Feasibility for Formation Stabilization of Multiple Unicycles

Zhiyun Lin, Bruce Francis, and Manfredi Maggiore
Department of Electrical and Computer Engineering
University of Toronto
Toronto, Canada, M5S3G4
linz, francis, maggiore@control.utoronto.ca

***Abstract*— The feasibility problem is studied of achieving a specified formation among a group of autonomous unicycles by local distributed control. The directed graph defined by the information flow plays a key role. Necessary and sufficient conditions are presented for formation stabilization of multiple unicycles to a point and a line. A similar result is given for formation stabilization to more general geometric arrangements with the condition that a group of unicycles have a common sense of direction.**

I. Introduction

The problem of coordinated control of a group of autonomous wheeled vehicles is of recent interest in control and robotics due to the broad range of applications of multi-vehicle systems in space missions, operations in hazardous environments, and military missions. Distributed vehicles can readily exhibit the attractive characteristics of structural flexibility, reliability through redundancy, and simple hardware as compared to sophisticated individual vehicles.

Despite the many advantages inherent in distributed multi-vehicle systems, there are challenges in coordination and control due to the absence of a centralized supervisor and global information. Individual vehicles in a distributed system must be capable of collectively accomplishing tasks using only locally sensed information and little or no direct communication. Coordinated control of multi-vehicle systems includes many aspects, one of the most important and fundamental being formation control.

Over the past decade, many researchers have worked on formation control problems with differences regarding the types of agent dynamics, the varieties of the control strategies, and the types of tasks demanded. In 1990, Sugihara and Suzuki [26] proposed a simple algorithm for a group of point-mass type robots to form approximations to circles and simple polygons. And in the years following, distributed algorithms were presented in [3], [2], and [27] with the objective of getting a group of such robots to congregate at a common location. Moving synchronously in discrete-time steps, the robots iteratively observe neighbors within some visibility range and follow simple rules to update their positions. Lately in [16], both synchronous and asynchronous maneuvering strategies are described for the multi-agent rendezvous problem.

Convergence to a common point is an example of an *agreement problem*: Agents initially without a common reference frame eventually come to agree upon a reference point by virtue of having arrived at one. Besides being of relevance to the rendezvous problem, convergence to a common point is also of interest because if it is feasible, then so is convergence to other formations, as shown for example in [17]. In [11], Jadbabaie et al. studied a different agreement problem in discrete-time: Getting autonomous agents in the plane to move in a common direction. Within a discrete-time setting, Moreau in [22] generalized and extended the work in [11] by presenting necessary and sufficient conditions for the convergence of the individual agents' states to a common value.

In addition to the references mentioned so far, there have been many results on the mathematical analysis of formation control. In [23], formation stabilization of a group of agents with linear dynamics (double-integrator) is studied using structural potential functions. An alternative is to use artificial potential functions and virtual leaders as in [15]. The approach known as leader-following has been used in maintaining a desired formation while moving, e.g., [6], [7]. In [19], stability of asynchronous swarms with a fixed communication topology is studied, where stability is used to characterize the cohesiveness of a swarm. In [28] and [29], Tanner et al. investigate the stable flocking of mobile agents for fixed topology and dynamic topology, respectively. Similarly to [11], Lin et al. [17] focus on the problem of achieving a specified formation among a group of mobile autonomous agents by distributed control when the communication topology is dynamic because agents come into and go out of sensor range. In [12], Justh and Krishnaprasad study achievable equilibrium formations of unicycles each moving at unit speed and subject to steering control and presents stabilizing control laws, wherein each unicycle senses all others. And in [20], a circular formation is achieved for a group of unicycles using the strategy of cyclic pursuit.

Motivated by the fact that no continuous time-invariant feedback control law can stabilize a nonholonomic system to a point, in 1991 Samson and Ait-Abderrahim [25] showed that smooth time-varying feedback can stabilize a unicycle. Since then, much work (for instance, [21], [24]) has focused on smooth time-varying feedback control of nonholonomic systems, and averaging theory has been used to study stability [21] and also motion planning [10]. On the question of formations, in [31], [32] a time-varying feedback control law is proposed for multiple Hilare-type mobile robots and averaging theory is used to analyze stability.

As a natural extension of our previous work [20], [17], and motivated by the proposed strategy in [32], [31], in this paper the feasibility problem is studied of achieving a specified formation among a group of unicycles by distributed control. Each unicycle is equipped with an onboard sensor, by which it can measure relative displacements to certain neighbors; in particular, we do not assume that the unicycles possess a common reference frame.

