TO SIMULATE CLOSED LOOP SYSTEM (METHOD #1)

Plant

$$\dot{x} = Ax + Bu$$
$$y = Cx$$
$$\dot{y} = CAx \qquad (\text{assume } CB = 0)$$

Controller

$$\dot{\eta} = A_c\eta + B_c y + \bar{B}_c \dot{y} + E_c y_{ref}$$
$$u = C_c\eta + D_c y + \bar{D}_c \dot{y} + F_c y_{ref}$$

← This describes almost all possible controllers which can be used. (1)

CLOSED LOOP SYSTEM

$$\boxed{\begin{aligned} \begin{pmatrix}\dot{x}\\ \dot{\eta}\end{pmatrix} &= \begin{pmatrix} A + BD_cC + B\bar{D}_cCA & BC_c \\ B_cC + \bar{B}_cCA & A_c \end{pmatrix}\begin{pmatrix}x\\ \eta\end{pmatrix} + \begin{pmatrix} BF_c \\ E_c\end{pmatrix} y_{ref} \\ y &= \begin{pmatrix} C & 0 \end{pmatrix}\begin{pmatrix}x\\ \eta\end{pmatrix} \\ u &= \begin{pmatrix} D_cC + \bar{D}_cCA & C_c \end{pmatrix}\begin{pmatrix}x\\ \eta\end{pmatrix} + F_c y_{ref} \end{aligned}} \qquad (2)$$

Example • consider $u = k\left(1 + \frac{k_1}{s}\right)(y_{ref} - y)$; this can be described by:

$$\dot{\eta} = 0\eta + (y_{ref} - y)$$
$$u = kk_1\eta + k(y_{ref} - y)$$

• consider $u = k\left(1 + \frac{h_1}{s}\right)(y_{ref} - y) + kh_2\dot{y}$; this can be described by:

$$\dot{\eta} = 0\eta + (y_{ref} - y)$$
$$u = kh_1\eta + k(y_{ref} - y) + kh_2\dot{y}$$

both controllers of which are described by (*)

NOTATION Let (2) be described by

$$\dot{\tilde{x}} = \mathcal{A}\tilde{x} + \mathcal{B}y_{ref}$$
$$y = \mathcal{C}\tilde{x} + \mathcal{D}y_{ref}$$
$$u = \mathcal{C}_u\tilde{x} + \mathcal{D}_u y_{ref}$$

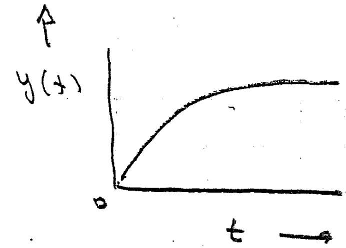
SIMULATION OF CLOSED LOOP SYSTEM

Given (2), a unit step response in y_{ref} can be simulated by

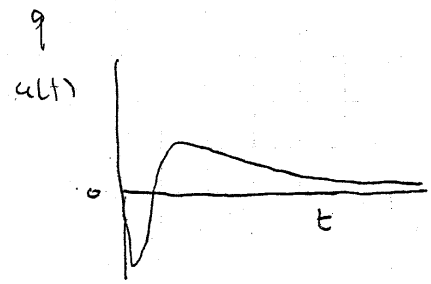
$$[y, x, t] = \text{step}(A, B, C, D, 1) \;;\; [u, \bar{x}, \bar{t}] = \text{step}(A, B, C_u, D_u, 1)$$

which can be plotted using:

plot(t, y) $\Rightarrow$



plot($\bar{t}$, u) $\Rightarrow$



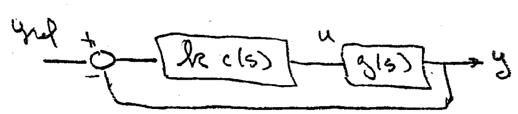
<u>NOTE</u>: THE DATE FOR SUBMISSION OF ASSIGNMENT #3 WAS ORIGINALLY FRI, MARCH 30, 2001 $\Rightarrow$ <u>THE NEW DATE OF SUBMISSION IS WED APRIL 4, 2001, NOON HOUR</u> IN THE BOX OPPOSITE ROOM GB 344.

