3.40 Satellite Tracking Antenna

![Diagram of satellite tracking antenna]

a) Find the transfer function

\[ J\ddot{\theta} + B\dot{\theta} = T_c \]
\[ Js^2\dot{\theta} + Bs\dot{\theta} = \dot{T}_c \]
\[ \hat{P}(s) = \frac{\dot{\theta}}{T_c} = \frac{1}{s^2J + sB} \]

b) Find the closed loop transfer function

\[ \dot{\theta} = (\dot{\theta}_r - \dot{\theta})K\hat{P}(s) \]
\[ \frac{\dot{\theta}}{\dot{\theta}_r} = \frac{K\hat{P}(s)}{1 + K\hat{P}(s)} \]
\[ \hat{H}(s) = \frac{K}{s^2J + sB} \]
\[ \hat{H}(s) = \frac{1}{1 + \frac{s^2J + sB}{K/J}} \]
\[ \hat{H}(s) = \frac{K/J}{s^2 + (B/J)s + K/J} \]

c) Find K for the \( M_p < 0.1 \)

\[ M_p < e^{-\frac{\pi^2\zeta^2}{\sqrt{1-\zeta^2}}} \]
\[ (\ln 0.1)^2 < \frac{\pi^2\zeta^2}{1-\zeta^2} \]
\[ 5.30 < (\pi^2 + 5.30)\zeta^2 \]
\[ \zeta > 0.591 \]
\[ \frac{B}{J} = 2\zeta\omega_n \]
\[ \sqrt{K} = \frac{B}{2\zeta J} \]
\[ K < 477 \]

d) Find K for \( t_r < 80s \)

\[ t_r = \frac{1.8}{\omega_n} \]
\[ \frac{1.8}{\sqrt{K/J}} < 80s \]
\[ K > 304 \]

e) Use matlab to verify

Here is a very simple bit of matlab code that will show you the step response for the system.

```matlab
K= 477;
J= 600000;
B= 20000;
num= [0 0 K/J];
den= [1 B/J K/J];
[Y,X,T]=STEP(num,den);
plot(T,Y)
```
4.43 Use Routh’s Stability Criterion on the following system

\[ y = KG(s)(u - y) \]

\[ KG(s)u = (1 + KG(s))y \]

\[ H(s) = \frac{y}{u} = \frac{KG(s)}{1 + KG(s)} \]

f) For the open loop system:

\[ KG(s) = \frac{4(s + 2)}{s(s^3 + 2s^2 + 3s + 4)} \]

Find the closed loop transfer function

\[ H(s) = \frac{4(s + 2)}{s(s^3 + 2s^2 + 3s + 4)} + \frac{4(s + 2)}{1 + s(s^3 + 2s^2 + 3s + 4)} \]

\[ = \frac{4(s + 2)}{s} \]

\[ = \frac{4s + 8}{s^4 + 2s^3 + 3s^2 + 8s + 8} \]

The characteristic equation is

\[ s^4 + 2s^3 + 3s^2 + 8s + 8 = 0 \]

Note that all of the \( a_i \)'s are positive non-zero
we now form the Routh table

| \( s^5 \) | 1 | 3 | 8 | 0 |
| \( s^4 \) | 2 | 8 | \( -1 \times 8 - (3 \times 2) \) | 2 |
| \( s^3 \) | -1 | \( -1 \times 0 - (8 \times 2) \) | 0 |
| \( s^2 \) | 24 | \( -1 \times 0 - (8 \times 24) \) | 24 |
| \( s^1 \) | 8 | \( -1 \times 0 - (8 \times 24) \) | 24 |

The sign changes from + to – and back to +
so there are 2 roots in the RHP.

note: from matlab the roots are

\[ \{0.4711 + 1.7994i, 0.4711 - 1.7994i, -1.4711 + 0.3851i, -1.4711 - 0.3851i\} \]

g) For the open loop system

\[ KG(s) = \frac{2(s + 4)}{s^2(s + 1)} \]