Central to a discussion of formation control is the nature of the information flow throughout the formation. This information flow can be modeled by a *sensor digraph* (directed graph), where a link from node i to node j indicates that vehicle i can sense the position of vehicle j—but only with respect to the local coordinate frame of vehicle i. In this paper, we assume the sensor digraph is static—the dynamic case, where ad hoc links can be established or dropped, is harder. Our analysis relies on several tools from algebraic graph theory [9], non-negative matrix theory [5], [14], and averaging theory [1], [13], [21].

Our first main result is that formation stabilization to a common point is feasible if and only if the sensor digraph has a globally reachable node (a node to which there is a directed path from every other node). That is, there exists at least one unicycle that is viewable, perhaps indirectly by hopping from one unicycle to another, by all other unicycles. This is precisely the degree of connectedness required and is much weaker than strong connectedness of the sensor digraph (as in cyclic pursuit [20], for example). Our second main result concerns formation stabilization to a line. This turns out to be feasible if and only if there are at most two disjoint closed sets of nodes in the sensor digraph. In addition, we introduce a special sensor digraph which guarantees that all vehicles converge to a line segment, equally spaced. This is an extension to unicycles of a line-formation scheme of Wagner and Bruckstein [30]. Finally, we show how formation stabilization to a common point can be adapted to any geometric pattern under the assumption that a group of vehicles have a common sense of direction.

The proofs of the main results appear in [18] and hence are omitted.

II. Problem Statement and Main Results

In this section, we introduce the problems addressed in this paper and then present three of our main results: point formation, line formation, and any geometric pattern formation.

A. Problem Description

Before treating unicycles, it is perhaps illuminating to give a result for the much simpler case of point masses. Consider n "point-mass robots" whose positions are modeled by complex numbers, $z_1, \dots, z_n$, in the plane. Assume a kinematic model of velocity control: $\dot{z}_i = u_i$. Assume each robot senses the relative positions of a subgroup, N_i, of the other robots. Let y_i denote the vector whose components are the relative positions $z_m - z_i$, as m ranges over N_i. Thus y_i, a vector of dimension the cardinality of N_i, represents the information available to u_i. We allow controllers of the form $u_i = F_i y_i$, or $u_i = 0$ if N_i is empty. Thus, $y_i = 0 \implies u_i = 0 \implies \dot{z}_i = 0$; that is, robot i does not move if all robots it senses are collocated with it (or if it does not sense any other robot). The problem of convergence to a common point is this:

Problem 1: Find, if possible, $F_1, \dots, F_n$ such that

$$(\forall\ i \in \{1, \dots, n\})(\forall\ z_i(0))(\exists\ z_{ss})\ \ \lim_{t\to\infty} z_i(t) = z_{ss}.$$

Now define the sensor graph $\mathcal{G}$ for this setup: There is a directed edge from node i to node m if and only if $m \in N_i$.

Before giving our results, we review some notions in graph theory. For a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, if there is a path in $\mathcal{G}$ from one node v_i to another node v_j, then v_j is said to be *reachable* from v_i, written $v_i \to v_j$. If not, then v_j is said to be not reachable from v_i, written $v_i \nrightarrow v_j$. If a node v_i is reachable from every other node in the digraph, then we say it is *globally reachable*. If $\mathcal{U}$ is a nonempty subset of $\mathcal{V}$ and $u \nrightarrow v$ for all $u \in \mathcal{U}$ and $v \in \mathcal{V} - \mathcal{U}$, then $\mathcal{U}$ is said to be *closed*. More information can be found in [4], [8].

Theorem 1: 1) *Problem 1 is solvable if and only if $\mathcal{G}$ has a globally reachable node.*
2) *When Problem 1 is solvable, one solution is*

$$F_i = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}.$$

Now we turn to the main topic of unicycles. We can identify the real plane, $\mathbb{R}^2$, and the complex plane, $\mathbb{C}$, by identifying a column vector, z_i, and a complex number, $\mathbf{z}_i$. Now consider a wheeled vehicle with coordinates (x_i, y_i, θ_i) with respect to a global frame ${}^g\Sigma$ (see Fig. 1).

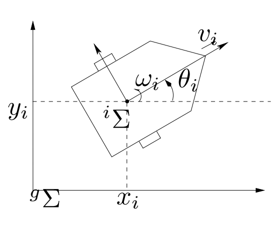


Fig. 1. Wheeled vehicle.