TO SIMULATE CLOSED LOOP SYSTEM (METHOD #2)

Given plant $g(s)$, assume a controller $u = k\,c(s)(y_{ref} - y)$ has been found to satisfy requirements 1), 2), 3) using Root Locus Diagrams.

Let $kg(s) = \dfrac{n(s)}{d(s)}$ & $c(s) = \dfrac{\bar{n}(s)}{\bar{d}(s)}$.

To simulate $y(t)$



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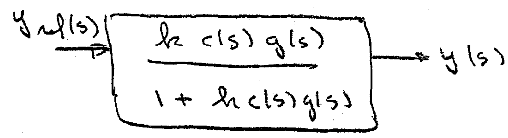

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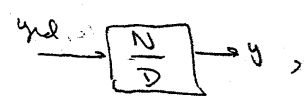
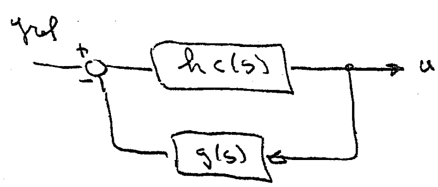


, $N = n(s)\bar{n}(s)$

$D = d(s)\bar{d}(s) + n(s)\bar{n}(s)$

& $[y, x, t] = \text{step}(\tilde{N}, \tilde{D})$ (see matlab for details)

To simulate $u(t)$



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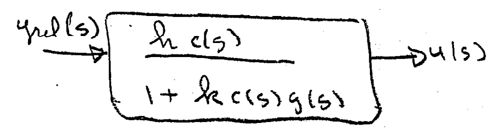

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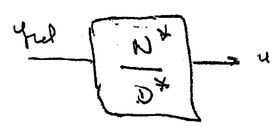


, $N^* = \bar{n}(s)$

$D^* = d(s)\bar{d}(s) + n(s)\bar{n}(s)$

&

$[u, \bar{x}, \bar{t}] = \text{step}(\tilde{N}^*, \tilde{D}^*)$ (see matlab for details)

NOTE: Here $\tilde{N}$ is a row vector containing polynomial coefficients of $N(s)$

" $\tilde{D}$ " " " " " $D(s)$

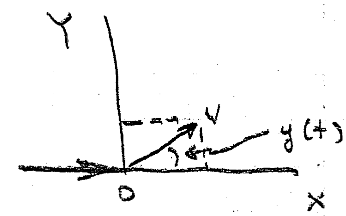
" $\tilde{N}^*$ " " " " " $N^*(s)$

" $\tilde{D}^*$ " " " " " $D^*(s)$

PLOTTING OF SHIP'S TRAJECTORY

Given the output simulation of the ship's heading angle $y(t)$ obtained above: $y(t) = \begin{pmatrix} y(t_1) \\ y(t_2) \\ y(t_3) \\ \vdots \end{pmatrix}$ for $t = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ \vdots \end{pmatrix}$, then the trajectory of the ship can be obtained by noting that:

$$\dot{X} = V \cos(y(t)) \;;\quad x(0) = 0 \;;\quad t > 0 \qquad (3)$$
$$\dot{Y} = V \sin(y(t)) \qquad Y(0) = 0$$



(ignoring v) & this can be integrated by using the simple Euler Procedure

given $\dot{x} = f(x,t) \Rightarrow \dot{x} \doteq \frac{x_{t+h} - x_t}{h} \Rightarrow \frac{x_{t+h} - x_t}{h} = f(x_t, t)$

$$\Downarrow$$

$$x_{t+h} = x_t + h f(x_t, t) \;;\; t = 0, t_1, t_2, t_3 \ldots \text{ (} h \text{ is a "small" number)}$$

$\Rightarrow$ (3) can be written as

$$X_{t+h} = X_t + h \cos[y(t)] \;;\quad X_0 = 0 \;;\quad t = 0, t_1, t_2, t_3 \ldots$$
$$Y_{t+h} = Y_t + h \sin[y(t)] \qquad Y_0 = 0 \qquad h = \text{"small" number}$$

from which the trajectory of the ship can be obtained on letting

$$X = \begin{pmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \\ \vdots \end{pmatrix}, \; Y = \begin{pmatrix} Y_0 \\ Y_1 \\ Y_2 \\ Y_3 \\ \vdots \end{pmatrix} \Rightarrow \text{plot}(X, Y)$$