Find the closed loop transfer function

\[ H(s) = \frac{2(s + 4)}{s^2(s + 1) + 2(s + 4)} \]

\[ = \frac{2s + 8}{s^3 + s^2 + 2s + 8} \]

The characteristic equation is

\[ s^3 + s^2 + 2s + 8 = 0 \]

Note that all of the \( a_i \)'s are positive non-zero
we now form the Routh table

| \( s^3 \) | 1 | 2 |
| \( s^2 \) | 1 | 8 |
| \( s^1 \) | -6 | \( 1 \times 0 - (2 \times 1) \) |
| \( s^0 \) | 8 | \( 1 \times 0 - (8 \times 6) \) |

The sign changes from + to – and back to +
so there are 2 roots in the RHP.

note: from matlab the roots are

\[ \{-2.0000, 0.5000 + 1.9365i, 0.5000 - 1.9365i\} \]
4.43 Use Routh’s Stability Criterion on the following system (cont.)

\[ y = KG(s)(u - y) \]

\[ KG(s)u = (1 + KG(s))y \]

\[ H(s) = \frac{y}{u} = \frac{KG(s)}{1 + KG(s)} \]

h) For the open loop system:

\[ KG(s) = \frac{4(s^3 + 2s^2 + s + 1)}{s^2(s^3 + 2s^2 - s - 1)} \]

Find the closed loop transfer function

\[ H(s) = \frac{4(s^3 + 2s^2 + s + 1)}{s^2(3s^3 + 2s^2 - s - 1) + 4(s^3 + 2s^2 + s + 1)} = \frac{4s^3 + 8s^2 + 4s + 4}{s^5 + 2s^4 + 3s^3 + 3s^2 + 4s + 4} \]

The characteristic equation is

\[ s^5 + 2s^4 + 3s^3 + 3s^2 + 4s + 4 = 0 \]

Note that all of the \(a_i\)’s are positive non-zero
we now form the Routh table

\[
\begin{array}{ccc}
1 & 3 & 4 \\
\frac{1}{2} & (2) & (2) \\
\frac{1}{3} & (3) & (3) \\
\frac{2}{3} & \frac{1}{3} & 0 \\
-16 & -16 & 0 \\
\end{array}
\]

The sign changes from + to – and back to + so there are 2 roots in the RHP.

Note: from matlab the roots are

\[-0.8780 + 1.1968i\]
\[-0.8780 - 1.1968i\]
\[0.4776 + 1.1340i\]
\[0.4776 - 1.1340i\]
\[-1.1992\]
4.50 Routh Stability criterion with varying parameters

Given the characteristic equation for a closed loop system.

\[ s^4 + (11 + K_2) s^3 + (121 + K_1) s^2 + (K_1 + K_2 + 110K_2 + 210)s + 11K_1 + 100 = 0 \]

Constraints on K1 and K2 which guarantee a stable system can be found as follows.

1. All of the coefficients of powers of s must be strictly greater than zero.

   \[
   K_2 > -11 \\
   K_1 > -\frac{100}{11} \\
   K_1 + K_2 + 110K_2 + 210 > 0
   \]

2. Also, all of the first column entries of the Routh table must be strictly greater than zero.

   \[
   \begin{align*}
   s^4 & : 1 & a_2 = 121 + K_1 & a_4 = 11K_1 + 100 \\
   s^3 & : a_1 = 11 + K_2 & a_3 = K_1 + K_2 + 110K_2 + 210 & a_2 = 0 \\
   s^2 & : b_1 = \frac{10K_1 + 11K_2 + 1121}{11 + K_2} & b_2 = 11K_1 + 100 & b_3 = 0 \\
   s^1 & : c_1 = \frac{b_1a_3 - a_1b_2}{b_1} & c_2 = 0 \\
   s^0 & : d_1 = b_2
   \end{align*}
   \]

These constraints are very complicated and it is necessary to use a numerical approach to find the solution. On the course web page you can find a link to some matlab code that generates the region of stability.