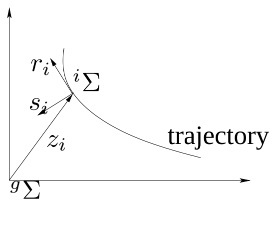


Fig. 2. Frenet-Serret frame.

The location of the vehicle in the plane is

$$z_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix} \quad \text{or} \quad \mathbf{z}_i = x_i + j y_i.$$

The vehicle has the nonholonomic constraint of pure rolling and non-slipping and is described kinematically as

$$\begin{cases} \dot{x}_i = v_i \cos(\theta_i), \\ \dot{y}_i = v_i \sin(\theta_i), \\ \dot{\theta}_i = \omega_i, \end{cases} \quad \text{or} \quad \begin{cases} \dot{\mathbf{z}}_i = v_i e^{j\theta_i}, \\ \dot{\theta}_i = \omega_i. \end{cases} \tag{1}$$

Following [12], we construct a moving frame ${}^i\Sigma$, the Frenet-Serret frame, that is fixed on the vehicle (see Fig. 2). Let r_i be the unit vector tangent to the trajectory at the current location of the vehicle (r_i is the normalized velocity vector) and let s_i be r_i rotated by $\pi/2$. Since the vehicle is moving at speed v_i, $v_i r_i = \dot{z}_i$, and so in complex form

$$\mathbf{r}_i = e^{j\theta_i}, \quad \mathbf{s}_i = j\mathbf{r}_i.$$

Thus

$$\dot{\mathbf{r}}_i = \frac{d}{dt}\left(e^{j\theta_i}\right) = je^{j\theta_i}\dot{\theta}_i = \mathbf{s}_i\omega_i, \quad \dot{\mathbf{s}}_i = j\dot{\mathbf{r}}_i = -\mathbf{r}_i\omega_i.$$

The kinematic equations using the Frenet-Serret frame are

$$\begin{cases} \dot{z}_i = v_i r_i, \\ \dot{r}_i = \omega_i s_i, \\ \dot{s}_i = -\omega_i r_i. \end{cases} \tag{2}$$

Now consider n wheeled vehicles, indexed by i. We refer to the individual vehicles as nodes and the information flows as links. Although the vehicles in the group are dynamically decoupled, meaning the motion of one vehicle does not directly affect any of the other vehicles, they are coupled through the sensor information flow. A natural way to model the interconnection topology is a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with $\mathcal{V} = \{1, 2 \ldots, n\}$ in which the vehicles are the nodes and the links are directed edges. The directed edge from node i to node m is one of the graph's edges just in case vehicle i can sense vehicle m. (Thus, information flows in the direction opposite to the orientation of the edges.) We refer to this as a *sensor digraph*.

Let N_i denote the set of labels of those vehicles sensed by vehicle i. In this paper, we assume N_i is time-invariant, meaning the information flow topology is static. In the control law that we study, no vehicle can access the absolute positions of other vehicles or its own. Specifically, vehicle i can measure only the relative positions of sensed vehicles with respect to its own Frenet-Serret frame (see Fig. 3),

$$\begin{cases} x_{im} = (z_m - z_i) \cdot r_i, \\ y_{im} = (z_m - z_i) \cdot s_i, \end{cases} \quad m \in N_i, \tag{3}$$

where dot denotes dot product. This leads to the following definition.

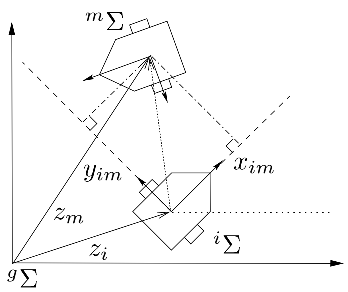


Fig. 3. Local information.

Definition 1: A controller (v_i, w_i), $i = 1, \ldots n$, is said to be a *local information controller* if

$$\begin{cases} v_i &= g_i(t, x_{im}, y_{im})|_{m \in N_i}, \\ \omega_i &= h_i(t, x_{im}, y_{im})|_{m \in N_i} \end{cases} \quad i = 1, \ldots, n$$

where g_i, h_i are smooth functions of their arguments, and g_i is such that $\{(\forall\ m \in N_i)\ z_m = z_i\} \Rightarrow \{v_i = 0\}$.

Notice that in our definition a vehicle does not translate (but it can rotate) when either it cannot sense any other vehicle or its neighbors have all converged to its position.

