u>SISO system</u> $u \to \boxed{S} \to y$ where $u \in \mathbb{R}^1, y \in \mathbb{R}^1$

described by

$$\ddot{z} + 4\dot{z}z + 2z = \cos(t)$$
$$y = z$$

then we can obtain a state space model of the system in the following way

Let $x_1 := z$
$x_2 := \dot{z}$ be the <u>state variables</u> of the system

$\Rightarrow$ $\dot{x}_1 = \dot{z} = x_2$

$\dot{x}_2 = \cos t - 4\dot{x}_1 x_1 - 2x_1$

$= \cos t - 4x_1 x_2 - 2x_1$

$\Rightarrow$ system is described by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -2(1+2x_2) & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \cos(t) \end{pmatrix} = \begin{pmatrix} x_2 \\ -2x_1(1+2x_2) + \cos(t) \end{pmatrix}$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1$$

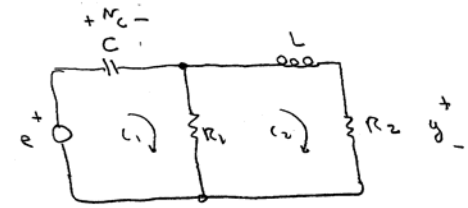
<u>Remark</u> 90% of time, a system can be described by

$$\left.\begin{aligned} \dot{x} &= A(t)x + B(t)u \\ y &= C(t)x + D(t)u \end{aligned}\right\}$$ a <u>linear time-varying (LTV) system</u>

or by

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}\right\}$$ a <u>linear time-invariant (LTI) system</u>

Example



$v_L = L \dfrac{di_L}{dt}$

$i_c = C \dfrac{dv_c}{dt}$

Let $\left.\begin{array}{l} x_1 = i_1 \\ x_2 = i_2 \end{array}\right\}$ be state variables (ad hoc choice)

From circuit analysis $\quad e = \frac{1}{C}\int i_1 d\tau + R_1(i_1 - i_2)$

$$0 = R_1(i_2 - i_1) + L\frac{di_2}{dt} + R_2 i_2$$

$\Rightarrow \quad \dot{e} = \frac{1}{C}\dot{i}_1 + R_1(\dot{i}_1 - \dot{i}_2)$

Let $x_1 = i_1$, $x_2 = i_2$ $\Rightarrow$ get

$$\begin{pmatrix}\dot{x}_1 \\ \dot{x}_2\end{pmatrix} = \begin{bmatrix} \frac{R_1}{L} - \frac{1}{R_1 C} & -\frac{R_1+R_2}{L} \\ \frac{R_1}{L} & -\frac{R_1+R_2}{L}\end{bmatrix}\begin{pmatrix}x_1 \\ x_2\end{pmatrix} + \begin{pmatrix}\frac{1}{R_1} \\ 0\end{pmatrix}\dot{e}$$

note in this case the input which plays a significant role is $\dot{e}$ not e.

$$y = \{0 \quad R_2\}\begin{pmatrix}x_1 \\ x_2\end{pmatrix}$$

where $\begin{pmatrix}x_1(0) \\ x_2(0)\end{pmatrix} = \begin{pmatrix}i_1(0) \\ i_2(0)\end{pmatrix}$ are given at $t = 0$.