In what follows, we present two main problems investigated in this paper, together with necessary and sufficient conditions for their solution. In Section II-C we also introduce a special sensor digraph which guarantees that all vehicles converge to a line segment, equally spaced. In Section II-D we show how our solution to Problem 2 can be employed to achieve formation stabilization to any geometric pattern.

B. Formation Stabilization to a Point

Problem 2: (Formation Stabilization to a Point) *Find, if possible, a local information controller such that for all* $(x_i(t_0), y_i(t_0), \theta_i(t_0)) \in \mathbb{R}^3$, $i = 1, \ldots n$, *and all* $t_0 \in \mathbb{R}$, *there exists* $z_{ss} \in \mathbb{R}^2$ *such that* $\lim_{t\to\infty} z_i(t) = z_{ss}$ *for all* i.

Theorem 2: Problem 2 is solvable if and only if the sensor digraph has a globally reachable node.

The above result shows that, if and only if a certain graphical condition holds, all the wheeled vehicles globally asymptotically converge to a point formation (or we say they achieve an agreement about a common point) through simple local actions by proper choice of controller. One possible time-varying feedback controller to solve this problem is given by

$$\begin{cases} v_i(t) = k \sum\limits_{m \in N_i} x_{im}(t), \\ \omega_i(t) = \cos(t), \end{cases} \quad i = 1, 2, \ldots, n, \tag{4}$$

where $k > 0$ is small enough.

An alternative choice of controller is

$$\begin{cases} v_i(t) = \sum\limits_{m \in N_i} x_{im}(t), \\ \omega_i(t) = \gamma\cos(\gamma t), \end{cases} \quad i = 1, 2, \ldots, n. \tag{5}$$

By applying a time scaling, $\tau = \frac{t}{\gamma}$, one can use Theorem 2 and conclude that if the sensor digraph has a globally reachable node, there exists a positive constant γ^* such that, for all $\gamma^* < \gamma < \infty$, (5) achieves formation stabilization to a point.

Notice that the sensor digraph is required to have just one globally reachable node. So if we treat a beacon placed at the proper location as one member of the group of vehicles and it is the globally reachable node that is viewable, perhaps indirectly by hopping from one vehicle to another, by all other vehicles, the local actions of each individual vehicle result in the group gathering at the beacon.

Fig. 4 depicts a simulation result for formation stabilization to a point of ten wheeled vehicles using the smooth time-varying feedback control law (4) with the choice of $k = 1$. The initial conditions are randomly produced and the sensor digraph is given in Fig. 5. For example, node 5 is globally reachable, while node 1 is not.

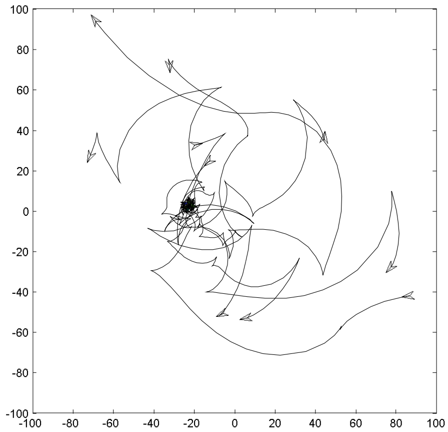


Fig. 4. Ten wheeled vehicles gather at a common position.

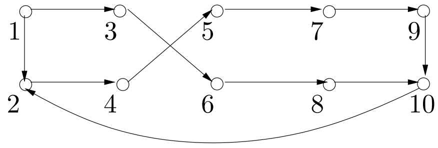


Fig. 5. The sensor digraph of a group of ten wheeled vehicles.

C. *Formation Stabilization to a Line*

Problem 3: (Formation Stabilization to a Line) *Find, if possible, a local information controller such that for all* $(x_i(t_0), y_i(t_0), \theta_i(t_0)) \in \mathbb{R}^3$*,* $i = 1, \dots n$*, and all* $t_0 \in \mathbb{R}$*, all vehicles converge to form a line.*

Theorem 3: Problem 3 is solvable if and only if there are at most two disjoint closed sets of nodes in the sensor digraph.

Theorem 3 has an interesting special case when the two disjoint closed sets of nodes in the sensor digraph both have only one member, say nodes 1 and n. Vehicles 1 and n are called *edge leaders*. The edge leaders here are not necessarily wheeled vehicles. They can be virtual beacons or landmarks. But the vehicles respond to these edge leaders much like they respond to real neighbor vehicles. The purpose of the edge leaders is to introduce the mission: to direct the vehicle group behavior. We emphasize that the edge leaders are not central coordinators. They do not broadcast instructions. They only play the role of individual vehicles, but cannot sense other vehicles or communicate with them. As for the remaining vehicles, i, $i = 2, \dots, n-1$, we assume that each agent can sense agents $i-1$ and $i+1$. This gives the sensor digraph in Fig. 6. It is readily seen that the

Fig. 6. The sensor digraph for a group of wheeled vehicles with two edge leaders.

digraph in Fig. 6 has exactly 2 disjoint closed sets of nodes. We now state that in this special case all vehicles converge to a uniform distribution on the line segment specified by the two edge leaders.

Theorem 4: Consider a group of n *wheeled vehicles with two stationary edge leaders labeled* 1 *and* n*. Then, there exists a positive constant* k^* *such that for all* $0 < k < k^*$*, the following smooth time-varying feedback control law*

$$\begin{cases} v_i(t) = k \sum_{j \in N_i} x_{ij}(t), & N_i = \{i-1, i+1\}, \\ \omega_i(t) = \cos(t), & i = 2, \dots, n-1 \end{cases} \tag{6}$$

guarantees that all the vehicles converge to a uniform distribution on the line segment specified by the two edge leaders.

Fig. 7 shows a simulation result of four wheeled vehicles and two stationary edge leaders to form a uniform distribution on a line using the control law (6) with the choice of $k = 1$.

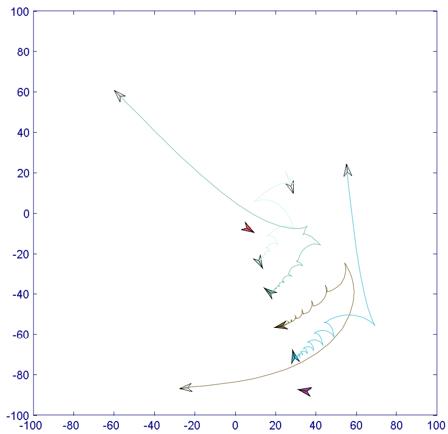


Fig. 7. Formation stabilization to a line of four wheeled vehicles and two edge leaders.

D. *Formation Stabilization to Any Geometric Pattern*

In this section, we turn our attention to the problem of formation stabilization to any geometric pattern. Following [27], we let Π be a predicate describing a geometric pattern, such as a point, a regular polygon, a line segment, etc. Such a predicate specifies a formation up to translation and rotation. By *formation stabilization of a group of* n *vehicles to* Π, we mean that the vehicles (globally exponentially) converge to a distribution satisfying Π.

We suppose that a group of wheeled vehicles have a *common sense of direction*, represented by the angle ψ in Fig. 8 . For instance, each vehicle carries a navigation device such as a compass. Alternatively, all vehicles initially agree on their orientation and use it as the common direction. The common direction may not coincide with the positive x-axis of the global frame. Let $\phi_i = \theta_i - \psi$ (see Fig. 8). We assume that vehicle i can measure its own ϕ_i.

It is worth noting that, for all $(R, b) \in SE(2)$, the vectors $\hat{c}_i = Rc_i + b$ describe the same geometric formation as the

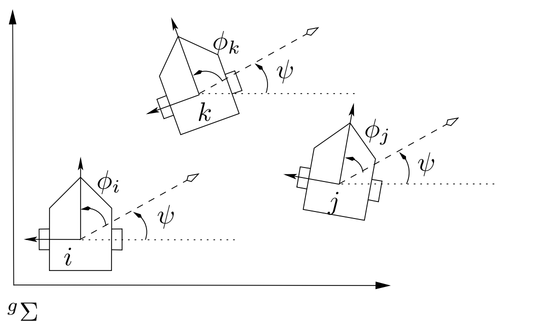
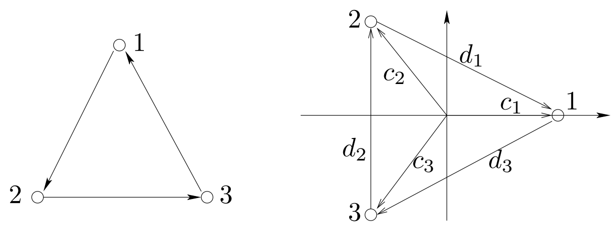


Fig. 8. A group of wheeled vehicles have a common sense of direction.

one specified by c_i. So given a desired geometric formation pictured by $c_i,\ i = 1, \ldots, n$, our objective is to stabilize the position state z_i of each vehicle to $\hat{c}_i = Rc_i + b,\ i = 1, \ldots, n$ for some R and b. To achieve a desired geometric formation characterized by $c = [c_1^T \ \cdots \ c_n^T]^T$, we can simply translate the formation vector c into a control offset $d = L_{(2)}c$ so that the forward control velocity is 0 when the group of vehicles has achieved a formation. We denote the offset for each vehicle by

$$d_i = \begin{bmatrix} d_{x_i} \\ d_{y_i} \end{bmatrix} \quad \text{or} \quad \mathbf{d}_i = d_{x_i} + j d_{y_i}.$$

As an example, consider the simple sensor digraph shown in Fig. 9(a) and a triangle formation described by $c_i,\ i = 1, 2, 3$. The corresponding control offsets $d_i,\ i = 1, 2, 3$ are shown in Fig. 9(b).



(a) Sensor digraph (b) Illustration for c_i and d_i

Fig. 9. An example of a simple triangle formation.

Next, we show that the time-varying control law for each unicycle $i \in \{1, \ldots, n\}$

$$\begin{cases} v_i(t) = k \left\{ \begin{bmatrix} 1 & 0 \end{bmatrix} R(-\phi_i(t)) d_i + \sum_{m \in N_i} x_{im}(t) \right\}, \\ \omega_i(t) = \cos(t), \end{cases} \tag{7}$$

where R is a rotation matrix defined by

$$R(\phi) = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix},$$

achieves formation stabilization to Π.

Theorem 5: Let Π be a desired geometric formation described by c. Suppose a group of n wheeled vehicles have a common sense of direction and formation stabilization to a point is feasible. Then there exists a positive constant k^* such that for all $0 < k < k^*$, the smooth time-varying feedback control law (7) with $d = L_{(2)}c$ guarantees global exponential formation stabilization to Π.

As an example, a simulation result for a circle formation of ten wheeled vehicles with the sensor digraph in Fig. 5 is shown in Fig. 10. The circle formation is described by $\mathbf{c}_i = 75e^{(j\frac{2(i-1)\pi}{10})}, i = 1, \ldots, 10$. The initial conditions are

$$\begin{aligned} x(0) &= [10, 10, 10, 0, 0, 0, 0, -10, -10, -10]^T, \\ y(0) &= [-5, 0, 5, 10, 3, -3, -10, -5, 0, 5]^T, \\ \theta(0) &= \left[\tfrac{\pi}{6}, \tfrac{\pi}{6}, \tfrac{\pi}{6}, \tfrac{\pi}{6}, \tfrac{\pi}{6}, \tfrac{\pi}{6}, \tfrac{\pi}{6}, \tfrac{\pi}{6}, \tfrac{\pi}{6}, \tfrac{\pi}{6}\right]^T, \end{aligned}$$

and the control law (7) is used with the parameters $k = 1$, $\mathbf{d}_1 = 61.74 - j107.72$, $\mathbf{d}_2 = 78.26 - j25.43$, $\mathbf{d}_3 = 91.63 + j66.57$, $\mathbf{d}_4 = 35 + j25.43$, $\mathbf{d}_5 = j82.29$, $\mathbf{d}_6 = -48.37 + j66.57$, $\mathbf{d}_7 = -78.26 + j25.43$, $\mathbf{d}_8 = -78.26 - j25.43$, $\mathbf{d}_9 = -35 - j25.43$, $\mathbf{d}_{10} = -j82.29$.

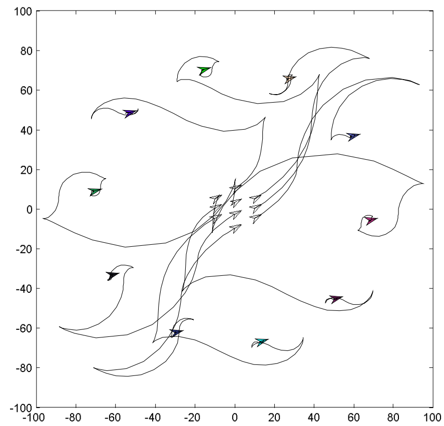


Fig. 10. Ten wheeled vehicles form a circle formation.

III. Conclusions

In this paper, the feasibility problem of achieving a specified geometric formation of a group of unicycles was investigated. Necessary and sufficient graphical conditions for the existence of local information controller to assure the asymptotic convergence of the closed system were derived. Further research issues include developing more general results for the dynamic sensor graph case, where ad hoc links can be established and dropped.

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